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The Priest-Klein Hypotheses: Proofs and
Generality

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JEL Classification Codes: D81, K, K4, K41

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1. Introduction

Priest and Klein's 1984 article, "The Selection of Disputes for Litigation," famously hypothesized that there will be a "tendency toward 50 percent plaintiff victories" among litigated cases (p.20). Their article has been one of the most influential legal publications, and its influence is growing as empirical work on law has become more common. Compare Shapiro and Pearse (2012) to Shapiro (1996). Even with the introduction of asymmetric information models of settlement, Priest and Klein's article continues to be cited by sophisticated empiricists and in respected peer-reviewed journals. See Hubbard (2013); Gelbach (2012); Atkinson (2009); Bernardo, Talley and Welch (2000); Waldfogel (1995); Siegelman and Donohue (1995).

Despite the passage of more than thirty years since the publication of Priest and Klein's original article, their results have never been rigorously proved, and doubts remain about the assumptions needed to sustain their conclusions. Because Priest and Klein supported their argument only with simulations and an informal, graphical proof, the precise statement and scope of their claims have not been entirely clear. Waldfogel (1995) formalized Priest and Klein's model, following carefully their original set-up and notation, but without proving any results. Shavell (1996 p. 499, nn. 19-20) provided a sketch of a proof in a footnote. Part of the challenge is that the formalization involves a double integral over a region of integration that is only implicitly defined. (Waldfogel (1995), p.237). Although Hylton and Lin (2012) also formalize and prove some of Priest and Klein's claims, they do so using a model substantially different from, and in many ways less general than, Priest and Klein's.¹

This paper provides the first set of rigorous proofs of various hypotheses that can be attributed to Priest and Klein, while remaining faithful to Priest and Klein's original set-up. Nevertheless, before setting out the formal analysis, it is helpful to distinguish six hypotheses plausibly attributable to the Priest and Klein (1984):

¹ Hylton and Lin's model differs from Priest and Klein's in that Priest and Klein assume that case strength is measured by a real number, $Y' \in (-\infty, \infty)$, where plaintiff prevails if $Y' > Y^*$, and where Y^* represents the decision standard or legal rule. In contrast, Hylton and Lin assume that case strength is a probability. Although this difference might seem small, it has major implications. In addition, Hylton and Lin make use of an extrinsic "censoring function"—the function determining which cases will be litigated—to arrive at their result. In Priest and Klein's model, this function, which we call the "litigation probability function," is derived from other aspects of the model. See *infra* Section 2.

THE TRIAL SELECTION HYPOTHESIS. “[D]isputes selected for litigation (as opposed to settlement) will constitute neither a random nor a representative sample of the set of all disputes” (p.4). This proposition is probably the most important contribution of their article.

THE FIFTY-PERCENT LIMIT HYPOTHESIS. “[A]s the parties’ error diminishes” there will be a “convergence towards 50 percent plaintiff victories” (pp.18). This hypothesis is often called *the Priest-Klein hypothesis*.

THE ASYMMETRIC STAKES HYPOTHESIS. If the defendant would lose more from an adverse judgment than the plaintiff would gain, then the plaintiff will win less than fifty percent of the litigated cases. Conversely, if the plaintiff has more to gain, then the plaintiff will win more than fifty-percent (see pp. 24-26). This hypothesis is most plausibly, like the Fifty-Percent Limit Hypothesis, a statement about the limit percentage of plaintiff victories as the parties become increasingly accurate in predicting trial outcomes.

THE IRRELEVANCE OF THE DISPUTE DISTRIBUTION HYPOTHESIS. The plaintiff trial win rate will be “unrelated . . . to the shape of the distribution of disputes” (pp. 19 and 22). Like the two previous hypothesis, this hypothesis is about the limit as the parties become increasingly accurate in predicting trial outcomes. This hypothesis is closely related to the Fifty-Percent Limit Hypothesis, but more fundamental. It is also more general, because it also applies when the stakes are unequal.

THE NO INFERENCES HYPOTHESIS. Because selection effects are so strong, no inferences can be made about the law or legal decisionmakers from the plaintiff trial win rate. Rather, “the proportion of observed plaintiff victories will tend to remain constant over time regardless of changes in the underlying standards applied.” (p. 31).

THE FIFTY-PERCENT BIAS HYPOTHESIS. Regardless of the legal standard, the plaintiff trial win rate will exhibit “a strong bias toward . . . fifty percent” as compared to the

percentage of cases plaintiff would have won if all cases went to trial (pp. 5 and 23).

That is, the plaintiff trial win rate will be closer to fifty percent than the plaintiff win rate that would be observed if all cases went to trial.

This paper explores the mathematical validity of each of these hypotheses, except the No Inferences Hypothesis. Klerman and Lee (2014) showed that the No Inferences Hypothesis is false under Priest and Klein's original model as well as under the canonical asymmetric information models. Because the No Inferences Hypothesis is analyzed extensively elsewhere, it will not be discussed further in this article.

Because this paper is concerned with the results under Priest and Klein's original model, we remain faithful to their original set-up. In a separate paper, Lee & Klerman (2015), we consider two extensions to the original model. First, we raise a novel critique of Priest and Klein's original model—that it is non-Bayesian – and show how the model can be modified to make the parties' behavior consistent with Bayes Rule. Most of the hypotheses set out above remain valid under this modified model. Second, Priest and Klein's model has been criticized for lacking an incentive-compatible mechanism. In Lee & Klerman (2015), we address the possibility of *ex post* bargaining inefficiency by coupling Priest and Klein's model with an incentive-compatible mechanism—specifically the Chatterjee-Samuelson mechanism,. Under this model, we show that there will always be at least one symmetric equilibrium that will yield a fifty-percent plaintiff trial win rate, even when stakes are slightly asymmetric. Moreover, this and other results continue to hold even under the Bayesian modification.

The rest of the paper proceeds as follows. Section 2 explores a formalized version of Priest and Klein's model, and Sections 3 through 6 are devoted to analyzing the five different hypotheses. The Appendix contains technical proofs and additional results.

2. Formalization of Priest and Klein's Original Model

This section assumes familiarity with Priest and Klein (1984) and follows Waldfogel's (1995) formalization. Although there have been other attempts to formalize Priest and Klein's model (see Wittman (1985) and Hylton and Lin (2012)), Waldfogel offers the formalization that is most faithful to the model in Priest and Klein's original article. See Hylton and Lin (2012), n.5. We begin by first presenting the formalization in the most general manner possible and then

introduce additional assumptions as needed for each hypothesis. We do not, however, introduce assumptions that conflict with the original model's set-up.

The merits of a case are represented by a real number Y' , and the decision standard is denoted Y^* , where the defendant prevails if $Y' \leq Y^*$, and the plaintiff prevails if $Y' > Y^*$. For example, in a negligence case, Y' might be the efficient level of precaution expenditures minus defendant's actual precaution, in which case $Y^* = 0$. Priest and Klein, for simulation purposes, assume Y' is distributed according to a standard normal distribution. We do not impose that restriction. Instead, we assume only that Y' is distributed according to a probability density function, $g(Y')$, that is bounded above everywhere and locally continuous and nonzero at Y^* . Note that if all disputes were litigated, the plaintiff win rate would be $\int_{Y^*}^{\infty} g(Y') dY'$. If $G(Y')$ is the corresponding cumulative distribution, then the plaintiff win rate can be rewritten as $1 - G(Y^*)$.

If a case goes to trial, the court observes the true Y' and gives judgment to the plaintiff if and if $Y' > Y^*$. The plaintiff and the defendant themselves make unbiased estimates of Y' , $Y_p = Y' + \epsilon_p$ and $Y_d = Y' + \epsilon_d$, respectively, where ϵ_p and ϵ_d have mean zero, standard deviations σ_p and σ_d , respectively, and are distributed according to the joint probability distribution $f_{\sigma_p, \sigma_d}(\epsilon_p, \epsilon_d)$. Priest and Klein assume that ϵ_p and ϵ_d are independent and that $f_{\sigma_p, \sigma_d}(\epsilon_p, \epsilon_d)$ is bivariate normal with $\sigma_p = \sigma_d$. In other words, they assume $f_{\sigma_p, \sigma_d}(\epsilon_p, \epsilon_d) = f_{\sigma_p}(\epsilon_p) f_{\sigma_d}(\epsilon_d)$, where $f_{\sigma}(\cdot)$ is the normal distribution with standard deviation σ . We relax these assumptions and assume instead that ϵ_p and ϵ_d are not necessarily independent, but are distributed with mean zero and standard deviations σ_p and σ_d , respectively, according to a joint probability density function $f_{\sigma_p, \sigma_d}(\epsilon_p, \epsilon_d)$ that may not be normal, but which has the following properties. First, $f_{\sigma_p, \sigma_d}(\epsilon_p, \epsilon_d)$ has full support over the entire \mathbf{R}^2 . Second, $f_{\sigma_p, \sigma_d}(\epsilon_p, \epsilon_d)$ satisfies the following condition: $f_{1,1}(x, y) = \sigma_x \sigma_y f_{\sigma_x, \sigma_y}(\sigma_x x, \sigma_y y)$. Third, the corresponding univariate marginal distributions for ϵ_p and ϵ_d are $f_{p, \sigma_p}(\epsilon_p)$ and $f_{d, \sigma_d}(\epsilon_d)$ such that $f_{p,1}(x) = \sigma_x f_{p, \sigma_x}(\sigma_x x)$ and $f_{d,1}(y) = \sigma_y f_{d, \sigma_y}(\sigma_y y)$. The second assumption, in particular, indicates that $f_{\sigma_p, \sigma_d}(\epsilon_p, \epsilon_d)$ belongs to a family of mean-zero probability density functions that vary parametrically with σ_p and σ_d and can be standardized with proper rescaling. The reason for

making this assumption is that some of the hypotheses require taking the limit as σ_p and σ_d approach zero. Therefore, there needs to be a well-defined family of distributions as the standard deviation parameter varies. A bivariate normal distribution with mean zero certainly satisfies all these conditions, but a host of other distributions also satisfy these conditions.² Initially, we do allow for σ_p and σ_d to be different. For example, σ_p may be greater than σ_d if the defendant is systematically superior in estimating the true merit of the case than the plaintiff. This may be the case if there is asymmetric information. Therefore, we let $\sigma_p = \sigma_d/\beta = \sigma$ for a fixed $\beta > 0$, and where necessary, we will explicitly assume $\beta = 1$. Finally, let $F_p[\cdot]$ and $F_d[\cdot]$ be the corresponding cumulative distributions for $\sigma_p = \sigma_d = 1$.

In order to estimate the probability with which the plaintiff will prevail, both the plaintiff and the defendant need to take into account the fact that their estimates of case merit, $Y' + \epsilon_p$ and $Y' + \epsilon_d$, are not wholly accurate. Therefore, they must estimate both the mean and standard deviation of their estimates of Y' . Priest and Klein (1984) assume that plaintiff estimates the mean of sampling distribution of Y' to be $Y' + \epsilon_p$ and the standard deviation to be σ_p .

Waldfoegel (1995) notes that under this set-up the plaintiff's subjective estimate of the probability

it will prevail, $P_p = P(Y' \geq Y^* | Y' + \epsilon_p)$, will simply be $P_p = F_p \left[\frac{Y' + \epsilon_p - Y^*}{\sigma_p} \right]$. Similarly, the

defendant estimates the mean of Y' to be $Y' + \epsilon_d$, and the standard deviation to be σ_d . So the

defendant estimates the probability that the plaintiff prevails to be $P_d = F_d \left[\frac{Y' + \epsilon_d - Y^*}{\sigma_d} \right]$. Although

this assumption is consistent with Priest and Klein's original model, this set-up implies that the parties do not take the underlying distribution of disputes into consideration in calculating their subjective beliefs. In Lee and Klerman (2015), we reconsider this assumption and modify the model to allow for the parties to use Bayes' rule to update their subjective beliefs, and show that nearly all of the results of this paper remain valid.

