Optimal Sentences to Induce Prisoners to Behave Well

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Abstract: The focus of this article is on controlling the behavior of prisoners who, in the model studied, can choose whether to behave well or badly. Good behavior, which requires some effort by prisoners, reduces the state’s cost of operating prisons. Ideally, prisoners for whom the required effort is sufficiently low will behave well. I derive the optimal sentence for the initial offense and the optimal sentence reduction if a prisoner behaves well, taking into account that the decision to commit an offense depends on the individual’s behavior in prison if he is caught. The main result is that the optimal sentence reduction for good behavior in prison is always positive. This is true even though inducing prisoners to behave well leads to their bearing a shorter sentence and lessens the deterrence of crime. I also show that when the upper bound on sentence length is binding, the optimal sentence reduction for good behavior in prison decreases with the severity of the harm from the offense and goes to zero as the level of harm goes to infinity. This result reflects that the state’s savings in operating prisons becomes less important relative to the deterrence of crime as the severity of the crime increases. But the motive to reduce prison costs by inducing good behavior is never completely dominated by the motive to reduce crime.

Key words: imprisonment; prison costs; prisoner behavior; deterrence; sanctions

JEL codes: H23; K14; K42

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1. Introduction

Prisoners are, almost by definition, an unruly group. Common infractions include causing disruptions during meals and recreational activities, buying and selling drugs, verbally abusing and physically assaulting guards, and homosexually raping or attacking other prisoners. Worse yet, but less common, are prison riots.

Controlling prisoners is expensive, requiring prison guards and, in some cases, solitary confinement cells or maximum security prisons. In fiscal year 2001, $35.1 billion was spent by state and federal correctional agencies, an average of $22,710 per inmate.¹ These costs could be reduced if prisoners would behave better, and the prison system provides numerous incentives to prisoners to do so. Good behavior may be rewarded, for example, by television privileges, greater access to recreational facilities, or sentence reductions. Bad behavior may be punished by the loss of privileges, solitary confinement, or augmented sentences. The cost of controlling badly-behaved prisoners beyond that required to control well-behaved prisoners is substantial — commonly fifty percent more, and sometimes twice as much. In Ohio, for example, the average prisoner costs $63 a day to house, whereas a prisoner in a maximum-security prison costs $101 a day, and a prisoner in a “supermax” prison costs $149 a day.²

In this article I examine the desirability of rewarding prisoners for good behavior by reducing their sentences. (As explained below, it is immaterial whether the problem is framed in terms of a reduced sentence for good behavior or an additional sentence for bad behavior.) I also consider individuals’ incentives to commit crime initially. The two issues are intertwined

¹ See Criminal Justice Institute, Inc. (2003, pp. 95, 107). This report lists the average daily cost per inmate as $62.22 (p. 107), which I have converted to an annual figure. (Fiscal year 2001 is the latest year for which these data are available.)

because the effort required by a prisoner to maintain good behavior, and the reward for such behavior (or punishment for bad behavior), affect the disutility of the prison sentence and thereby affect the incentive to commit crime. I derive both the optimal sentence for the initial offense and the optimal sentence reduction if a prisoner behaves well.

The main result is that the optimal sentence reduction for good behavior in prison is always positive. This is true even though inducing prisoners to behave well leads to their bearing a shorter sentence and lessens the deterrence of crime. I also show that when the upper bound on sentence length is binding, the optimal sentence reduction for good behavior in prison decreases with the severity of the harm from the offense and goes to zero as the level of harm goes to infinity. This result reflects that the state’s savings in operating prisons becomes less important relative to the deterrence of crime as the severity of the crime increases. But the motive to reduce prison costs by inducing good behavior is never completely dominated by the motive to reduce crime.

