On the winner-take-all principle in innovation races

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Abstract

What is the optimal allocation of prizes in an innovation race? Should the winner take all, or is it preferable that first inventors share the market with late independent duplicators? Several papers have argued that the latter, more permissive regime is socially preferable. We re-examine that issue, finding that the winner-take-all system can in fact be socially optimal in a broad set of circumstances, much broader than claimed by the earlier literature. In our baseline model, two firms race for an innovation in continuous time. In the winner-take-all system, as soon as one firm innovates the other stops investing in R&D; in the alternative, more permissive system, the laggard keeps investing to duplicate the innovation and when it also succeeds the market becomes a duopoly. The winner-take-all system is preferable in highly innovative industries, whereas the permissive system is more likely to be optimal in mature industries, where product market competition is strong and innovation is a relatively rare occurrence. We analyze several extensions of this baseline model to better clarify why we arrive at different results than the earlier literature.

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1 Introduction

What is the optimal allocation of prizes in an innovation race? Should the winner take all, or is it preferable that first inventors share the market with independent duplicators, if and when they materialize? Following La Manna, MacLeod and De Meza (1989), a recent literature (reviewed below) has addressed that issue, arriving at the conclusion that a “permissive” regime, in which late independent inventors are allowed to practice the innovation and compete with the first, is generally preferable to the winner-take-all system. This paper cautions that this conclusion is based on restrictive assumptions, and points out that the winner-take-all system can in fact be socially optimal in a broad set of circumstances.

Our analysis sheds light on a number of controversial legal rules and policy issues. For example, a major difference between patents and other protection mechanisms, such as copyrights and trade secrets, is that independent invention is a defense against copyright infringement or trade secret misappropriation, but is not a defense against patent infringement (Maurer and Scotchmer (2002), Cugno and Ottoz (2004), Shapiro (2006), and Henry (2007). For a different perspective, see Kultti et al. (2006, 2007).

In the last decade, economists have also devoted a lot of attention to the issue of the allocation of prizes in contests; see e.g. Taylor (1995), Fullerton and McAfee (1999), Moldovanu and Sela (2001) and Che and Gale (2003). Innovation races differ from research contests in two ways. First, research contests end on a specified date, whereas an innovation race ends whenever the innovation is achieved. As a consequence, “in a contest the timing is fixed but the amount of innovative knowledge produced is variable, whereas in an innovation race the R&D output is fixed but the timing of innovation is variable” (Taylor, 1995, p. 875). Second, the prize in a contest is typically a sum of money, whereas in an innovation race it amounts to some degree of market power. While the social cost of raising a fixed sum of money is independent of its division, the deadweight losses caused by innovators’ market power generally depend on the degree of exclusivity they enjoy.
rer and Scotchmer, 2002). The patent system, that is to say, largely works on the winner-take-all principle, whereas secrets and copyrights are more permissive as protection mechanisms. Thus, by determining whether new technological fields are eligible to patent protection or else innovators must rely on secrecy or copyrights, policy can affect the division of the prize.

Even though the patent system is the prototypical winner-take-all regime, recently the introduction of an independent-invention defense into the patent law has itself become a topical policy issue. Moreover, the extent to which the patent system effectively exhibits the winner-take-all property depends on the “breadth” of patent protection, which is determined by various legal provisions and choices made by patent offices and the courts.

3 Prominent examples are software patents, business methods patents, and gene patents. All of these today exist in the US, whereas in Europe business methods and software patents are still controversial, and gene patents are much harder to uphold than in the US. Databases, by contrast, are protected by strong exclusive intellectual property rights in Europe, but not in the US.

4 Building on the economics literature cited in footnote 1, several law scholars have argued in favor of a general reduction of patent holders’ prerogatives with respect to infringing followers. To be sure, there are many difficulties in the practical implementation of an independent-invention defense, as argued at length by Blair and Cotter (2002). However, Vermont (2006) and Lemley (2007) counter that such difficulties may not be insurmountable. A related issue pertains to patent holders’ right to exclude prior inventors who have not patented their innovations. Bills introducing a first-inventor defense have been repeatedly put on the floor in the Congress over the last decade, including HR 1908 which passed the House of Representatives on September 7, 2007. It must be said, however, that an independent-invention defense would protect second inventors who duplicated patented innovations independently, whereas a first-inventor defense (or prior user right) would protect first inventors who concealed their innovations (perhaps because they did not believe them to be patentable) against the claims of second-inventor patentees: see Denicolò and Franzoni (2004).