² A partial list of distributions (with full support over \mathbf{R}^2) that satisfy this condition include bivariate distributions composed of generalized normal distributions, Laplace distributions, and logistic distributions. Given any univariate probability density function $f(x)$ with mean zero and standard deviation 1, one can always construct such a family of bivariate density function by setting $f_{\sigma_p, \sigma_d}(\epsilon_p, \epsilon_d) = \frac{f(\frac{\epsilon_p}{\sigma_p})f(\frac{\epsilon_d}{\sigma_d})}{\sigma_p \sigma_d}$.

Priest and Klein assume that the parties go to trial³ if $P_p J - C_p + S_p > P_d J + C_d - S_d$, where $J > 0$ is the damages that the defendant pays the plaintiff if the case is litigated and the plaintiff prevails, C_p and C_d are litigation costs for the plaintiff and the defendant, respectively, and S_p and S_d are settlement costs for the plaintiff and the defendant, respectively. This condition for litigation makes sense, because settlement can only happen if both parties perceive the payoffs to settlement to be higher than the payoffs to litigation. The litigation condition can be rewritten as $(P_p - P_d)J > C - S$, where $C = C_p + C_d$ and $S = S_p + S_d$. $(P_p - P_d)J > C - S$ is known as the Landes-Posner-Gould condition for litigation, after the three scholars who formulated it. Priest and Klein simulate their results with $\frac{C-S}{J} = 1/3$. We assume $0 < (C - S)/J \leq 1$. Priest and Klein assume that the plaintiff always has a credible threat to go to trial and thus can litigate or settle even when $P_p J < C_p$. We retain that assumption, even though it is unrealistic. Relaxing it would complicate the math, but have little effect on the main conclusions.

Priest and Klein are silent about how the parties bargain to arrive at a settlement. Technically, the Landes-Posner-Gould condition is merely a sufficient condition for litigation, not a necessary one. Litigation might happen even if the condition is violated, because parties might not be able to agree on the settlement amount, even if there is a range of settlement amounts that would be in their perceived mutual interest. As modern mechanism design research has shown, bargaining is frequently inefficient. See Myerson and Satterthwaite (1983); but see McAfee and Reny (1992). Nevertheless, Priest and Klein (1984) and others using the divergent expectations model have assumed that the Landes-Posner-Gould condition is necessary as well as sufficient for litigation. We retain this assumption because this is consistent with Priest and Klein's original model. In Lee and Klerman (2015), we graft an incentive-compatible bargaining mechanism onto Priest and Klein's model and show that most of the results of the results remain valid for symmetric equilibria.

Priest and Klein allow for the possibility that parties may have asymmetric stakes and suggest there will be a deviation from fifty-percent in such cases. For example, the defendant

³ Priest and Klein and much of the later literature assume that "litigate" and "go to trial" are synonymous, because they assume that all cases either settle or go to trial. More recent work explores the fact that many cases are resolved by motions to dismiss or summary judgment. Gelbach (2012); Hubbard (2013). Cases resolved by such motions are litigated, but did not go to trial. This article, however, retains the simplifying assumption that all litigated cases go to trial. The term "disputes" or "all disputes" means both cases that settle and cases that are litigated.

may be more concerned about its reputation or an adverse precedent, so it may lose more from an adverse judgment than the plaintiff gains from prevailing. As Priest and Klein point out, asymmetric stakes can be formalized by assuming that the plaintiff would win αJ if it prevailed and the defendant would lose J if the plaintiff won. If α is greater than 1, then the plaintiff faces a greater stake in the litigation than the defendant, and vice versa. Taking into account the possibility of asymmetric stakes, the trial condition becomes $\alpha P_p - P_d > (C - S)/J$.

3. Trial Selection Hypothesis and Irrelevance of the Dispute Distribution Hypothesis

Let $P_{\alpha, \sigma_p, \sigma_d}(Y'; Y^*)$ denote the probability that a dispute Y' goes to trial when the decision standard is Y^* and where the parties predict case merit with errors ϵ_p and ϵ_d that are distributed with mean zero and standard deviations σ_p and σ_d . We shall call this the “litigation probability function.” When $\sigma_p = \sigma_d = \sigma$, we will simply denote this probability as $P_{\alpha, \sigma}(Y'; Y^*)$. $P_{\alpha, \sigma_p, \sigma_d}(Y'; Y^*)$ can be written as the probability that

$$\alpha F_p \left[\frac{Y' + \epsilon_p - Y^*}{\sigma_p} \right] - F_d \left[\frac{Y' + \epsilon_d - Y^*}{\sigma_d} \right] > \frac{C - S}{J}.$$

In other words, $P_{\alpha, \sigma_p, \sigma_d}(Y'; Y^*) = \iint_{R_{\alpha, \sigma_p, \sigma_d}(Y'; Y^*)} f_{\sigma_p, \sigma_d}(\epsilon_p, \epsilon_d) d\epsilon_p d\epsilon_d$, where

$$R_{\alpha, \sigma_p, \sigma_d}(Y'; Y^*) = \left\{ (\epsilon_p, \epsilon_d) \in R^2 \mid \alpha F_p \left[\frac{Y' + \epsilon_p - Y^*}{\sigma_p} \right] - F_d \left[\frac{Y' + \epsilon_d - Y^*}{\sigma_d} \right] > \frac{C - S}{J} \right\}.$$

$P_{\alpha, \sigma_p, \sigma_d}(Y'; Y^*)$ can be expressed as a double integral over a region of integration that is implicitly defined by the inequality.

Figures 1a and 1b depict examples of litigation probability functions, $P_{\alpha, \sigma_p, \sigma_d}(Y'; Y^*)$ for large and small for σ for cumulative normal distributions and $\alpha = 1$, $\sigma_p = \sigma_d = \sigma$, and $Y^* = 1$. As these figures show, when $\alpha = 1$ the probability of litigation is single-peaked and symmetric around Y^* . Therefore, disputes close to Y^* are the most likely to be litigated. In addition, as σ becomes smaller, the probability of litigation becomes highly concentrated near Y^* .

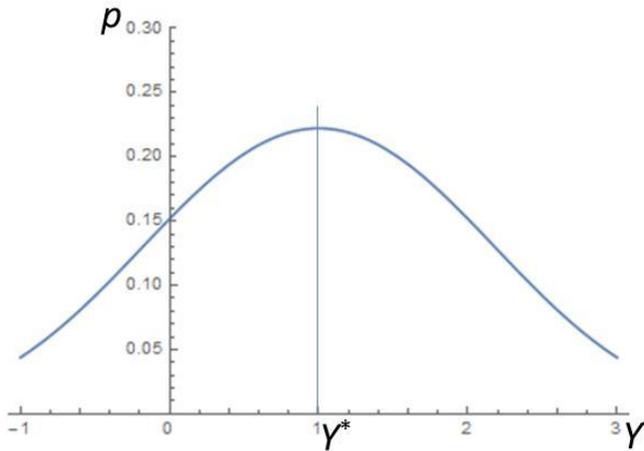


Figure 1a. $P_{1,\sigma}(Y'; 1)$ for $\sigma = 1$.

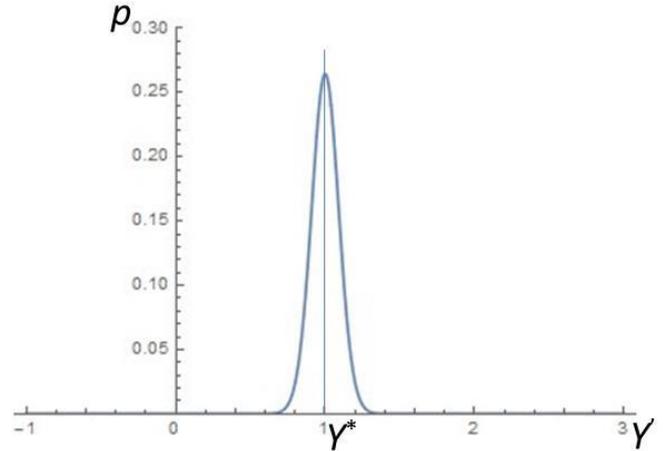


Figure 1b. $P_{1,\sigma}(Y'; 1)$ for $\sigma = 0.1$.

The fraction of cases litigated is $\int_{-\infty}^{\infty} P_{\alpha,\sigma_p,\sigma_d}(Y'; Y^*)g(Y')dY'$. This value approaches zero as σ_p and σ_d approach zero. The plaintiff trial win rate then is

$$W_{\alpha,\sigma_p,\sigma_d}(Y^*) = \frac{\int_{Y^*}^{\infty} P_{\alpha,\sigma_p,\sigma_d}(Y'; Y^*)g(Y')dY'}{\int_{-\infty}^{\infty} P_{\alpha,\sigma_p,\sigma_d}(Y'; Y^*)g(Y')dY'}$$

When $\sigma_p = \sigma_d = \sigma$, we denote the plaintiff trial win rate as simply $W_{\alpha,\sigma}(Y^*)$. Since $W_{\alpha,\sigma_p,\sigma_d}(Y^*)$ is mathematically different from $\int_{Y^*}^{\infty} g(Y')dY'$, the plaintiff trial win rate if all disputes were litigated, we can readily see that *the set of litigated cases is not simply a random set of all disputes*. The Trial Selection Hypothesis is therefore clearly correct.

Figure 2 shows how $P_{\alpha,\sigma_p,\sigma_d}(Y'; Y^*)$ and $g(Y')$ interact for different σ values. The graphs on the left show the distribution of all disputes (the solid line) and the probability of litigation (the dotted line). The graphs on the right show the distribution of *litigated* disputes, which is derived by multiplying the density of the dispute distribution times the probability of litigation. The top right panel shows the result when $\sigma = 0.5$. The bottom right panel shows the result when $\sigma = 0.2$. The graphs show that the distribution of litigated disputes becomes more symmetric around $Y^* = 1$ as σ approaches zero.