The intuition behind the main result can be explained as follows. Raising the sentence reduction for good behavior always lowers prison costs. The only countervailing consideration is that deterrence declines because prisoners who behave well serve less time. But, starting at a sentence reduction of zero, there are no prisoners who are behaving well (there is no incentive to do so) and therefore no prisoners who would be deterred less if the sentence reduction is raised.3 This implies that it is always optimal to raise the sentence reduction for good behavior above zero. However, once the sentence reduction is positive, there are some prisoners who will behave well and who will be deterred less if the sentence reduction is increased. The optimal

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3 No one else is affected either. The marginal prisoners who switch from bad to good behavior are not deterred less because, by definition, they are indifferent between bad and good behavior. The prisoners who continue to act badly obviously are not affected because they do not receive the sentence reduction.
sentence reduction for good behavior balances the beneficial effect on prison costs with this
detrimental effect on deterrence.

The conclusion that it is always optimal to reward good behavior in prison is quite robust.
For reasons explained below, it holds regardless of whether individuals are risk neutral, risk
averse, or risk preferring with respect to prison sentences; regardless of whether the probability
of detection is treated as an exogenous parameter or a policy instrument; and regardless of
whether fines are used together with imprisonment sentences.

The results in this article can be interpreted either in terms of a reduced sentence for good
behavior (the interpretation I employ) or an enhanced sentence for bad behavior. Let $s$ be the
sentence for the crime at issue, and $r$ the sentence reduction if the offender is caught and then
behaves well in prison. Thus, the sentence is $s$ if the prisoner behaves badly and $s - r$ if the
prisoner behaves well. Alternatively, one could impose a sentence of $s - r$ for the crime and then
increase the sentence by $r$ if the prisoner behaves badly, resulting in the same outcomes. I
employ the language of reward because actual penal practice tends to be couched in terms of
time off for good behavior.

Section 2 presents the model used to undertake the analysis. Section 3 derives the main
results in two cases — when the upper bound on sentence length is not binding, and when it is
binding. Section 4 provides a numerical example. Section 5 discusses prior literature related to
the present analysis, and section 6 concludes with a discussion of the robustness of the results.

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4 The two articles focused on are those by Miceli (1994) and Garoupa (1996).
2. The Model

Individuals are assumed to be risk neutral. They contemplate committing a harmful act in order to obtain a benefit that varies among them. They are assumed to have no wealth, so the only sanction that can be imposed on them is a prison sentence,\(^5\) which cannot exceed a certain level (say due to limited lifetimes). The probability of detection is exogenous. If an individual is caught after committing the harmful act, he can choose whether to behave well or badly in prison. Good behavior requires effort by a prisoner and reduces the state’s cost of operating prisons. The effort needed to be good varies among individuals. For example, it would be much harder for individuals with impulse-control or anger-management problems to behave well than for individuals with more even temperaments.

Let

\[ h = \text{harm caused if the offense is committed; } h > 0; \]

\[ b = \text{benefit to an individual from committing the offense; } b \geq 0; \]

\[ \nu(b) = \text{distribution of } b \text{ among individuals; } \nu(b) > 0 \text{ for all } b \geq 0; \]

\[ p = \text{probability of catching an offender; } p > 0; \]

\[ s = \text{prison sentence for the offense; } s > 0; \]

\[ s = \text{maximal prison sentence; } s \leq \bar{s}; \]

\[ r = \text{sentence reduction if a prisoner behaves well; } r \geq 0; \]

\[ e = \text{effort required per unit time by a prisoner to behave well; } e \geq 0; \]

\[ z(e) = \text{distribution of } e \text{ among individuals; } z(e) > 0 \text{ for all } e \geq 0; \]

\[ c_G = \text{cost to the state per unit time of prison if prisoners are good; } c_G > 0; \text{ and } \]

\[ c_B = \text{cost to the state per unit time of prison if prisoners are bad; } c_B > c_G. \]
Sentences are measured in units of time corresponding to one dollar of disutility. The distributions of benefit from the offense and of effort to behave well are assumed to be independent. I also assume that individuals know $b$ and $e$ before they decide whether to commit the offense.

First consider the decision of a prisoner whether to behave well. If he behaves badly, his disutility from time in prison is $s$. If he behaves well, his disutility is $(s - r)(1 + e)$. Hence, he will behave well if and only if $(s - r)(1 + e) < s$, or $e < r/(s - r)$. Naturally, the incentive to behave well increases with $r$. It decreases with $s$ because the total effort required to behave well, $(s - r)e$, increases with $s$.