5 If patents are narrow in scope, there may be plenty of room for duplicating the innovation lawfully; if instead patents have a broad coverage, the monopoly they create is more persistent: see Gilbert and Shapiro (1990), Klemperer (1990), Gallini (1992) and Denicolò (1996). Let us consider, for instance, the prevalence of so-called “me-too” drugs in pharmaceuticals. Arguably, among the causes of this phenomenon is the narrowness of pharmaceutical patents, which usually protect innovative molecules rather than the
It is important to note that technological effects, such as learning by doing or network externalities, can also affect the ability of an innovating firm to capture the whole market. Let us consider, for instance, an industry characterized by network externalities. Here, a short lead time may suffice to create an effective barrier to entry, even in the absence of strong, exclusive intellectual property rights: the strategy for the first inventor is to build an installed base large enough that late independent duplicators can enter the market only by designing compatible products. If inter-operability information is kept secret, such late entry may be very difficult, if not impossible, and the winner of the race effectively obtains the whole prize. But even in this case policy can play a role: if the first inventor were compelled to disclose inter-operability information, followers could achieve compatibility much more easily, and the winner would have to share the market with laggards.\(^6\)

Which regime is socially preferable? Generally, a switch from one regime to another will impact the size of the prize as well as its division, and so will change the overall incentives to innovate. To assess the final effect on social welfare one needs some estimate of the elasticity of the supply of inventions (Denicolò, 2007). Following the earlier literature, here we abstract from this issue and focus on the structure of the reward. To this end, we assume that a therapeutic process itself. Broader patents (e.g., patents covering therapeutic processes) might discourage duplicative research in the pharmaceutical sector and, conversely, might create an incentive to devote more resources to really innovative projects.

\(^6\)Recent litigation between the European Commission and Microsoft shows that such mandatory disclosure of inter-operability information is a very concrete risk for firms holding dominant positions in Europe.
move from one regime to another is accompanied by suitable policy changes that guarantee that the overall incentive to innovate is unaltered. From this perspective, the relevant question becomes, Which regime minimizes the social cost per unit of incentive to innovate it provides?\footnote{This approach was pioneered by Kaplow (1984), whose analysis led to the development of so-called “ratio tests,” which compare the ratio of deadweight losses to profits under various scenarios.}

To address this issue, the earlier literature has almost invariably equated the incentive to innovate with industry profits. Since in a winner-take-all system the innovator obtains all of its profits as a monopolist, whereas in a more permissive regime part of its overall reward will consist of oligopoly profits, the question of which regime is more efficient then boils down to the question of whether it is less distorting to raise an euro of profits under oligopoly or under monopoly. It turns out that oligopoly is relatively less distorting, except when the demand function is extremely convex. Hence the conclusion that the winner-take-all system is generally inefficient.

We contend that in an innovation race the incentive to innovate depends not only on industry profits, but also on the division of the profits between early and late innovators. Typically, the equilibrium R&D expenditure in an innovation race increases with both the prize to the winner (the \textit{profit incentive}) and the difference between the prize to the winner and to the losers (the \textit{competitive threat}) (Beath et al., 1989). One virtue of the winner-take-all system is that it maximizes the competitive threat by making the “consolation prize” in the innovation race vanish.
Another virtue of the winner-take-all system is that it prevents wasteful duplication of efforts. In a regime in which late inventors are allowed to share the market with the first inventor, there is an incentive to invest in R&D even after the first inventor has succeeded. If the late invention reproduces the original innovation identically, these duplication efforts are completely wasteful from the social viewpoint. Even if duplication results in differentiated products or devices, the incentive to engage in duplicative activity may be excessively high from a social viewpoint because of a business stealing effect, and so preventing such activity may be socially valuable (Gallini, 1992).