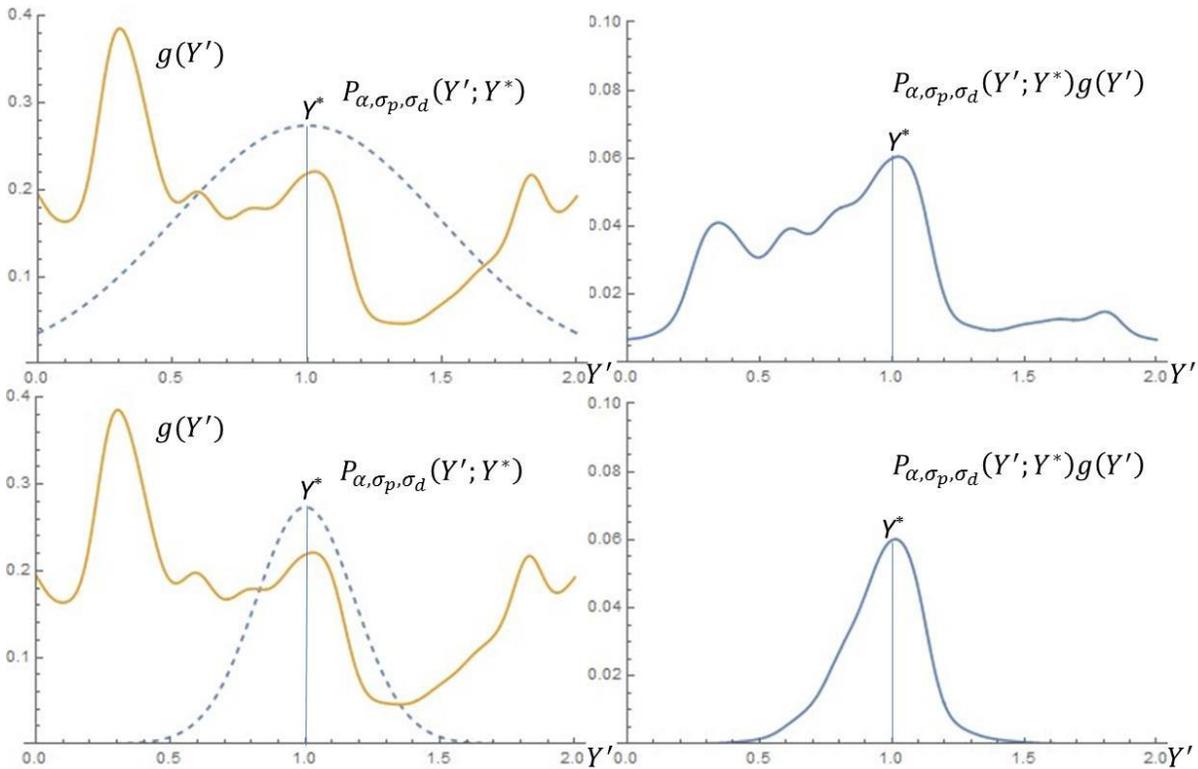


Figure 2. Distribution of Disputes and Litigated Cases

The plaintiff trial win rate, $W_{\alpha, \sigma_p, \sigma_d}(Y^*)$, is the area under the product graph with $Y' > Y^*$ divided by the area under the entire graph. Because the graphs become increasingly symmetric around Y^* , it makes sense that this ratio converges to fifty percent as σ approaches zero. Nevertheless, the fifty-percent result is not entirely obvious, because, as the graph becomes increasingly symmetric, both the area to the right of Y^* and the entire area under the curve approach zero. In terms of the fraction in $W_{\alpha, \sigma_p, \sigma_d}(Y^*)$, the numerator and the denominator are both approaching zero, and the limit must be calculated. Furthermore, because both the numerator and the denominator include $P_{\alpha, \sigma_p, \sigma_d}(Y'; Y^*)$, which is a double integral with the region of integration defined implicitly by an inequality, L'Hopital's rule cannot readily be used to simplify the fraction. For this reason, it is not clear that the limit will be exactly fifty-percent (assuming, of course, $\alpha = 1$). Hence, an analysis of the behavior at the limit is warranted.

Note that $W_{\alpha, \sigma_p, \sigma_d}(Y^*)$, as written, depends on many parameters, including the shape of the distribution of disputes, $g(Y')$. In the limit as σ_p and σ_d approach zero, however, the plaintiff trial win rate will not vary with the distribution of disputes. The following Proposition

provides the specific functional form for the limit of the plaintiff trial win rate and establishes the Irrelevance of the Dispute Distribution Hypothesis.

PROPOSITION 1: IRRELEVANCE OF THE DISPUTE DISTRIBUTION HYPOTHESIS. *The limit value of the plaintiff trial win rate under the Priest-Klein model will not depend on the distribution of disputes. More specifically, suppose $\sigma_p = \sigma_d/\beta = \sigma$ for a fixed $\beta > 0$, $g(Y')$ is bounded above everywhere and locally continuous and nonzero at Y^* , and ϵ_p and ϵ_d are distributed with mean zero according to $f_{\sigma_p, \sigma_d}(\epsilon_p, \epsilon_d)$ such that $f_{1,1}(x, y) = \sigma_x \sigma_y f_{\sigma_x, \sigma_y}(\sigma_x x, \sigma_y y)$ with full support over R^2 . Then the plaintiff trial win rate is given by*

$$W_{\alpha, \sigma_p, \sigma_p}(Y^*) = \frac{\int_{Y^*}^{\infty} P_{\alpha, \sigma_p, \sigma_d}(Y'; Y^*) g(Y') dY'}{\int_{-\infty}^{\infty} P_{\alpha, \sigma_p, \sigma_d}(Y'; Y^*) g(Y') dY'}$$

where $P_{\alpha, \sigma_p, \sigma_d}(Y'; Y^*) = \iint_{R_{\alpha, \sigma_p, \sigma_d}(Y'; Y^*)} f_{\sigma_p, \sigma_d}(\epsilon_p, \epsilon_d) d\epsilon_p d\epsilon_d$ and

$$R_{\alpha, \sigma_p, \sigma_d}(Y'; Y^*) = \left\{ (\epsilon_p, \epsilon_d) \in R^2 \left| \alpha F_p \left[\frac{Y' + \epsilon_p - Y^*}{\sigma_p} \right] - F_d \left[\frac{Y' + \epsilon_d - Y^*}{\sigma_d} \right] > \frac{C - S}{J} \right. \right\}.$$

As a result:

1. For $\alpha \leq \frac{C-S}{J}$, the parties never litigate, so the plaintiff trial win rate is undefined.

2. For $\frac{C-S}{J} < \alpha < 1 + \frac{C-S}{J}$, the limit of this value as σ approaches 0 reduces to an

expression independent of $g(Y')$: $\lim_{\sigma \rightarrow 0^+} W_{\alpha, \sigma_p, \sigma_d}(Y^*) = \frac{\int_0^{\infty} P_{\alpha, 1, \beta}(z; 0) dz}{\int_{-\infty}^{\infty} P_{\alpha, 1, \beta}(z; 0) dz}.$

3. For $\alpha > 1 + \frac{C-S}{J}$, the limit value will always be 1 regardless of $g(Y')$.

4. For $\alpha = 1 + \frac{C-S}{J}$, then regardless of $g(Y')$, the limit value will be $\frac{\int_0^{\infty} P_{\alpha, 1, \beta}(z; 0) dz}{\int_{-\infty}^{\infty} P_{\alpha, 1, \beta}(z; 0) dz}$ if

$$\lim_{t \rightarrow \infty} \frac{t}{F_d^{-1} \left[\left(1 + \frac{C-S}{J} \right) F_p \left[\frac{t}{J} \right] - \frac{C-S}{J} \right]} > \beta \text{ and } 1 \text{ otherwise. If the univariate marginal}$$

distributions are equal (i.e., $f_{p, \sigma}(x) = f_{d, \sigma}(x)$) and they have an increasing hazard rate,⁴

$$\text{then } \lim_{t \rightarrow \infty} \frac{t}{F_d^{-1} \left[\left(1 + \frac{C-S}{J} \right) F_p \left[\frac{t}{J} \right] - \frac{C-S}{J} \right]} = 1.$$

⁴ All log-concave probability density functions have increasing hazard rates. See Bagnoli and Bergstrom (2004).

Before we go on, we make a few observations. First, although $W_{\alpha, \sigma_p, \sigma_d}(Y^*)$ is well-defined for all *positive* σ values, it is undefined at $\sigma = 0$.⁵ This is because when $\sigma = 0$, each party knows whether it will win or lose with 100-percent certainty and *no disputes will go to trial*. Proposition 1 is therefore a statement about the limit value of a function at a point at which it is not continuous (since undefined). Second, for $\frac{C-S}{J} < \alpha < 1 + \frac{C-S}{J}$, the limit value of the plaintiff trial win rate does not depend on the shape of the distribution of disputes, $g(Y')$, but only on the shape of the litigation probability function, $P_{\alpha, 1, \beta}(z; 0)$, which in turn depends only on the shape of $f_{1, \beta}(x, y)$ and the region of integration (affected by α). Third, when $\alpha > 1 + \frac{C-S}{J}$, the limit value of plaintiff trial win rate will also not depend on the distribution of disputes because it will be 1 regardless of α , β , and $g(Y')$. The intuition for this last point is as follows. When $\alpha > 1 + \frac{C-S}{J}$ and thus the plaintiff has a significantly greater stake than the defendant, there is a threshold case estimate for the plaintiff, $Y_p^0 > Y^*$, above which the defendant will be unable to make an attractive settlement offer even for disputes the defendant is sure the plaintiff will win. Thus, as σ goes to zero, all disputes that are intrinsically stronger cases for plaintiffs than $Y' = Y_p^0$ will litigate, but no cases below the threshold value will get litigated. Therefore, in the limit, plaintiffs will win all litigated cases, and the limit value of the plaintiff trial win rate is 1. When $\alpha = 1 + \frac{C-S}{J}$, the plaintiff trial win rate becomes more complex, and we can only prove the irrelevance of the dispute distribution with additional assumptions. Nevertheless, this complexity is of little real-world significance, because it is relevant only when α is precisely $1 + \frac{C-S}{J}$, and there is no reason to think that α , a parameter that can take any value between zero and infinity, takes on the precise value of $1 + \frac{C-S}{J}$ with any empirically significant frequency.

The formal proof is included in the Appendix. We include only a portion of the proof here. Since $\sigma_p = \sigma_d / \beta = \sigma$ for a fixed $\beta > 0$, as σ_p approaches zero, σ_d necessarily approaches zero. We begin by normalizing the variables. For a given $\sigma > 0$, let $u = \frac{Y' + \epsilon_p - Y^*}{\sigma_p} =$

$\frac{Y' + \epsilon_p - Y^*}{\sigma}$, $v = \frac{Y' + \epsilon_d - Y^*}{\sigma_d} = \frac{Y' + \epsilon_d - Y^*}{\beta \sigma}$, $z = \frac{Y' - Y^*}{\sigma}$. Then we have

⁵ This does not affect integrability over z since the point of discontinuity has measure zero.

$$\begin{aligned} f_{\sigma_p, \sigma_d}(\epsilon_p, \epsilon_d) d\epsilon_p d\epsilon_d &= f_{\sigma_p, \sigma_d}(\sigma_p(u-z), \sigma_d(v-z/\beta)) \sigma_p \sigma_d dudv \\ &= f_{1, \beta}\left(u-z, v-\frac{z}{\beta}\right) dudv \end{aligned}$$

Meanwhile, $R_{\alpha, \sigma_p, \sigma_d}(Y'; Y^*) = \left\{ (u, v) \mid \alpha F_p[u] - F_d[v] > \frac{c-s}{j} \right\} = R_\alpha(u, v)$. Therefore, for each $\sigma > 0$,

$$P_{\alpha, \sigma_p, \sigma_d}(Y'; Y^*) = \iint_{R_\alpha(u, v)} f_{1, \beta}\left(u-z, v-\frac{z}{\beta}\right) dudv$$

This normalization of the variables is useful. Prior to normalization, $P_{\alpha, \sigma_p, \sigma_d}(Y'; Y^*)$, as a function of Y' , was a double integral of a fixed bivariate distribution over a region of integration in the $\epsilon_p \epsilon_d$ -plane that depended on three parameters: Y' , σ_p , and σ_d . After the change of variables, $P_{\alpha, \sigma_p, \sigma_d}(Y'; Y^*) = P_{\alpha, \sigma, \beta \sigma}(\sigma z + Y^*; Y^*)$, as a function of z , is a double integral of a bivariate distribution over a region of integration in the uv -plane that depends on only one parameter: z .

Figures 3a and 3b illustrate the region of integration when $\beta = 1$. Figure 3a shows the region of integration for $\alpha = 1$. The most important thing is that the region of integration, $R_\alpha(u, v)$, is always bounded by a horizontal asymptote above and by a vertical asymptote on the left. In other words, the region of integration can be contained by a *translated* fourth quadrant. As discussed in greater detail in Section 3.3 and Lemma A1 and as illustrated in Figures 5a and 5b, the region of integration will, in fact, have that property whenever $\frac{c-s}{j} < \alpha < 1 + \frac{c-s}{j}$. In contrast, as illustrated in Figure 3b and proved in Lemma A1, for $\alpha > 1 + \frac{c-s}{j}$, $R_\alpha(u, v)$ will not be bounded by any horizontal asymptote, but will be characterized by two vertical asymptotes.