An individual who will behave badly in prison will commit the offense if and only if $b > ps$, while an individual who will behave well in prison will commit the offense if and only if $b > p(s - r)(1 + e)$. Note that individuals who will behave well in prison are deterred less: $(s - r)(1 + e) < s$ for $e < r/(s - r)$. This makes sense intuitively because prisoners who choose to behave well do so only if the total burden of imprisonment, including the effort to be good, is thereby reduced.

Social welfare is the sum of the benefits obtained by individuals who commit the offense, less the harm done, less the private disutility of imprisonment, and less the public cost of imprisonment:

\[ \text{Social welfare} = \sum \text{(benefits)} - \text{(harm)} - \text{(private disutility)} - \text{(public cost)}. \]

\[5\] Since many criminals have little or no wealth, this assumption often is reasonable. In any event, the results would not be affected if fines were used as well (see section 6).

\[6\] Nothing hinges on the assumption that a prisoner behaves badly if he is indifferent. I make similar assumptions below without further comment.
The first term applies to individuals who will be good in prison, for whom $e < r/(s-r)$. Such individuals commit the offense if $b > p(s-r)(1 + e)$. Each offender obtains a benefit $b$, causes harm $h$, incurs expected disutility from imprisonment of $p(s-r)(1 + e)$, and causes the state to incur expected prison costs of $p(s-r)c_G$. The second term applies to individuals who will be bad in prison, and is interpreted similarly, though there is no effort cost incurred by prisoners and the state’s cost of imprisonment is higher.

The state’s problem is to choose $s$ and $r$ so as to maximize (1) subject to the constraints that $s \leq \bar{s}$ and $r \leq s$. Asterisks are used to denote the optimal values. I assume that $s^* > 0$ (otherwise the problem is uninteresting) and that the usual second-order conditions for a maximum hold.

Before proceeding, it will be useful to state the derivative of social welfare with respect to the sentence reduction, $r$, which is

$$
\int_{0}^{\infty} \left\{ \left[ h + p(s-r)c_G \right] \left[ dV(p(s-r)(1 + e))/dr \right] + p(1 + e + c_G) \left[ 1 - V(p(s-r)(1 + e)) \right] \right\} z(e) de
\right.

+ p [sc_B - (s-r)c_G] [1 - V(ps)] [dZ(r/(s-r))/dr].
$$

The first line is, for individuals who would behave well in prison, the detriment of reduced deterrence and the benefit of reduced social cost of imprisonment, as a result of raising $r$. The second line is the reduction in the public cost of operating prisons as a result of some bad
prisoners switching to being good prisoners. (Raising $r$ does not affect the behavior of prisoners who continue to act badly since they do not receive a sentence reduction.)

3. Analysis

As noted, the main result is

**Proposition 1.** The optimal sentence reduction for good behavior in prison is positive:

$$r^* > 0.$$  

**Proof:** This follows immediately from observing that the derivative of social welfare with respect to $r$ is, at $r = 0$,

$$p[s(c_B - c_G)][1 - V(ps)][dZ(0)/dr] > 0.$$  

Hence, $r^* > 0$ regardless of $s$.  

This result can be explained as follows. Obviously, raising the sentence reduction for good behavior, $r$, has the beneficial effect of inducing more prisoners to behave well, thereby reducing the state’s cost of operating prisons. In general, raising $r$ also has the detrimental effect of increasing crime because prisoners who behave well bear shorter sentences. Initially, however, when $r = 0$, all prisoners are acting badly, so there are no well-behaved prisoners whose sentences would decline if $r$ is raised. Moreover, the marginal prisoners who switch from acting badly to acting well are, by definition, indifferent between good and bad behavior and consequently are not deterred less. Thus, the only effect of raising $r$, starting at $r = 0$, is the beneficial savings of prison costs, implying that $r^* > 0$.  