Our analysis accounts for these effects and combines them with those already identified in the literature. In our baseline model, two firms race for an innovation. Firms choose their R&D expenditures, which determine the expected date of successful completion of their R&D projects according to a Poisson discovery process. In the winner-take-all system, as soon as one firm innovates the other stops investing in R&D since it will be precluded from exploiting the innovation anyway. In the alternative, more permissive system, the laggard may keep investing to duplicate the innovation. When it also succeeds, the market becomes a duopoly. In this framework, we develop a new ratio test for the dominance of the winner-take-all system. As it turns out, this ratio test is passed under broad circumstances. In particular, the winner-take-all system is more likely to be optimal if product market competition is weak, the innovation race is intense, and duplication costs
Moving beyond the baseline model, we analyze several extensions allowing for licensing, the possibility that duplicative activity may not be completely wasteful, free entry, and the case in which R&D expenditures are an up-front payment rather than flow expenditures. In some extensions the permissive regime fares better than in the baseline model, but the results of the earlier literature are re-obtained only in extreme cases.

The remainder of the paper is organized as follows. Section 2 develops the baseline model. Section 3 derives our ratio test and argues that the winner-take-all system is preferable in a broad set of circumstances. Section 4 discusses the related literature and analyzes several extensions, showing how the ratio test changes as our assumptions are relaxed. Section 5 concludes the paper. All proofs are relegated to an Appendix.

2 Model outline

In this section we develop our baseline model. We first describe the innovation race, and then the downstream product market.

2.1 The innovation race

Two symmetric firms, A and B, race in continuous time to obtain an innovation. The timing of the innovation, the nature of which is exogenous, is a probabilistic function of the amount invested in R&D. For each firm $i$, the R&D effort determines the expected time of successful completion of
the R&D project according to a Poisson discovery process with a hazard rate equal to \( x_i \) (\( i = A, B \)). While exerting effort \( x_i \), firm \( i \) sustains a flow cost \( c(x_i) \). The projects of the two firms are independent; thus, the aggregate instantaneous probability of success is simply the sum of the individual probabilities.

In the winner-take-all regime, innovative activity ends as soon as the first inventor succeeds. In the permissive regime, by contrast, the innovation can be duplicated. Like innovation, duplication occurs according to a Poisson process whose hazard rate \( y \) depends on the laggard’s duplication effort. Let \( s(y) \) be the duplication cost function. One could imagine that \( s(.) = c(.) \), i.e., a firm’s innovative capabilities are not affected by its competitor’s success. However, to keep the model more general we allow these cost functions to differ, making only the standard regularity assumption that both are twice differentiable, increasing, and convex. That is, \( s'(y) > 0, c'(x_i) > 0, s''(y) > 0 \) and \( c''(x_i) > 0 \), for \( i = A, B \).

Let us consider the innovation race in more details. The R&D efforts \( x_i \) are determined as the Nash equilibrium of a simultaneous moves game between firm A and firm B.\(^9\) Firm \( i \)'s expected profit is

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\(^8\)In particular, it might be argued that the arrival of the innovation may provide useful information that facilitates the follower’s research, reducing the cost that must be borne to duplicate the innovation.

\(^9\)In principle, firms could choose different levels of R&D efforts at different points in time. Note, however, that the game is stationary in the sense that at each point in time, given no success to date, firms face exactly the same payoff functions as at time 0. Subgame perfection then ensures that equilibrium R&D efforts will be constant over time until one firm innovates: see Reinganum (1989) for details.
\[ \Pi_i(x_i, x_j) = \frac{x_i P_W + x_j P_L - c(x_i)}{x_i + x_j + r}, \]  
(1)

where \( r \) is the interest rate, \( x_i \) is own R&D effort, \( x_j \) is the competitor’s R&D effort, \( P_W \) is the reward to winner of the race and \( P_L \) to the loser, both to be determined presently. The best reply function of firm \( i \) is implicitly given by:

\[
\frac{P_W}{\text{profit incentive}} r + (P_W - P_L) x_j - c'(x_i) (x_i + x_j + r) + c(x_i) = 0. \tag{2}
\]

The first term in (2) is proportional to the profit incentive \( P_W \); this would be the only determinant of the incentive to innovate if firm \( i \) raced alone \((x_j = 0)\). The second term is proportional to the competitive threat \( P_W - P_L \), which, by contrast, captures the incentive to take over the competitor and turn from second to the first prize. The larger is the instantaneous probability that firm \( j \) succeeds, relegating firm \( i \) to the second position, the more important is the competitive threat.