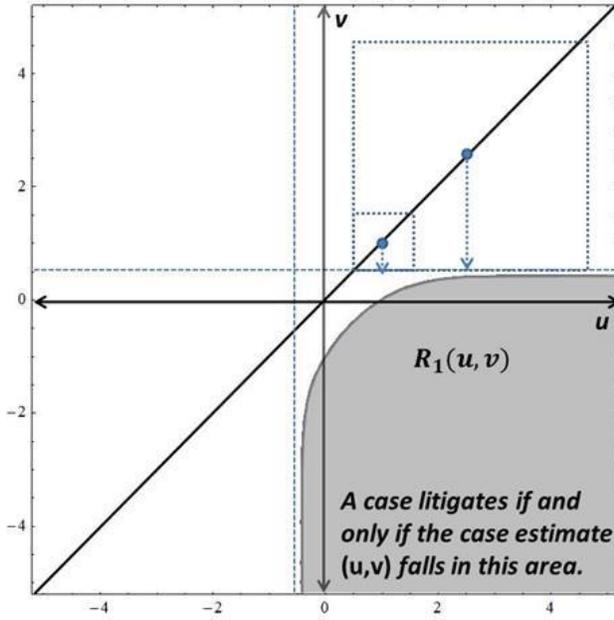


Figure 3a. Litigation Set ($\alpha = 1$)

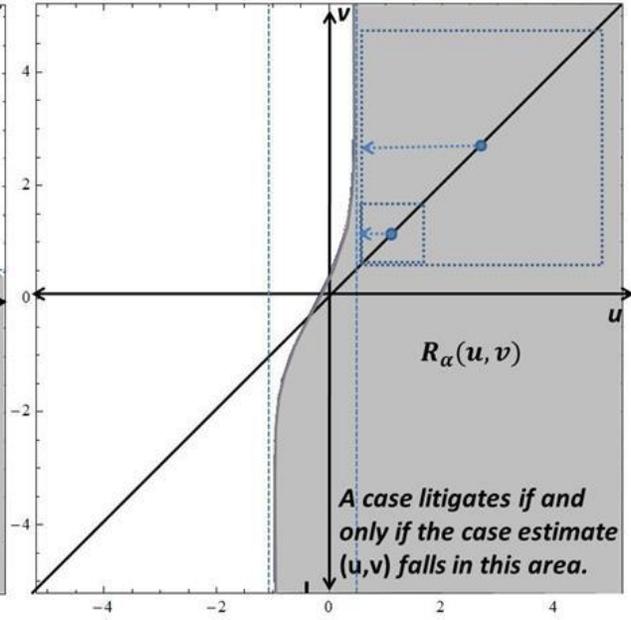


Figure 3b. Litigation Set ($\alpha > 1 + \frac{c-s}{J}$)

The integral in the numerator of $W_{t,\sigma_p,\sigma_d}(Y^*)$ is evaluated by centering the bivariate distribution of errors at each point along the line $u = v$ in the first quadrant and integrating it over the shaded area. Two such center points are illustrated in Figures 3a and 3b. So, for all $\sigma > 0$, we can rewrite $P_{\alpha,\sigma,\beta\sigma}(\sigma z + Y^*; Y^*)$ as $P_{\alpha,1,\beta}(z; 0)$. Therefore,

$$W_{t,\sigma_p,\sigma_d}(Y^*) = \frac{\int_{Y^*}^{\infty} P_{\alpha,\sigma_p,\sigma_d}(Y'; Y^*) g(Y') dY'}{\int_{-\infty}^{\infty} P_{\alpha,\sigma_p,\sigma_d}(Y'; Y^*) g(Y') dY'} = \frac{\int_0^{\infty} P_{\alpha,1,\beta}(z; 0) g(\sigma z + Y^*) dz}{\int_{-\infty}^{\infty} P_{\alpha,1,\beta}(z; 0) g(\sigma z + Y^*) dz}$$

For the main result when $\frac{c-s}{J} < \alpha < 1 + \frac{c-s}{J}$, we are done if we can take the limits under the integral by applying Lebesgue's Dominated Convergence Theorem. Then

$$\lim_{\sigma \rightarrow 0^+} \frac{\int_0^{\infty} P_{\alpha,1,\beta}(z; 0) g(\sigma z + Y^*) dz}{\int_{-\infty}^{\infty} P_{\alpha,1,\beta}(z; 0) g(\sigma z + Y^*) dz} = \frac{\int_0^{\infty} P_{\alpha,1,\beta}(z; 0) dz}{\int_{-\infty}^{\infty} P_{\alpha,1,\beta}(z; 0) dz}$$

since $g(Y')$ is locally continuous at Y^* and since $g(Y^*) \neq 0$. Application of Lebesgue's Dominated Convergence Theorem, however, requires existence of a Lebesgue-integrable function that dominates $P_{\alpha,1,\beta}(z; 0)g(\sigma z + Y^*)$. Lemma A4 in the Appendix shows that when $\alpha < 1 + \frac{c-s}{J}$ such construction is possible by taking the same integral but over a region of integration that properly contains $R_{\alpha}(u, v)$.

To show this upper-bound function is Lebesgue-integrable—in other words, that the integral of the constructed function is indeed finite—we make use of the bivariate version of Chebyshev’s Inequality. Here is the intuition. Note from Figure 3a that $P_{\alpha,1,\beta}(z; 0)$ is calculated as follows: for each point $(z, z/\beta)$, along the line $v = u/\beta$, the integral of $f_{1,\beta}(u, v)$ is centered at $(z, z/\beta)$ and is integrated over $R_\alpha(u, v)$. Since $g(Y')$ is bounded above, we need only show that $P_{\alpha,1,\beta}(z; 0)$ is dominated by a Lebesgue-integrable function. But notice that after some threshold point z_0 in either direction, $R_\alpha(u, v)$ will always be properly contained in the set that is the *complement* of a square box centered at $(z, z/\beta)$, whose length increases as z increases. Therefore, $P_{\alpha,1,\beta}(z; 0)$, eventually, will always be strictly smaller than the integral of $f_{1,\beta}(u, v)$ centered at $(z, z/\beta)$ but integrated over the complement of the square box with an increasing length. Because Chebyshev’s Inequality gives us an inverse quadratic relation between the distance from the mean and the integration of any probability distribution away from the mean by that distance, the integral of $f_{1,\beta}(u, v)$ centered at $(z, z/\beta)$ over the complement of the square box must eventually decrease at least as fast as the speed of z^{-2} , which must converge to a finite value. Hence, a Lebesgue-integrable dominating function is constructed, and we can take the limits under the integral.

In contrast, when $\alpha > 1 + \frac{c-s}{j}$, it is easy to see that $R_\alpha(u, v)$ is characterized as a region to the right of a graph that is characterized by two vertical asymptotes (shown in Figure 3b). In this case, $R_\alpha(u, v)$ will properly contain a region defined by $u \geq u_0$ for some $u_0 > 0$. To get the limit, we employ Chebyshev’s Inequality to the complement of $R_\alpha(u, v)$ for the numerator. We can show that the numerator and the denominator must diverge to infinity for positive z at the same pace, and thus the limit will equal 1.

Proposition 1 is stated in a highly general terms. For most of the rest of the paper, we will assume that the parties’ errors have a common standard deviation (that is, $\sigma_p = \sigma_d$ and therefore $\beta = 1$). Thus we will drop the subscript β , which measures the extent to which plaintiff or defendant formed more accurate estimates of the true case value. Priest and Klein did not consider the possibility that the parties differ in their ability to estimate case quality. Nevertheless, by varying the value of β , we can explore the effect of asymmetric information. See Lee & Klerman (2015).

4. Fifty-Percent Limit Hypothesis

The fact that we are able to derive the functional form of the limit value is useful, because it means that, for $\alpha \in \left(\frac{C-S}{J}, 1 + \frac{C-S}{J}\right)$, the limit value will equal fifty percent only when

$\frac{\int_0^\infty P_{\alpha,1}(z;0)dz}{\int_{-\infty}^\infty P_{\alpha,1}(z;0)dz}$ is equal to fifty percent, and not otherwise. Therefore, the limit value of the plaintiff trial win rate will be determined entirely by the shape of $P_{\alpha,1}(z;0)$. For example, if $P_{\alpha,1}(z;0)$ is symmetric around zero, the limit value will equal exactly fifty percent; and if $P_{\alpha,1}(z;0)$ is not symmetric around zero, the limit value will not equal exactly fifty percent except by coincidence. It turns out that when the stakes are equal ($\alpha = 1$), parties are equally well informed ($\beta = 1$), and $f_{\sigma,\sigma}(\epsilon_p, \epsilon_d)$ is symmetric in ϵ_p , in ϵ_d , and with respect to ϵ_p and ϵ_d (that is, $f_{\sigma,\sigma}(\epsilon_p, \epsilon_d) = f_{\sigma,\sigma}(\epsilon_d, \epsilon_p)$), then $P_{1,1}(z;0)$ will in fact be symmetric around zero (see Appendix). Thus, we have the following proposition.

PROPOSITION 2: FIFTY-PERCENT LIMIT HYPOTHESIS. *Suppose $g(Y')$ is bounded above everywhere and locally continuous and nonzero at Y^* , the stakes are equal (i.e., $\alpha = 1$), the parties are equally well informed (i.e., $\beta=1$), the parties' prediction errors, ϵ_p and ϵ_d , are distributed according to the same probability density function (i.e., $f_{p,\sigma} = f_{d,\sigma}$) and according to a joint probability density function that is symmetric around 0 and symmetric with respect to each other, then the limit value of the plaintiff trial win rate is fifty percent.*

Proposition 2 is more general than Priest and Klein's original version. Priest and Klein's simulations assume that $f_{\sigma,\sigma}(\epsilon_p, \epsilon_d)$ is independent bivariate normal and that the distribution of disputes, $g(Y')$ is normal. Those assumptions, while reasonable, are not necessary.

5. Asymmetric Stakes Hypothesis

In this section, we discuss disputes with asymmetric stakes, $\alpha \neq 1$. The defendant might have more at stake, $\alpha < 1$, in cases involving product liability, where an adverse judgment would damage the defendant's reputation or could be used against it in other cases. Conversely, but less commonly, the plaintiff would have more at stake, $\alpha > 1$, in cases alleging patent infringement, where a judgment invalidating the patent would bar suits against other alleged

infringers. In these cases, the litigation probability function will not be symmetric and will not center around Y^* . Instead, if the plaintiff has less at stake than the defendant, the litigation probability will peak before Y^* ; and if the plaintiff has more at stake than the defendant, the litigation probability function will peak after Y^* . Figures 4a and 4b depict litigation probability functions for $\alpha = 0.5$ and $\alpha = 1.5$.

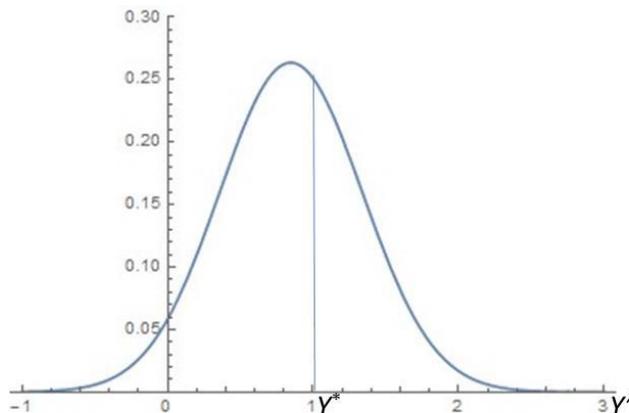


Figure 4a. $P_{\alpha, \sigma, \sigma}(Y'; 1)$ for $\alpha = 0.5$.