Once $r$ is positive, however, there will be some well-behaved prisoners whose sentences will decline further as $r$ is raised, causing a reduction in deterrence. The optimal $r$ balances the beneficial savings in prison costs against this detrimental increase in crime. Everything else
equal, the greater the prison cost savings due to good behavior, the higher is the optimal \( r \); and the higher the harm from crime, \( h \), the lower is the optimal \( r \).\(^7\)

In general, however, the optimal sentence reduction for good behavior, \( r^* \), does not necessarily decline as the harm \( h \) increases. This is because the optimal sentence for the offense, \( s^* \), also changes as \( h \) increases, and would be expected to rise. If \( s \) rises, then there is a tendency for \( r \) to rise too in order to maintain the same incentive for prisoners to behave well — recall that the critical value of effort below which prisoners will behave well, \( r/(s - r) \), is decreasing in \( s \) and increasing in \( r \). I provide an example in the next section in which both \( s^* \) and \( r^* \) rise as \( h \) rises.

That the optimal sentence for the offense, \( s^* \), eventually rises as the level of harm increases is demonstrated by the following proposition.

\textit{Proposition 2.} The optimal sentence for the offense goes to infinity as harm goes to infinity: \( \lim_{h \to \infty} s^* = \infty \).

\textit{Proof:} To see that \( \lim_{h \to \infty} s^* = \infty \), suppose otherwise. In other words, suppose that \( s^* \) is bounded from above as \( h \to \infty \). Let \( \hat{s} \) represent this upper bound. I will show that it is possible to pick an \( s \) exceeding \( \hat{s} \) that leads to higher social welfare for \( h \) sufficiently high, contradicting the premise that \( s^* < \hat{s} \). If \( s \leq \hat{s} \), it is clear from (1) that social welfare is strictly less than

\[ \int_{0}^{\infty} [bv(b)db - (1 - V(ps))]h \leq \int_{0}^{\infty} [bv(b)db - (1 - V(ps))]h. \tag{4} \]

Now compute social welfare at \( s = \hat{s} + \varepsilon \) for some \( \varepsilon > 0 \) and \( r = 0 \):

\[ \int_{0}^{\infty} [bv(b)db - (1 - V(ps))]h \leq \int_{0}^{\infty} [bv(b)db - (1 - V(ps))]h. \]

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\(^7\) It is straightforward to show that, holding \( s \) constant, \( dr_B / dh > 0 \) and \( dr_B / dh < 0 \). The statement in the text also suggests that \( dr_G / dh < 0 \); however, if \( c_G \) rises, it may be desirable to raise \( r \) to reduce the public cost of imprisoning well-behaved individuals, even though \( c_B - c_G \) has declined.
\[
\int [b - h - p(\hat{s} + \varepsilon)(1 + c_B)]v(b)db. \\
p(\hat{s} + \varepsilon)
\]

Subtract the right-hand side of (4) from (5) to obtain

\[
p(\hat{s} + \varepsilon) \\
- \int [b v(b)db + [V(p(\hat{s} + \varepsilon)) - V(p\hat{s})]h - p(\hat{s} + \varepsilon)(1 + c_B)(1 - V(p(\hat{s} + \varepsilon))],
\]

which is positive for \( h \) sufficiently high. This establishes the contradiction.

Thus, for sufficiently high harm, the upper bound on sentence length, \( \bar{s} \), will be a binding constraint on the optimal sentence for the offense. The final proposition examines how the optimal sentence reduction for good behavior varies with the harm when the sentence for the offense is at the upper bound.

**Proposition 3.** If the optimal sentence for the offense is maximal, \( s^* = \bar{s} \), then

(a) the optimal sentence reduction for good behavior in prison is decreasing in harm: \( dr^*/dh < 0 \); and

(b) the optimal sentence reduction goes to zero as harm goes to infinity: \( \lim_{h \to \infty} r^* = 0 \).