Since firms are symmetric, in equilibrium \( x_A = x_B = x^* \) (Nti, 1999). In such a symmetric equilibrium,

\[
P_W r + (P_W - P_L) x^* - c'(x^*) (2x^* + r) + c(x^*) = 0 \tag{3}
\]

Assuming that the equilibrium is stable,\(^\text{10}\) we have (the proof of this and other results is in the Appendix):

\(^\text{10}\)Stability of the equilibrium requires the condition \( \left| \frac{dx_i}{dx_j} \right| < 1 \) for \( i,j = A,B \), \( i \neq j \) (see Lee and Wilde, 1980, Beath et al., 1989, and Nti, 1999).
Lemma 1 If the equilibrium is stable (i.e., if $\left| \frac{dx_i}{dx_j} \right| < 1$), equilibrium R&D efforts increase with the prize to the winner $P_W$ and decrease with the prize to the loser $P_L$:

$$\frac{\partial x^*}{\partial P_W} > 0 \text{ and } \frac{\partial x^*}{\partial P_L} < 0.$$ 

The consolation prize $P_L$ impacts negatively the incentive to innovate because it lowers the competitive threat.\textsuperscript{11}

2.2 The product market

We now turn to the product market where the innovation is used. To keep the analysis as general as possible, initially we do not make any specific assumption on the nature of the innovation, demand, and the competition in the product market. We only assume that when the first inventor innovates, it starts to earn a flow monopoly profit $\pi_m$. In the winner-take-all regime, at some finite date (e.g., when patent protection expires) the innovation falls into the public domain and competition drives profits to zero. In the permissive regime, by contrast, when the laggard also succeeds the industry becomes a duopoly and each firm obtains $\pi_d$ per period. For simplicity, we assume that in the permissive regime duopoly profits lasts indefinitely.\textsuperscript{12}

\textsuperscript{11}The Lemma takes the prizes $P_W$ and $P_L$ as given, but variables that affect the consolation prize may also indirectly impact the prize to the winner by speeding up or slowing down the duplication process, as we shall see in the next section.

\textsuperscript{12}One can easily extend our results to the case where duopoly profits may also end, because the innovation may be superseded by exogenous technical progress, or because it may leak in the public domain. All that matters is that duopoly profits in the permissive regime last, on average, more than monopoly profits in the winner-take-all regime: otherwise, it would be impossible to compare the two regimes holding the incentive to innovate constant.
Let \( v \) be the flow social value of the innovation once it is in the public domain. If the innovation is used exclusively by one firm, society suffers a flow monopoly deadweight loss \( \Delta_m \) and so the social benefit from the innovation is only \( v - \Delta_m \) per period. When both firms practice the innovation and the product market is a duopoly, the deadweight loss is generally lower, \( \Delta_d \leq \Delta_m \).

Apart from any dynamics associated with duplication or patent expiration, the environment is stationary, with a constant discount rate equal to \( r \). For the time being, we assume that the first inventor does not license the innovative knowledge to the other firm; this assumption will be relaxed below.

## 3 The ratio test

We now derive the equilibrium in the two policy regimes and develop the welfare comparison. We start with the permissive regime in which independent inventors are allowed to practice the innovation and compete with the first inventor.

### 3.1 The permissive regime

How are the rewards \( P_W \) and \( P_L \) determined? In the permissive regime, the loser earns discounted duopoly profits upon duplication, less duplication costs:

\[
P_L^{HD} = \max_y \left[ \frac{y \frac{\pi_d}{y} - s(y)}{y + r} \right],
\]

(4)
where IID stands for independent-invention defense. Let $y^* = \arg \max_y \left[ \frac{y^d - s(y)}{y + r} \right]$ denote the optimal duplication effort, which, by implicit differentiation, increases with $\pi_d$. Define $q = \frac{y^*}{y^* + r} < 1$ as the “discounting adjusted” probability of duplication: with a Poisson duplication process, the innovation will eventually be duplicated with probability one, but since there is discounting, a delayed duplication counts less than instant duplication. Then, we can re-write (4) as:

$$
P_{LIID} = q \frac{\pi_d}{r} - \left(1 - q\right) \frac{s(y^*)}{r}.
$$

(5)

Turning to the winner’s reward, this is given by

$$
P_{WIID} = \frac{\pi_m + y^* \frac{\pi_d}{r}}{y^* + r} = (1 - q) \frac{\pi_m}{r} + q \frac{\pi_d}{r}.
$$

(6)