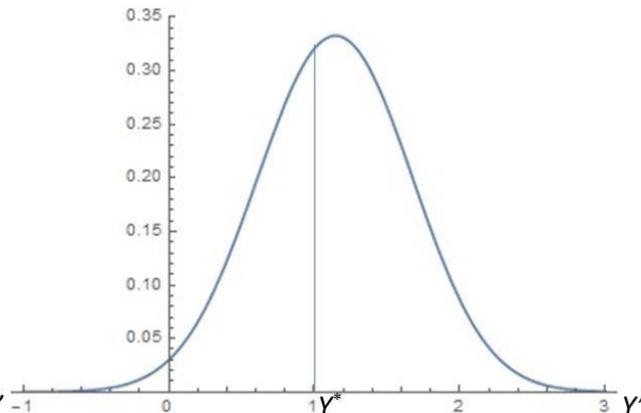


Figure 4b. $P_{\alpha, \sigma, \sigma}(Y'; 1)$ for $\alpha = 1.5$.

In addition, if $\alpha \neq 1$, the region of integration, $R_\alpha(u, v)$, will not be symmetric with respect to the line $v = -u$, and thus the limit value will not equal to fifty percent. In fact, we show in the Appendix that when $\frac{c-s}{J} < \alpha < 1$ the region of integration will be *biased towards the third-quadrant*. As shown in Figure 5a, when $\frac{c-s}{J} < \alpha < 1$, the top boundary of $R_\alpha(u, v)$, when reflected across the line $v = -u$ is bounded left by the left boundary of $R_\alpha(u, v)$. The reflection of the top boundary is represented by the dotted line. This shows that $R_\alpha(u, v)$ is composed of a subregion that is symmetric around $v = -u$ and a separate subregion that lies entirely below $v = -u$ (the side more favorable toward the plaintiff). Conversely, when $1 < \alpha < 1 + \frac{c-s}{J}$, the region of integration will be *biased towards the first-quadrant*. As shown in Figure 5b, this time the left boundary of $R_\alpha(u, v)$, when reflected across the line $v = -u$ is bounded above by the top boundary of $R_\alpha(u, v)$, indicating more litigated cases for the defendant.

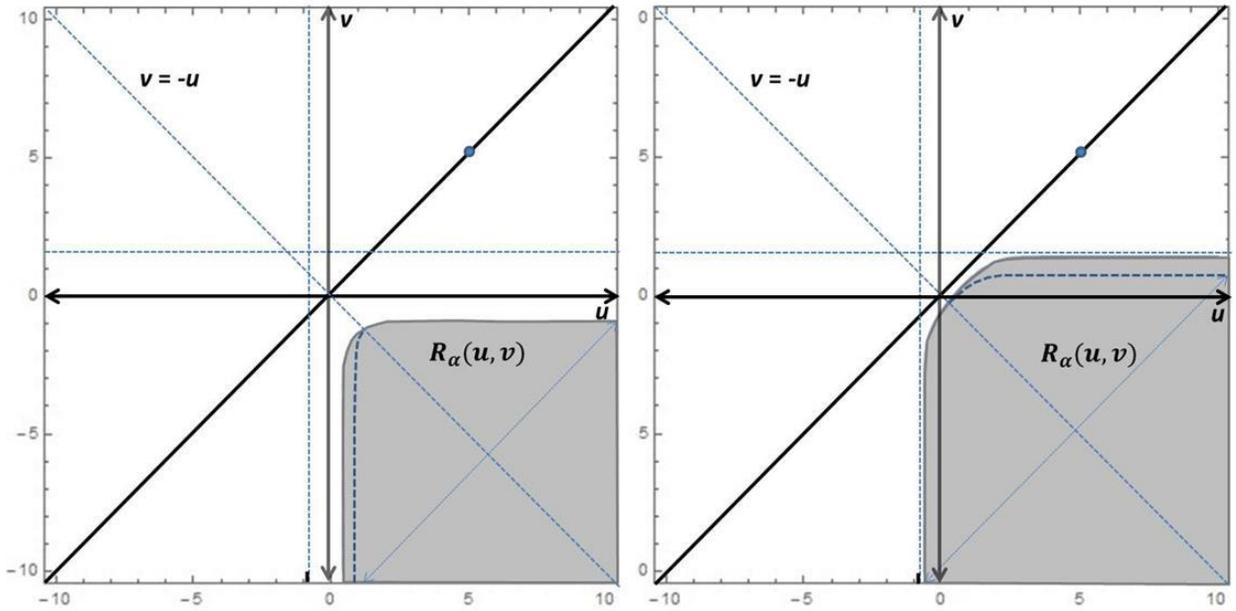


Figure 5a. Litigation Set ($\frac{c-s}{J} < \alpha < 1$)

Figure 5b. Litigation Set ($1 < \alpha < 1 + \frac{c-s}{J}$)

These graphs suggest the following proposition, which is proved in the Appendix.

PROPOSITION 3: ASYMMETRIC STAKES HYPOTHESIS. *Suppose the conditions of Proposition 2 are true except $\alpha \neq 1$, then for $\alpha > \frac{c-s}{J}$, the limit value of plaintiff trial win rate will be greater than fifty percent for $\alpha > 1$ and less than fifty percent for $\alpha < 1$. In other words, if the plaintiff's stake is larger than the defendant's, then the limit value of the plaintiff trial win rate will be greater than fifty percent. Conversely, if the defendant's stake is larger than the plaintiff's stake, then the limit value of the plaintiff trial win rate will be less than fifty percent. When $\alpha \leq \frac{c-s}{J}$, there will be no trials, so the plaintiff trial win rate is undefined.*

Figure 6 illustrates a *Mathematica* simulation for the win-rate for using normal distributions.⁶ Interestingly, the result is not strictly monotonic for $1 < \alpha < 1 + \frac{c-s}{J}$. Instead, there is a slight dip in the beginning.

⁶ Waldfoegel (1995) does not assume $\frac{c-s}{J}$ varies in relations to α . See Waldfoegel (1995), p.236. The simulation likewise maintains this assumption. The result of Proposition 3, however, holds more generally and is independent of this assumption.

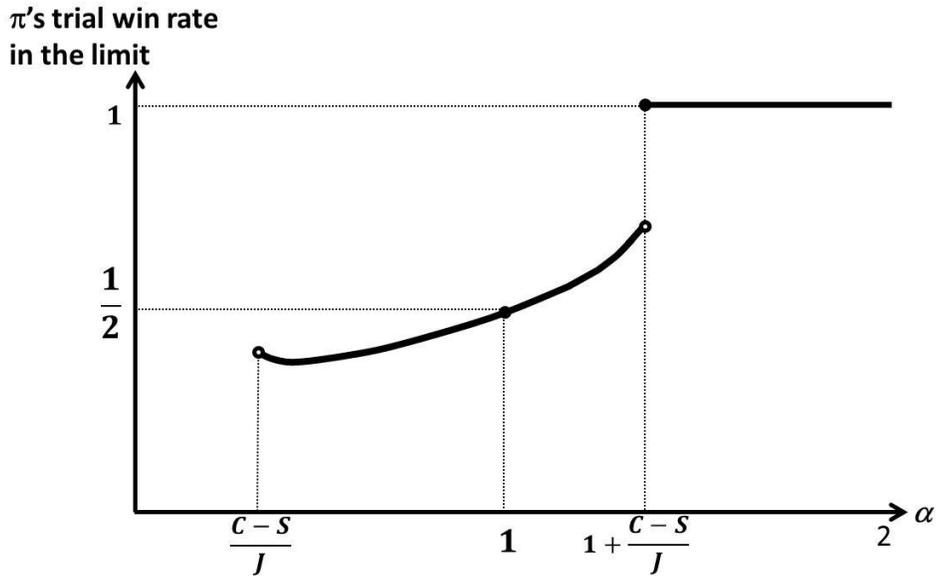


Figure 6. The Plaintiff Trial Win Rate (at the Limit) for Asymmetric Stakes

6. Fifty-Percent Bias Hypothesis

Thus far we have limited our analysis to hypotheses relating to the limit as parties become increasingly accurate in predicting trial outcomes. Proposition 1, however, also offers some insight into the Fifty-Percent Bias Hypothesis. That hypothesis says that the plaintiff trial win rate will be *closer to* fifty percent than the percentage of cases that plaintiff would have won if all cases had been litigated and none had settled.

Note, however, that this hypothesis is generally plausible only if the following two conditions are met: first, the stakes are symmetric ($\alpha = 1$) and both parties are equally well informed ($\beta = 1$); and second, the plaintiff win rate if all cases were litigated is not itself fifty percent. If the first condition is not satisfied, Proposition 1 tells us that the limit value of the plaintiff win rate will not itself be fifty percent (except by coincidence), and therefore, the Fifty-Percent Bias Hypothesis is likely to be false for sufficiently small values of σ .⁷ On the other hand, if the first condition is satisfied, the plaintiff trial win rate will converge to fifty percent, and thus it is reasonable to think that the plaintiff trial win rate in the real world (where most cases settle) will be closer to its limit value than the plaintiff trial win rate in a counterfactual

⁷ It is of course possible that α and β are not equal to 1, but offset each other in such a way that the limit value, $\frac{\int_0^\infty P_{\alpha,1,\beta}(z;0)dz}{\int_{-\infty}^\infty P_{\alpha,1,\beta}(z;0)dz}$, comes out to be exactly $1/2$. If so, the Fifty-Percent Bias Hypothesis might be true.

world where no cases settle. The second condition is also necessary (and plausible) because if the plaintiff win rate if no cases were settled happens to be fifty percent, it is impossible for plaintiff trial win rates to be closer to fifty percent.

If these two conditions are satisfied, as a corollary to the Fifty-Percent Limit Hypothesis, we can conclude that for σ values that are sufficiently small, the Fifty-Percent Bias Hypothesis *must* be true.

On the other hand, if σ is sufficiently high, the Fifty-Percent Bias Hypothesis will not be generally true unless we make more restrictive assumptions about $g(Y')$. We show that if $g(Y')$ is symmetric (not necessarily around Y^*) and is logarithmically concave—conditions which are satisfied, for example, by normal and Laplace distributions—the Fifty-Percent Bias Hypothesis is true. These are sufficient conditions, rather than necessary conditions. This is clear since a small perturbation on the distribution of disputes would be unlikely to thwart the overall selection bias. Nevertheless, we also show that neither symmetry nor logarithmically concave cumulative distribution by itself is sufficient. See the Appendix for proofs.

PROPOSITION 4: FIFTY-PERCENT BIAS HYPOTHESIS. *Under the conditions of Proposition 2, the Fifty-Percent Bias Hypothesis is true for sufficiently small values of σ . That is, the plaintiff trial win rate will be closer to fifty percent than the plaintiff win rate among all disputes. In other words, under these conditions, $|W_{1,\sigma}(Y^*) - 1/2| \leq |\int_{Y^*}^{\infty} g(Y')dY' - 1/2|$ for all $Y^* \in R$. For large values of σ , the Fifty-Percent Bias Hypothesis will be true if the conditions of Proposition 2 are true and if $g(Y')$ is symmetric and logarithmically concave. In addition, if $g(Y')$ is strictly logarithmically concave, the inequality is strict unless $\int_{Y^*}^{\infty} g(Y')dY' = 1/2$. Finally, neither symmetry nor logarithmic concavity alone is sufficient to sustain the Fifty Percent Bias Hypothesis for large values of σ .*

7. Conclusion

This paper provides a rigorous analysis of Priest and Klein's conclusions about the selection of disputes for litigation. It distinguishes between several hypotheses plausibly attributable to Priest and Klein, and proves or disproves them. We conclude that the Fifty-Percent Limit Hypothesis and four other hypotheses attributable to Priest and Klein (1984) are mathematically well-founded and true under the assumptions made by Priest and Klein. In fact,

they are true under a wider array of assumptions. More specifically, under Priest and Klein's original model, the Trial Selection Hypothesis, Fifty-Percent Limit Hypothesis, Asymmetric Stakes Hypothesis, and Irrelevance of Dispute Distribution Hypothesis are true for any distribution of disputes that is bounded and both positive and continuous near the decision standard, even if the parties' prediction errors are not independent. The Fifty-Percent Bias Hypothesis is true when the parties are very accurate in estimating case outcomes, but only sometimes true when parties are less accurate.