**Proof:** (i) Let \( W(r(h), h) \) represent the expression for social welfare derived by substituting \( \bar{s} \) for \( s \) in (1). The first-order condition with respect to \( r \) is \( W_1(r(h), h) = 0 \). Totally differentiate this condition with respect to \( h \) and solve for \( dr/dh = -W_{12}/W_{11} \). Since \( W_{11} < 0 \) by the second-order condition, the sign of \( dr/dh \) is the same as the sign of \( W_{12} \). With \( s = \bar{s} \), differentiate (2) with respect to \( h \) to obtain

\[
W_{12} = \int [dV(p(\bar{s} - r)(1 + e))/dr]z(e)de < 0.
\]

This proves part (a).
(ii) To demonstrate that \( \lim r^* = 0 \) as \( h \to \infty \), I will show that if the limit of \( r^* \) exceeds zero, social welfare can be increased by setting \( r = 0 \). Suppose that \( \lim r^* = \hat{r} > 0 \). Social welfare at \( s = \bar{s} \) and \( r = \hat{r} \) can be expressed as

\[
\hat{r}(\bar{s} - \hat{r}) K' - \left\{ \int [1 - V(p(\bar{s} - \hat{r})(1 + e))] z(e) de + [1 - Z((\hat{r}/(\bar{s} - \hat{r})))[1 - V(p\bar{s})]] \right\} h, \quad (8)
\]

where \( K' \) is a term that does not depend on \( h \). Social welfare at \( s = \bar{s} \) and \( r = 0 \) can be expressed as

\[
K'' - [1 - V(p\bar{s})] h, \quad (9)
\]

where \( K'' \) is a term that does not depend on \( h \). Subtracting (8) from (9) yields

\[
\hat{r}(\bar{s} - \hat{r}) K'' - K' + \left\{ \int [V(p\bar{s}) - V(p(\bar{s} - \hat{r})(1 + e))] z(e) de \right\} h. \quad (10)
\]

The expression in braces is positive since \((\bar{s} - \hat{r})(1 + e) < \bar{s} \) for \( e < \hat{r}/(\bar{s} - \hat{r}) \). Hence, for \( h \) sufficiently high, (10) is positive, implying that the limit of \( r^* \) as \( h \to \infty \) cannot be positive. This demonstrates part (b).

An increase in harm from the offense implies, of course, that greater deterrence of crime is desirable. But if the sentence for the offense, \( s \), is at its upper bound, increased deterrence cannot be obtained by raising \( s \). Then the only way to reduce crime is by reducing the time off for good behavior, \( r \). This will be beneficial even though the costs of operating prisons will rise as a consequence. Although in the limit, as harm goes to infinity, \( r^* \) becomes arbitrarily small, it always remains positive for the reasons discussed after Proposition 1.
4. An Example

In this section I describe an example in which both the distribution of benefits from the offense and the distribution of effort are discrete. For simplicity, I assume that the upper bound on sentence length is not binding. 8

The benefits from the offense are 50, 150, and 600 with probabilities .5, .25, and .25, respectively; the effort costs per unit time in prison are .25 and 1, with probabilities .75 and .25, respectively; and the public cost of prison per unit time is .25 if prisoners behave well and .75 if they behave badly. Since good behavior saves the state .5 in prison costs per unit time, the first-best outcome is only for prisoners whose effort cost is .25 to behave well. The probability of detection is .1. Two levels of harm are considered, 200 and 300.

If the harm from the offense is 200, the optimal sentence for the offense, s*, is 500 and the optimal sentence reduction for good behavior in prison, r*, is 100. These sentences induce only low-effort prisoners to behave well, and deter only the lowest-benefit individuals. The resulting level of social welfare is 54.1. 9

If instead the harm is 300, then s* rises to 1,500 and r* rises to 300. These sentences also induce only low-effort prisoners to behave well, but now deter all but the highest-benefit individuals. The resulting level of social welfare is 24.8.

To see the social desirability of inducing the low-effort prisoners to behave well, consider the outcomes in the example if the sentence reduction for good behavior were zero. Then every prisoner would behave badly and the resulting levels of social welfare (given the optimal s when

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8 Because this example is a straightforward application of the general model, the details are omitted here.

9 Because of the discrete nature of the example, individuals who behave well in prison are not deterred less than individuals who behave badly in prison. The latter bear a sentence of s* = 500, while the former bear a burden of (s* − r*)(1 + e) = 400(1 + .25) = 500.
$r = 0$) would be 43.8 when the harm is 200 and 9.4 when the harm is 300, a 19.1 percent and 62.3 percent decline in social welfare, respectively.