The first line of (6) says that the winner earns monopoly profits until the loser duplicates, which happens with instantaneous probability $y^*$. After duplication, both firms obtain duopoly profits $\pi_d$. The second line shows that the innovator’s reward can be regarded as a weighted average of monopoly and duopoly discounted profits, with weights reflecting the “discounting adjusted” probability of duplication. Clearly, $P_{WIID} > P_{L IID}$; more precisely, the competitive threat is:

$$
P_{WIID} - P_{L IID} = (1 - q) \left[ \frac{\pi_m}{r} + \frac{s(y^*)}{r} \right].
$$

(7)
3.2 The winner-take-all regime

In the winner-take-all regime, by definition the prize for the second innovator is zero \( P_{L}^{WTA} = 0 \). The prize for the first innovator, assuming that it holds a monopoly for a time period of length \( T \), is

\[
P_{W}^{WTA} = \int_{0}^{T} \pi_{m} e^{-rt} dt = \tau \frac{\pi_{m}}{r},
\]

where \( \tau \equiv 1 - e^{-rT} \) is the normalized length of the first inventor’s monopoly.

3.3 Welfare

We use the standard definition of social welfare in a partial equilibrium framework, namely the sum of consumer and producer surplus. In the winner-take-all regime, expected social welfare is

\[
W^{WTA} = 2x^{s} \left[ \tau \frac{v - \Delta m}{r} + (1 - \tau) \frac{v}{r} \right] - 2c(x^{s}) \frac{2x^{s}}{2x^{s} + r},
\]

since monopoly ends when the innovation falls into the public domain. The term inside square brackets is the discounted social value of the innovation, accounting for the monopoly distortions that prevail for a period of discounted length \( \tau \). The coefficient \( \frac{2x^{s} \tau}{2x^{s} + r} \) is the “discounting adjusted” probability that the innovation is achieved (the hazard rate is \( 2x^{s} \) since two firms are racing but who innovates is a matter of indifference for society), and the last term is the total discounted R&D expenditure.

In the permissive regime, things are slightly more complicated. Expected
social welfare is

$$W^{IID} = \frac{2x^*}{2x^* + r} \left[ \frac{v - \Delta_m - s(y^*) + y^* \frac{v - \Delta_d}{r}}{y^* + r} \right] - \frac{2c(x^*)}{2x^* + r}$$

$$= \frac{2x^*}{2x^* + r} \left[ (1 - q) \frac{v - \Delta_m - s(y^*)}{r} + q \frac{v - \Delta_d}{r} \right] + \frac{2c(x^*)}{2x^* + r}. \tag{10}$$

The term inside square brackets reflects the fact that now monopoly prevails only until the innovation is duplicated, which occurs with an instantaneous probability $y^*$. After duplication, the market becomes a duopoly and society suffers a lower deadweight loss, $\Delta_d$, forever. As long as duplication has not occurred yet, however, the laggard also sustains the duplication cost $s(y^*)$.

The second line of (10) expresses these effects in terms of the discounting adjusted probability of duplication $q$.

### 3.4 Comparison

Generally speaking, the comparison between $W^{WTA}$ and $W^{IID}$ is complicated by the fact that the equilibrium R&D effort $x^*$ may differ across regimes. Since we are interested in ascertaining which rule, winner-take-all or independent-invention defense, provides incentives to innovate more efficiently, we develop the welfare comparison assuming that the incentives to innovate, and hence the R&D efforts, are the same in both regimes. Specifically, we assume that the level of protection in the winner-take-all regime, $\tau$, is adjusted so as to yield the same level of equilibrium R&D expenditure as in the permissive regime.
To proceed, note that the consolation prize can be rewritten as

$$P_L^{ID} = q^\frac{\pi d}{r}(1 - \Sigma),$$  \hspace{1cm} (11)

where

$$\Sigma = \frac{s(y^*)}{\frac{\pi d}{r}},$$  \hspace{1cm} (12)

can be interpreted as a relative index of the costliness of duplication. It represents the share of expected discounted duopoly profits, $q^\frac{\pi d}{r}$, which is absorbed by duplication costs. $\Sigma$ ranges from 0 (costless duplication)\(^{13}\) to 1 (duplication is so costly as to absorb all expected revenues).