Appendix

This Appendix contains proofs not included in the main text as well as some additional results referenced in the main text.

A.1. Priest-Klein Hypotheses under the Original Priest-Klein Model

The proof of Proposition 1 proceeds according to the sketch of the proof included in the text. We begin by showing several lemmas. Lemma A1 describes the region of integration for various values of α . Lemmas A2 and A3 then pave the way for constructing a Lebesgue-integrable function $P_{\alpha,1,\beta}^U(z; 0)$ that dominates $P_{\alpha,1,\beta}(z; 0)$ in Lemma A4. The function need not be continuous. It need only integrate to a finite value. We construct $P_{\alpha,1,\beta}^U(z; 0)$ by integrating $f_{1,\beta}\left(u - z, v - \frac{z}{\beta}\right)$ over an area that properly contains $R_\alpha(u, v)$.

LEMMA A1.

- $R_\alpha(u, v)$ is non-empty if and only if $\alpha > \frac{c-s}{J}$.
- For all $\alpha > \frac{c-s}{J}$, $R_\alpha(u, v)$ is bounded left by a vertical asymptote at $u = F_p^{-1}\left(\frac{c-s}{\alpha J}\right)$.
- For all $\alpha < 1 + \frac{c-s}{J}$, $R_\alpha(u, v)$ is bounded above by a horizontal asymptote $v = F_d^{-1}\left(\alpha - \frac{c-s}{J}\right)$.
- When $\alpha \geq 1 + \frac{c-s}{J}$, $R_\alpha(u, v)$ is not bounded above, and if $\alpha > 1 + \frac{c-s}{J}$, it is characterized by a region to the right of an increasing curve that has two vertical asymptotes, at $u = F_p^{-1}\left(\frac{c-s}{\alpha J}\right)$ and $u = F_p^{-1}\left(\frac{1 + \frac{c-s}{J}}{\alpha}\right)$.
- When $\alpha = 1 + \frac{c-s}{J}$, and $F_p[\cdot] = F_d[\cdot] = F[\cdot]$ has an increasing hazard rate, then the boundary of $R_\alpha(u, v)$ in the first quadrant is characterized by an asymptote with a slope of 1, and the boundary approaches it monotonically.

PROOF OF LEMMA A2. If $\alpha \leq \frac{c-s}{J}$, $R_\alpha(u, v)$ is empty since $\alpha F_p[u] - F_d[v] < \alpha \leq \frac{c-s}{J}$ for all (u, v) . On the other hand, if $\alpha > \frac{c-s}{J}$, for high enough u and low enough v , we can find some value such that $\alpha F_p[u] - F_d[v] > \frac{c-s}{J}$. Meanwhile, if $\alpha F_p[u] - F_d[v] > \frac{c-s}{J}$, then $\alpha F_p[u] > \frac{c-s}{J}$, and thus $R_\alpha(u, v)$ is bounded left by $u = F_p^{-1}\left(\frac{c-s}{\alpha J}\right)$. If $\alpha < 1 + \frac{c-s}{J}$, $F_d[v] < \alpha F_p[u] - \frac{c-s}{J} \leq \alpha - \frac{c-s}{J}$. Thus, $R_\alpha(u, v)$ is bounded above by $v = F_d^{-1}\left(\alpha - \frac{c-s}{J}\right)$. Meanwhile, if $\alpha \geq 1 + \frac{c-s}{J}$, then the boundary curve $\alpha F_p[u] - F_d[v] = \frac{c-s}{J}$ must continually increase as u increases. Furthermore, if $\alpha > 1 + \frac{c-s}{J}$, then the inequality must hold for all value of $u > F_p^{-1}\left(\frac{1 + \frac{c-s}{J}}{\alpha}\right)$ and all values of v .

Now suppose $\alpha = 1 + \frac{c-s}{J}$ and $f_{p,\sigma}(x) = f_{d,\sigma}(x) = f(x)$. Let $F_p[\cdot] = F_d[\cdot] = F[\cdot]$. Note first that $u > v$ for all $(u, v) \in R_\alpha(u, v)$. Therefore, $R_\alpha(u, v)$ must lie strictly under the line $v = u$, and the

boundary cannot have a slope greater than 1 *in the limit*. Now it suffices to show that, given any point $(u, v) \in R_\alpha(u, v)$, we must also have $(u + x, v + x) \in R_\alpha(u, v)$ for all $x \geq 0$. This means $R_\alpha(u, v)$ must wholly contain its own translation in the direction of $v = u$. This will ensure that the boundary will have slope at least 1 at all points. To see this, we show that all points in $R_\alpha(u, v)$ will be properly contained in $R_\alpha(u, v)$ when translated upward by x . This amounts to showing the following: if $(1 + k)F[u] - F[v] \geq k$, then $(1 + k)F[u + x] - F[v + x] \geq k$, where $k = \frac{c-s}{J}$ and $x \geq 0$. To show this, we rewrite the statement as follows: if $\left(\frac{1-F[v]}{1-F[u]}\right) \geq k + 1$, then $\left(\frac{1-F[v+x]}{1-F[u+x]}\right) \geq k + 1$. It is then sufficient to show that $\left(\frac{1-F[v+x]}{1-F[u+x]}\right) \geq \left(\frac{1-F[v]}{1-F[u]}\right)$, or simply that $\frac{1-F[v+x]}{1-F[u+x]}$ is increasing in x . Under the quotient rule, this will be true as long as $(1 - F[u + x])(-f(v + x)) - (1 - F[v + x])(-f(u + x)) \geq 0$, which can be rewritten as $\frac{f(u+x)}{(1-F[u+x])} \geq \frac{f(v+x)}{(1-F[v+x])}$. The last result is true (for $u > v$) when $f(x)$ has an increasing hazard rate. *Q.E.D.*

Lemma A1 tells us that, for $\frac{c-s}{J} < \alpha < 1 + \frac{c-s}{J}$, $R_\alpha(u, v)$ is properly contained by a translated fourth quadrant in the uv -plane (with the origin at $(F_p^{-1}(\frac{c-s}{\alpha J}), F_d^{-1}(\alpha - \frac{c-s}{J}))$). This set is obviously contained in all of R^2 . Moreover, for $z > \beta F_d^{-1}(\alpha - \frac{c-s}{J})$, the point $(z, \frac{z}{\beta})$ is at least $\frac{z}{\beta} - F_d^{-1}(\alpha - \frac{c-s}{J})$ away from $R_\alpha(u, v)$. For $z < F_p^{-1}(\frac{c-s}{\alpha J})$, $(z, \frac{z}{\beta})$ is at least $|z - F_p^{-1}(\frac{c-s}{\alpha J})|$ away from $R_\alpha(u, v)$. Therefore, we have the following two Lemmas.

LEMMA A2. Let $c = \max\{F_p^{-1}(\frac{c-s}{\alpha J}), \beta F_d^{-1}(\alpha - \frac{c-s}{J})\}$ and $d = \min\{F_p^{-1}(\frac{c-s}{\alpha J}), \beta F_d^{-1}(\alpha - \frac{c-s}{J})\}$. Then for $\frac{c-s}{J} < \alpha < 1 + \frac{c-s}{J}$, $R_\alpha(u, v)$ is properly contained in $S_\alpha(u, v; z)$ for every $z \in R$, where

$$S_\alpha(u, v; z) = \begin{cases} R^2 & \text{for } |z| < c, \\ \{(u, v) \mid |u - z| > |z| - d \text{ or } |v - \frac{z}{\beta}| > |\frac{z}{\beta}| - d\} & \text{for } |z| \geq c. \end{cases}$$

LEMMA A3. For $\frac{c-s}{J} < \alpha < 1 + \frac{c-s}{J}$ and for each $z \in R$, $P_{\alpha,1,\beta}(z; 0) < P_{\alpha,1,\beta}^U(z; 0)$ where

$$P_{\alpha,1,\beta}^U(z; 0) = \iint_{S_\alpha(u,v;z)} f_{1,\beta}\left(u - z, v - \frac{z}{\beta}\right) dudv.$$

LEMMA A4. $\int_0^\infty P_{\alpha,1,\beta}(z; 0)dz$ is finite for all $\frac{c-s}{J} < \alpha < 1 + \frac{c-s}{J}$ and $\int_{-\infty}^0 P_{\alpha,1,\beta}(z; 0)dz$ is finite for all $\alpha > \frac{c-s}{J}$. $\int_0^\infty P_{\alpha,1,\beta}(z; 0)dz$ is infinite for all $\alpha \geq 1 + \frac{c-s}{J}$.

PROOF OF LEMMA A4. For the first part, by Lemma A3, we need to show only that $\int_0^\infty P_{\alpha,1,\beta}^U(z; 0)dz$ is finite.

$$\int_0^\infty P_{\alpha,1,\beta}^U(z;0)dz = \int_0^c P_{\alpha,1,\beta}^U(z;0)dz + \int_c^\infty P_{\alpha,1,\beta}^U(z;0)dz < c + \int_c^\infty P_{\alpha,1,\beta}^U(z;0)dz$$

Meanwhile, for each $z > c$,

$$P_{\alpha,1,\beta}^U(z;0) \leq 1 - P\left(|u-z| < |z|-d \text{ and } \left|v-\frac{z}{\beta}\right| < \left|\frac{z}{\beta}\right|-d\right) \leq 1 - P\left(|u-z| < \left|\frac{z}{\max\{1,\beta\}}\right|-d \text{ and } \left|v-\frac{z}{\beta}\right| < \left|\frac{z}{\max\{1,\beta\}}\right|-d\right).$$

By bivariate Chebyshev's Inequality,

$$P\left(|u-z| < \left|\frac{z}{\max\{1,\beta\}}\right|-d \text{ and } \left|v-\frac{z}{\beta}\right| < \left|\frac{z}{\max\{1,\beta\}}\right|-d\right) \geq 1 - \frac{1 + \sqrt{1 + \text{Cor}(\epsilon_p, \epsilon_d)^2}}{\left(\left|\frac{z}{\max\{1,\beta\}}\right|-d\right)^2}.$$

Therefore,

$$P_{\alpha,1,\beta}^U(z;0) \leq \frac{1 + \sqrt{1 + \text{Cor}(\epsilon_p, \epsilon_d)^2}}{\left(\left|\frac{z}{\max\{1,\beta\}}\right|-d\right)^2},$$

which is quadratic in z in the denominator and therefore integrates to a finite value over $z \in [c, \infty)$. The integral over $z \in (-\infty, 0]$ can likewise be shown to be finite. However, in this case we need not assume $\alpha < 1 + \frac{c-s}{j}$ since $R_\alpha(u, v)$ is always bounded by a vertical asymptote.