5. Related Literature

To my knowledge, there are only two prior articles — Miceli (1994) and Garoupa (1996) — that consider the choice of sanctions to induce prisoners to behave well. Both are framed in terms of the optimal parole period — a period of time during which an offender is under judicial supervision outside of prison in lieu of spending more time in prison. In their models, the disutility to an individual is lower as a parolee than as a prisoner and the cost to the state of parole supervision is less than the cost of imprisonment. In Miceli’s model, but not in Garoupa’s, good behavior in prison lowers the public cost of imprisonment per unit time. In both models, good behavior results in an offender receiving a prison sentence reduction, with the reduced time being spent on parole. Depending on the relevant parameter values, Miceli concludes (p. 209) that the optimal punishment scheme “can involve either a prison term without chance of parole, a prison term followed by parole, or pure probation [in essence, parole alone].” Garoupa comes to essentially the same conclusion.10

Miceli and Garoupa and I consider different rewards for good behavior in prison: parole versus time off. While Miceli and Garoupa show that parole may or may not be socially desirable, I demonstrate that a policy of time off for good behavior always is desirable. Their analyses of parole are inconclusive because parole may or may not be a cost-effective method of accomplishing deterrence compared to imprisonment — whether it is depends on how much less

10 Although he states (p. 133) that “the optimal sanction . . . usually includes imprisonment . . . and parole . . . ”, his formal analysis, like Miceli’s, shows that this outcome is one of three possibilities (the others being prison alone and parole alone).
burdensome parole is for an individual compared to imprisonment (per unit time), and how much cheaper parole is for the state compared to imprisonment.\textsuperscript{11}

More generally, one could consider both parole and time off together. My conjecture is that a policy of time off for good behavior would still be used in the optimal sentencing scheme, essentially for the reasons developed here. Thus, if parole is desirable too, the optimal reward for good behavior in prison could be characterized as a prison sentence reduction, only part of which would be spent on parole.\textsuperscript{12}

6. Conclusion

There are several natural ways to generalize the model employed in this article.\textsuperscript{13} First, suppose that individuals are risk averse or risk preferring with respect to prison sentences, rather than, as assumed here, risk neutral. While this change will affect the magnitudes of the optimal sentences, it does not affect the conclusion that the optimal sentence reduction for good behavior in prison, $r^*$, is positive. The reasoning is essentially identical to that given previously — there is a benefit from raising $r$ above zero, in terms of reducing the cost of operating prisons, but no first-order reduction of deterrence.

\textsuperscript{11} In Miceli’s (1994) analysis, the desirability of parole also depends on the extent to which good behavior in prison — induced by the prospect of parole — lowers the state’s cost of imprisonment.

\textsuperscript{12} Lewis (1979) also studies the optimal parole period, though his focus is quite different than mine. He treats the sentence for the offense as exogenous and derives the optimal point during the sentence at which to release the prisoner to parole status, given assumptions about the likelihood of the parolee reverting to criminal behavior. His objective is to minimize the public’s cost of crime and punishment. There is no discussion of the behavior of prisoners. Landes and Posner (2007) study empirically the supply of and demand for presidential pardons and commutations (the shortening of a sentence currently being served), which, they note (p. 2), are sometimes granted “in recognition of postconviction good behavior by the individual.”

\textsuperscript{13} I have verified the claims in this section, but do not report the results formally because they are so similar to those already presented.
Second, suppose that the probability of detection is a policy instrument rather than an exogenous parameter, with greater enforcement costs incurred to detect a higher fraction of offenders. Then, depending on individuals’ risk preferences with respect to prison sentences, the optimal sentence for the offense, \( s^* \), might be maximal, with a relatively low probability of detection, or less than maximal, possibly with a higher probability of detection. The argument used to show that the optimal sentence reduction for good behavior, \( r^* \), is positive applies to this case as well, without modification.

Third, suppose that individuals do have some wealth and that both fines and prison sentences can be used as sanctions. In this case, if it is optimal to impose a prison sentence to control the offense, it also is optimal to give time off for good behavior. The usual reasoning applies.

Thus, the central result of this article — that it is optimal to reduce the sentences of prisoners who behave well in order to lower prison costs — holds quite generally.
References