With this definition, we can state:

**Proposition 1 (The Ratio Test)** The winner-take-all regime is preferable to the permissive regime in terms of social welfare if

$$\frac{\Delta d}{\pi_d} + \Sigma \left(1 - p \frac{\Delta m}{\pi_m}\right) \geq (1 - p) \frac{\Delta m}{\pi_m},$$  \hspace{1cm} (13)

where $p = \frac{s^*}{x^* + r}$ is the stand-alone, discounting-adjusted probability of success in the innovation race.

When is the ratio test more likely to be passed? Is the optimality of the winner-take-all principle a mere theoretical curiosum, as suggested by the earlier literature, or is it a realistic possibility? Unfortunately, inequality (13) looks complicated. Moreover, some of the variables that appear in (13),\(^{13}\)

\(^{13}\)The case $\Sigma = 0$ arises, for instance, when duplication is completely costless up to a certain level $y^*$ and is infinitely costly beyond that level. Then, duplicative activity will always occur at rate $y^*$ and $\Sigma = 0$.  

15
such as $p$ and $\Sigma$, are endogenous, and others, such as $\frac{\Delta x}{\pi_d}$ and $\frac{\Delta m}{\pi_m}$, depend on the shape of the demand curve and the intensity of competition. To proceed, one has to make further assumptions.

### 3.5 Linear demand

Let us consider, for instance, the case of a product innovation with a linear demand function. After suitable normalization, the demand function can be written as $P = 1 - Q$, where $P$ is price and $Q = q_A + q_B$ is total output. The unit production cost of the new product is normalized to 0, and demand and costs are assumed to be stationary.

At the monopoly equilibrium, we have $\pi_m = \frac{1}{4}$ and $\frac{\Delta m}{\pi_m} = \frac{1}{2}$. As for duopoly, the equilibrium depends on the intensity of product market competition. Using a conjectural variations reduced-form model where the conjectural variations parameter $\rho$ varies from $-1$ to $1$, we can allow for any degree of the intensity of competition between collusion ($\rho = -1$) and Bertrand competition ($\rho = 1$). The case $\rho = 0$ corresponds to the Cournot equilibrium. Standard calculations show that $Q = \frac{2}{3-\rho}$, $\pi_d = \frac{1}{(3-\rho)^2}$ and

\[ \frac{\Delta x}{\pi_d} \geq \frac{\Delta m}{\pi_m}, \]

Intuitively, the speed of innovation, $x^*$, and of duplication, $y^*$, depend positively on the magnitude of prospective profits and negatively on the difficulty of achieving the innovation, as captured by shift parameters in the cost functions $c(x)$ and $s(y)$. Using quadratic specifications such as $c(x) = \alpha x^2$ and $s(y) = \beta y^2$, for instance, one could easily re-express the ratio test (13) in terms of exogenous variables only, but the outcome would hardly be more transparent or easier to interpret.

One reason why this is a useful benchmark is that in this case the ratio test arrived at by the earlier literature, namely $\frac{\Delta x}{\pi_d} \geq \frac{\Delta m}{\pi_m}$, is never passed: see section 4 below.

In this example, the flow social value of the innovation is $v = \frac{1}{2}$.

In the limiting case of Bertrand competition ($\rho = 1$), with homogeneous products the duplicator’s profits vanish and so there is no investment in duplication anyway. The two regimes are then indistinguishable.

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17. In the limiting case of Bertrand competition ($\rho = 1$), with homogeneous products the duplicator’s profits vanish and so there is no investment in duplication anyway. The two regimes are then indistinguishable.
\[ \Delta \pi = \frac{1}{2} (1 - \rho). \]

Inserting the above formulas into (13), the condition for the winner-take-all system to be preferable becomes

\[ \rho \leq 2\Sigma + p (1 - \Sigma ). \]  

(14)

Inspection of (14) reveals that the winner-take-all system is more likely to be optimal:

(i) the lower the intensity of product market competition, \( \rho \);

(ii) the greater the costliness of duplication, \( \Sigma \);

(iii) the greater the intensity of the innovation race, \( p \) (and hence the lower the interest rate, \( r \), and the lower the expected waiting time to discovery, \( \frac{1}{2p} \)).