Meanwhile, for all $\alpha > 1 + \frac{c-s}{j}$, $R_\alpha(u, v)$ is defined by two vertical asymptotes. Therefore, $P_{\alpha,1,\beta}(z;0)$ can be bounded below by $P_{\alpha,1,\beta}^L(z;0)$ which integrates $f_{1,\beta}\left(u-z, v-\frac{z}{\beta}\right)$ over the complement of $S_\alpha(u, v; z)$, that is, over a rectangle containing $\left(z, \frac{z}{\beta}\right)$, which increases in z . Therefore, again by bivariate Chebyshev's Inequality, we can show that this lower bound integrand is bounded below by $\left(1 - \frac{1 + \sqrt{1 + \text{Cor}(\epsilon_p, \epsilon_d)^2}}{\left(\left|\frac{z}{\max\{1,\beta\}}\right|-d\right)^2}\right)$, which clearly integrates to infinity over all z . *Q.E.D.*

PROOF OF PROPOSITION 1. For $\frac{c-s}{j} < \alpha < 1 + \frac{c-s}{j}$, the rest of the proof is explained in the text.

For $\alpha > 1 + \frac{c-s}{j}$, note that

$$\frac{\int_0^\infty P_{\alpha,1,\beta}(z;0)g(\sigma z + Y^*)dz}{\int_{-\infty}^\infty P_{\alpha,1,\beta}(z;0)g(\sigma z + Y^*)dz} = \frac{\int_0^\infty P_{\alpha,1,\beta}(z;0)g(\sigma z + Y^*)dz}{\int_{-\infty}^0 P_{\alpha,1,\beta}(z;0)g(\sigma z + Y^*)dz + \int_0^\infty P_{\alpha,1,\beta}(z;0)g(\sigma z + Y^*)dz}$$

Here $\lim_{\sigma \rightarrow 0^+} \int_{-\infty}^0 P_{\alpha,1,\beta}(z;0)g(\sigma z + Y^*)dz = \int_{-\infty}^0 P_{\alpha,1,\beta}(z;0)g(Y^*)dz$ as before, since $R_\alpha(u, v)$ is bounded by a left asymptote. Meanwhile, we cannot apply Lebesgue's Dominated Convergence Theorem to $\int_0^\infty P_{\alpha,1,\beta}(z;0)g(\sigma z + Y^*)dz$ because $R_\alpha(u, v)$ contains all of the increasing large rectangles. Instead, we can write

$$\begin{aligned} \int_0^\infty P_{\alpha,1,\beta}(z;0)g(\sigma z + Y^*)dz &= \int_0^\infty \left(1 - Q_{\alpha,1,\beta}(z;0)\right)g(\sigma z + Y^*)dz \\ &= \int_0^\infty g(\sigma z + Y^*)dz - \int_0^\infty Q_{\alpha,1,\beta}(z;0)g(\sigma z + Y^*)dz \end{aligned}$$

where $Q_{\alpha,1,\beta}(z; 0) = \iint_{R_\alpha^c(u,v)} f_{1,\beta}(u-z, v-\frac{z}{\beta}) dudv$ where $R_\alpha^c(u, v)$ is the complement of $R_\alpha(u, v)$. Then we can apply Lebesgue's Dominated Convergence Theorem to $\int_0^\infty Q_{\alpha,1,\beta}(z; 0)g(\sigma z + Y^*)dz$ since the integral over the complement of $R_\alpha(u, v)$ can now be bounded above. Therefore,

$$\begin{aligned} & \lim_{\sigma \rightarrow 0^+} \frac{\int_0^\infty P_{\alpha,1,\beta}(z; 0)g(\sigma z + Y^*)dz}{\int_{-\infty}^\infty P_{\alpha,1,\beta}(z; 0)g(\sigma z + Y^*)dz} \\ &= \lim_{\sigma \rightarrow 0^+} \frac{\int_0^\infty g(\sigma z + Y^*)dz - \int_0^\infty Q_{\alpha,1,\beta}(z; 0)g(\sigma z + Y^*)dz}{\int_{-\infty}^0 P_{\alpha,1,\beta}(z; 0)g(\sigma z + Y^*)dz + \int_0^\infty g(\sigma z + Y^*)dz - \int_0^\infty Q_{\alpha,1,\beta}(z; 0)g(\sigma z + Y^*)dz} \\ &= \frac{\left(\lim_{\sigma \rightarrow 0^+} \int_0^\infty g(\sigma z + Y^*)dz\right) - \int_0^\infty Q_{\alpha,1,\beta}(z; 0)g(Y^*)dz}{\left(\lim_{\sigma \rightarrow 0^+} \int_0^\infty g(\sigma z + Y^*)dz\right) + \int_{-\infty}^0 P_{\alpha,1,\beta}(z; 0)g(Y^*)dz - \int_0^\infty Q_{\alpha,1,\beta}(z; 0)g(Y^*)dz} \end{aligned}$$

Since $\lim_{\sigma \rightarrow 0^+} \int_0^\infty g(\sigma z + Y^*)dz = \lim_{\sigma \rightarrow 0^+} \frac{1-G(Y^*)}{\sigma} = \infty$, while all other terms are finite, the limit value of the plaintiff win rate is 1.

When $\alpha = 1 + \frac{c-s}{j}$, notice that the same logic as above applies as long as the height of the boundary of the region of integration, $R_\alpha(u, v)$, in the limit as u goes to infinity lies strictly below some fraction of the line $v = u/\beta$. Let $M(u) = \frac{u}{F_d^{-1}\left[\left(1 + \frac{c-s}{j}\right)F_p[u] - \frac{c-s}{j}\right]}$. Then notice that the slope of the boundary of the region of integration, $R_\alpha(u, v)$, in the limit is $\lim_{u \rightarrow \infty} \frac{1}{M(u)}$. As long as this value in the limit is strictly below the slope of $v = u/\beta$, which is $1/\beta$, then a Lebesgue dominating function is similarly constructible. This case is similar to the case where $\alpha < 1 + \frac{c-s}{j}$. On the other hand, if it is strictly greater than the slope ($1/\beta$), a similar argument as the case $\alpha > 1 + \frac{c-s}{j}$ applies and so the limit will be 1. The path of integral will (eventually) lie wholly beneath the boundary or $R_\alpha(u, v)$ in the first quadrant, and will be increasingly farther away. The only special case is when $\beta = \lim_{u \rightarrow \infty} M(u)$. In this case, we cannot rely on Lebesgue's Dominated Convergence Theorem along the integral path in the first quadrant. Note however that $\int_{-\infty}^0 P_{\alpha,1,\beta}(z; 0)g(\sigma z + Y^*)dz$ will continue to be finite. We show that $\int_0^\infty P_{\alpha,1,\beta}(z; 0)g(\sigma z + Y^*)dz = \infty$. Note that if $\beta = \lim_{u \rightarrow \infty} M(u)$, then the limit of the slope of the boundary will approach the slope of the path of integration. In this case, for $z > 0$, $P_{\alpha,1,\beta}(z; 0)$ must eventually converge to a constant positive value rather than decay. This is because the boundary of the region of integration is in the limit parallel to the path of integration, and as z approaches infinity, the region of integration will become more like a half a plane that is only a certain distance away from z .

This is most directly seen in the special case when $F_p[\cdot] = F_d[\cdot] = F[\cdot]$ has an increasing hazard rate, but the same logic applies for other case. When $\alpha = 1 + \frac{c-s}{j}$ and $\beta = 1$, note that from Lemma A1, we must have $P_{\alpha,1,\beta}(z; 0) \geq P_{\alpha,1,\beta}(0; 0) > 0$ for all z . This is because $P_{\alpha,1,\beta}(0; 0)$ is equivalent to taking the double integral centered at (z, z) over a region that corresponds to $R_\alpha(u, v)$ translated by (z, z) , which would be wholly contained in the original $R_\alpha(u, v)$. Therefore, $\int_0^\infty P_{\alpha,1,\beta}(z; 0)g(\sigma z + Y^*)dz \geq \int_0^\infty P_{\alpha,1,\beta}(0; 0)g(\sigma z + Y^*)dz = P_{\alpha,1,\beta}(0; 0) \int_0^\infty g(\sigma z + Y^*)dz$. As shown above, $\lim_{\sigma \rightarrow 0^+} \int_0^\infty g(\sigma z + Y^*)dz = \infty$. Nevertheless, even when we don't have $F_p[\cdot] = F_d[\cdot] = F[\cdot]$ or increasing hazard rates, if

$\beta = \lim_{u \rightarrow \infty} M(u)$ then $P_{\alpha,1,\beta}(z; 0)$ will not decay but will converge to a constant positive value that is greater than $P_{\alpha,1,\beta}(0; 0)$. *Q.E.D.*

PROOF OF PROPOSITION 2. Since $f_{p,\sigma} = f_{d,\sigma}$, it follows that $F_p[\cdot] = F_d[\cdot]$, so we can drop the subscripts and simply use $F[\cdot]$ to denote the cumulative distributions, $F[\cdot] = F_p[\cdot] = F_d[\cdot]$. First, we show that for all Y' , $P_{1,\sigma}(Y'; Y^*) = P_{1,\sigma}(2Y^* - Y'; Y^*)$. Notice

$$P_{1,\sigma}(2Y^* - Y'; Y^*) = \iint_{R_{1,\sigma}(2Y^*-Y'; Y^*)} f_{\sigma,\sigma}(\epsilon_p, \epsilon_d) d\epsilon_p d\epsilon_d$$

where the region of integration is

$$\begin{aligned} R_{1,\sigma}(2Y^* - Y'; Y^*) &= \left\{ (\epsilon_p, \epsilon_d) \left| F \left[\frac{(2Y^* - Y') + \epsilon_p - Y^*}{\sigma} \right] - F \left[\frac{(2Y^* - Y') + \epsilon_d - Y^*}{\sigma} \right] > \frac{C - S}{J} \right\} \\ &= \left\{ (\epsilon_p, \epsilon_d) \left| \left(1 - F \left[\frac{-(Y^* - Y' + \epsilon_p)}{\sigma} \right] \right) - \left(1 - F \left[\frac{-(Y^* - Y' + \epsilon_d)}{\sigma} \right] \right) > \frac{C - S}{J} \right\} \\ &= \left\{ (\epsilon_p, \epsilon_d) \left| F \left[\frac{(Y' - Y^*) - \epsilon_d}{\sigma} \right] - F \left[\frac{(Y' - Y^*) - \epsilon_p}{\sigma} \right] > \frac{C - S}{J} \right\}. \end{aligned}$$

Since $f_{\sigma,\sigma}(\epsilon_p, \epsilon_d)$ is symmetric with respect to ϵ_p and ϵ_d , $f_{\sigma,\sigma}(\epsilon_p, \epsilon_d) = f_{\sigma,\sigma}(-\epsilon_d, -\epsilon_p)$. Thus we can swap ϵ_d and ϵ_p without changing the value of the integral. Thus, $P_{1,\sigma}(Y'; Y^*)$ is symmetric around Y^* , which implies $P_{1,1}(z; 0)$ is symmetric around zero. From Proposition 1, it follows that the limit value must be $1/2$. *Q.E.D.*

PROOF OF PROPOSITION 3. Since $\alpha > 1 + \frac{C-S}{J}$, Proposition 1 provides that the win rate limit will be exactly 1. So assume $\frac{C-S}{J} < \alpha < 1 + \frac{C-S}{J}$. As in the proof of Proposition 2, let $F_p[\cdot] = F_d[\cdot] = F[\cdot]$. Note first that the horizontal asymptote, $F^{-1}(\alpha - \frac{C-S}{J})$, occurs at a value that is more positive than the vertical asymptote, $F^{-1}(\frac{C-S}{\alpha J})$, is negative if and only $\alpha > 1$. In other words, $\left| F^{-1}(\alpha - \frac{C-S}{J}) \right|$ is greater (less) than $\left| F^{-1}(\frac{C-S}{\alpha J}) \right|$ if $\alpha > 1$ ($\alpha < 1$). This is obvious since $F\left(-F^{-1}\left(\frac{C-S}{\alpha J}\right)\right) = 1 - F\left(F^{-1}\left(\frac{C-S}{\alpha J}\right)\right) = \left(1 - \frac{C-S}{\alpha J}\right)$ and $F\left(F^{-1}\left(\alpha - \frac{C-S}{J}\right)\right) = \alpha\left(1 - \frac{C-S}{\alpha J}\right)$.

Suppose $\alpha > 1$. To show that the limit value of the plaintiff trial win rate is greater than $1/2$, it suffices to show that $R_\alpha(u, v)$ is skewed in the direction of the first quadrant in the following sense. Take $S_\alpha(u, v) = R_\alpha(u, v) \cap \{(u, v) | v < -u\}$. Then $S_\alpha(u, v)$ is the portion of $R_\alpha(u, v)$ that lies below the line $v = -u$. Let $S'_\alpha(u, v) = \{(-v, -u) | (u, v) \in S_\alpha(u, v)\}$. $S'_\alpha(u, v)$ is the reflection of $S_\alpha(u, v)$ over the line $v = -u$. We show that $S'_\alpha(u, v) \subset R_\alpha(u, v)$. This means that the entire region of integration that lies below $v = -u$ can be reflected across the line $v = -u$ and that reflection will be properly contained in $R_\alpha(u, v)$. Since $R_\alpha(u, v)$ contains additional regions above the line $v = -u$ (because the horizontal asymptote is more positive than the vertical asymptote is negative in this case), this shows that $R_\alpha(u, v)$ is skewed in the direction of the first quadrant, and this is sufficient to show that the plaintiff win rate will be higher than fifty percent. To see this, take away the portion that is symmetric, which is $S_\alpha(u, v) \cup S'_\alpha(u, v)$. What is left must lie entirely on the side of $v > -u$, and hence closer to the plaintiff's win side

than the defendant's win side. To show $S'_\alpha(u, v) \subset R_\alpha(u, v)$, we need to show the following: If (i) $\alpha > 1$, (ii) $\alpha F[u] - F[v] > \frac{c-s}{j}$, and (iii) $v < -u$, then $\alpha F[-v] - F[-u] > \frac{c-s}{j}$. In turn, it suffices to show that if (i) $\alpha > 1$ and (ii) $F[v] < F[-u]$, then $\alpha F[-v] - F[-u] > \alpha F[u] - F[v]$, since the last expression will be greater than $\frac{c-s}{j}$. But this last inequality rearranges as follows:

$$\begin{aligned} \alpha F[-v] - F[-u] > \alpha F[u] - F[v] &\Leftrightarrow \alpha(1 - F[v]) - F[-u] > \alpha(1 - F[-u]) - F[v] \\ &\Leftrightarrow (\alpha - 1)(F[-u] - F[v]) > 0, \end{aligned}$$

which is immediate. Therefore, the limit value of the plaintiff's win rate is greater than fifty percent. Notice by the same logic that the last inequality also holds when we have $\alpha < 1$ and $v > -u$. This shows that when $\alpha < 1$, $R_\alpha(u, v)$ is skewed in the direction of the third quadrant, and hence the limit value of the plaintiff win rate is lower than $\frac{1}{2}$.

Finally, if $\alpha = 1 + \frac{c-s}{j}$, there may not be a horizontal asymptote, but the above proof of the asymmetry of the region of integration for the general case when $\alpha > 1$ goes through. So according to Proposition 1, the limit value will be either 1 or if not, then greater than $\frac{1}{2}$ at any rate along the same line of argument. So the hypothesis still holds true when $\alpha = 1 + \frac{c-s}{j}$. *Q.E.D.*

PROOF OF PROPOSITION 4. Without loss of generality, assume $g(Y')$ is centered around 0. The inequality is clearly satisfied when $Y^* = 0$. Since $g(Y')$ is log-concave, (i) it is also single-peaked (including the possibility of a flat-top) and (ii) its cumulative distribution, $G(Y')$, is also log-concave. Without loss of generality, suppose $Y^* < 0$. Then we need to show that $|W_{1,\sigma}(Y^*) - 1/2| < \int_{Y^*}^{\infty} g(Y') dY' - \frac{1}{2}$. First notice $W_{1,\sigma}(Y^*) > \frac{1}{2}$. This can be seen as follows. $W_{1,\sigma}(Y^*)$ over all of $g(Y')$ is greater than $W_{1,\sigma}(Y^*)$ over a symmetric image of $g(Y')|_{Y' \leq Y^*}$ around Y^* . The latter graph lies completely under $g(Y')$ because $g(Y')$ is symmetric and single-peaked. And clearly, $W_{1,\sigma}(Y^*) = \frac{1}{2}$ over a symmetric image of $g(Y')|_{Y' \geq Y^*}$ around Y^* . Thus, we need only show $\int_{Y^*}^{\infty} g(Y') dY' > W_{1,\sigma}(Y^*)$. This is equivalent to showing

$$\frac{\int_{Y^*}^{\infty} g(Y') dY'}{\int_{-\infty}^{\infty} g(Y') dY'} = \int_{Y^*}^{\infty} g(Y') dY' > W_{1,\sigma}(Y^*) = \frac{\int_{Y^*}^{\infty} P_{1,\sigma}(Y'; Y^*) g(Y') dY'}{\int_{-\infty}^{\infty} P_{1,\sigma}(Y'; Y^*) g(Y') dY'}$$

This inequality rearranges to

$$\frac{\int_{-\infty}^{Y^*} P_{1,\sigma}(Y'; Y^*) g(Y') dY'}{\int_{-\infty}^{Y^*} g(Y') dY'} > \frac{\int_{Y^*}^{\infty} P_{1,\sigma}(Y'; Y^*) g(Y') dY'}{\int_{Y^*}^{\infty} g(Y') dY'}$$

By changing the variable to $Y' = Y^* - w$ for the left-side integral and $Y' = Y^* + w$ for the right-side integral and recognizing that $P_{1,\sigma}(Y'; Y^*)$ is symmetric around Y^* , we have

$$\begin{aligned} \int_0^{\infty} P_{1,\sigma}(Y^* - w; Y^*) \left(\frac{g(Y^* - w)}{\int_0^{\infty} g(Y^* - w) dw} \right) dw &= \int_0^{\infty} P_{1,\sigma}(Y^* + w; Y^*) \left(\frac{g(Y^* - w)}{\int_0^{\infty} g(Y^* - w) dw} \right) dw \\ &> \int_0^{\infty} P_{1,\sigma}(Y^* + w; Y^*) \left(\frac{g(Y^* + w)}{\int_0^{\infty} g(Y^* + w) dw} \right) dw \end{aligned}$$

Since $P_{1,\sigma}(Y^* + w; Y^*)$ is strictly decreasing in w , we need only show $\left(\frac{g(Y^*+w)}{\int_0^\infty g(Y^*+w)dw}\right)$ as a probability density function defined over $[0, \infty)$ first-order stochastically dominates $\left(\frac{g(Y^*-w)}{\int_0^\infty g(Y^*-w)dw}\right)$. Equivalently,

we need to show $\frac{\int_0^w g(Y^*-w)dw}{\int_0^\infty g(Y^*-w)dw} > \frac{\int_0^w g(Y^*+w)dw}{\int_0^\infty g(Y^*+w)dw}$ for all $w > 0$, or

$$\begin{aligned} \frac{G(Y^*) - G(Y^* - w)}{G(Y^*)} &> \frac{G(Y^* + w) - G(Y^*)}{1 - G(Y^*)} = \frac{(1 - G(-Y^* - w)) - (1 - G(-Y^*))}{G(-Y^*)} \\ &= \frac{G(-Y^*) - G(-Y^* - w)}{G(-Y^*)} \end{aligned}$$

since $1 - G(Y') = G(-Y')$ for all Y' . Since $-Y^* > Y^*$, it now suffices to show that $\frac{G(Y^*) - G(Y^* - w)}{G(Y^*)} = 1 - \frac{G(Y^* - w)}{G(Y^*)}$ is decreasing in Y^* , or that $\frac{G(Y^* - w)}{G(Y^*)}$ is increasing in Y^* for each $w > 0$. Under the Quotient Rule, this is true if

$$G(Y^*)g(Y^* - w) - g(Y^*)G(Y^* - w) = G(Y^*)G(Y^* - w) \left(\frac{g(Y^* - w)}{G(Y^* - w)} - \frac{g(Y^*)}{G(Y^*)} \right) > 0$$

which holds if $\frac{g(Y^*)}{G(Y^*)}$ is decreasing in Y^* , or put differently, if $G(Y^*)g'(Y^*) < (g(Y^*))^2$. But this last inequality holds since $G(Y')$ is strictly log-concave (which must be true since $g(Y')$ is log-concave). The proof goes through for weak inequality of $G(Y')$ is only weakly log-concave.

Meanwhile, for a general log-concave function or a single-peaked function that is not symmetric, the inequality generally will not hold for all σ at the mean (that is, for Y^* such that $G(Y^*) = \frac{1}{2}$) since $W_{1,\sigma,\sigma}(Y^*)$ will not always equal $\frac{1}{2}$. For a symmetric counter example, consider $g(Y')$, which equals zero everywhere but takes on the value of 1 on $[Y^* - \frac{1}{2}, Y^*]$ and $[Y^* + X - \frac{1}{2}, Y^* + X]$, where $X > \frac{1}{2}$. This function is clearly symmetric around $Y' = Y^* + \frac{X}{2}$. Then $\int_{Y^*}^\infty g(Y')dY' = 1/2$. But in this case, plaintiff trial win rate is

$$\frac{\int_{Y^*+X-\frac{1}{2}}^{Y^*+X} P_{1,\sigma}(Y'; Y^*)dY'}{\int_{Y^*-\frac{1}{2}}^{Y^*} P_{1,\sigma}(Y'; Y^*)dY' + \int_{Y^*+X-\frac{1}{2}}^{Y^*+X} P_{1,\sigma}(Y'; Y^*)dY'}$$

By Chebyshev's Inequality, the numerator can get arbitrarily small as X increases, while the denominator maintains a certain minimum value. Hence the plaintiff trial win rate can get arbitrarily small, even though the win rate if all cases had gone to trial would have been $\frac{1}{2}$. Since this example has a discontinuity at Y^* , the Fifty-Percent Limit Hypothesis, too, will not hold. Instead, as prediction errors go to zero, the plaintiff will lose zero percent of litigated cases. Nevertheless, it is possible to create a continuous version of this counter-example that closely approximates it, under which the Fifty-Percent Limit Hypothesis is true, but the Fifty-Percent Bias Hypothesis is false. The continuous version is the same as discussed above, except $g(Y')$ takes on the value 1 on $[Y^* - \frac{1}{2}, Y^* + \Delta]$ and $[Y^* + X - \frac{1}{2}, Y^* + X]$, where Δ is much smaller than $X - \frac{1}{2}$. In this continuous version, as the parties' prediction errors go to zero, the proportion of cases won by the plaintiff converges to fifty percent. Nevertheless, if prediction errors, σ , are large compared to Δ but small as compared to $X - \frac{1}{2}$, then the plaintiff trial win rate will be substantially smaller than fifty percent, even though the proportion of plaintiff victories, if all cases went

to trial, would be very close to fifty-percent. Thus, the Fifty-Percent Bias Hypothesis fails, even though the Fifty-Percent Limit Hypothesis would hold. *Q.E.D.*

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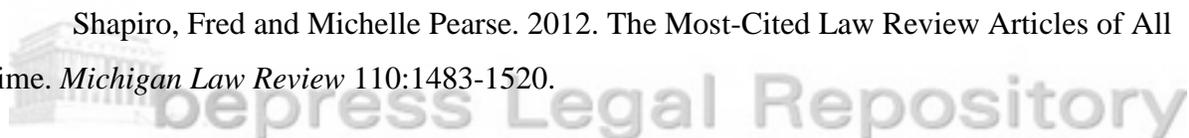
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