These comparative statics results are illustrated in Figure 1. The winner-take-all system is preferable below the upward sloping parallel lines. All these lines have equation (14), taken as an equality. Two such lines, corresponding to \( \Sigma = 0 \) and \( \Sigma = \frac{1}{2} \), are depicted. When \( \Sigma \geq \frac{1}{2} \), the winner-take-all system dominates the permissive system for all values of the intensity of competition and of the intensity of the race. The winner-take-all system is also preferable when the intensity of competition is Cournot or weaker (\( \rho \leq 0 \)), irrespective of the costliness of duplication and the speed of the innovation race.

Next, let us focus on the case where \( \Sigma < \frac{1}{2} \) and competition is tougher than Cournot (\( \rho > 0 \)). Figure 1 shows that when \( p \) is large, the winner-take-
Figure 1: The winner-take-all system is preferable below the upward sloping straight lines, which collapse to the north-east vertex of the rectangle when $\Sigma \geq \frac{1}{2}$.

All system can be optimal even if competition is substantially more intense that Cournot competition – even with zero duplication costs. To assess the likelihood that the winner-take-all system is still preferable, it is therefore important to get a sense of the magnitude of $p$. The variable $p = \frac{x^*}{x^* + r}$ depends on the real interest rate $r$ and the instantaneous probability of discovery $x^*$. Taking 10% as an upper bound for the real interest rate and 2% as a lower bound for the instantaneous probability of success,\(^{18}\) one gets a lower bound for $p$ of 0.2. In highly innovative industries, however, $p$ can

\(^{18}\)Note that $\frac{1}{x^*}$ is the expected time to discovery, and so a value of $x^* = 2\%$ means that a firm innovates every 50 years. Although $x^*$ is ostensibly an endogenous variable, it must be said that private firms rarely engage in research when the chances of success are so remote.
be significantly larger. With $p = 0.5$, for instance, the ratio test is passed when $\rho \leq \frac{1}{2}$, irrespective of the magnitude of duplication costs, or when $\Sigma \geq \frac{1}{3}$, irrespective of the intensity of product market competition. Thus, the baseline model suggests that it is quite likely that the winner-take-all principle may be preferable in highly innovative industries. Conversely, the permissive regime is more likely to be optimal in mature industries, where product market competition is strong and innovation is a relatively rare occurrence ($x^* \text{ low}$).

3.6 Creative duplication

So far we have assumed that products are homogeneous and so duplication is entirely wasteful. If the innovations targeted by the two firms are not identical, however, duplication may increase product variety and hence may be socially valuable. To allow for this possibility, we now suppose that the duplicator supplies a product that is different from that supplied by the innovator. More precisely, we assume that the two products are horizontally differentiated. To address the issue of “creative” duplication in its purest form, we assume that the first inventor cannot seek to discover the other variety and is restricted to produce only its own. This means that there are now two sources of monopoly deadweight losses: high prices and low variety.19

19Clearly, the assumption that the first inventor can produce only its own variety biases the comparison against the winner-take-all system: if this assumption were relaxed, the appropriate ratio test would be easier to pass.
Following the classic formulation of Singh and Vives (1984), let the inverse demand functions be:

\[ P_A = 1 - q_A - \theta q_B, \quad \text{and} \quad P_B = 1 - q_B - \theta q_A. \]  

(15)

where \( \theta \in [0, 1] \) is a parameter that captures the degree of substitutability between products. The two goods are independent for \( \theta = 0 \), and are perfect substitutes for \( \theta = 1 \) (the case considered above). Again, production costs are normalized to 0.

When both products are supplied and prices are set equal to marginal costs, we have \( q_A = q_B = \frac{1}{1+\theta} \) and so social surplus is \( v = \frac{1}{1+\theta} \). Prices, however, will generally exceed marginal costs, reducing the benefit society obtains from the innovations. Using again a conjectural variations reduced-form solution, we obtain

\[ \Delta_d = \frac{1 - \rho \theta}{1 + \theta}, \]  

(16)

where the conjectural variations parameter \( \rho \) now ranges from \( \rho = -1 \) (conclusion) to \( \rho = \theta \) (Bertrand competition).\(^{20}\)

As for monopoly, accounting for both sources of deadweight losses (high

\(^{20}\)Standard calculations show that \( q_A = q_B = \frac{1}{1+\theta} \). Duopoly equilibrium profits are \( \pi_d = \frac{1 - \rho d}{(2 + \theta - \rho \theta)^2} \), while consumer surplus can be easily calculated as \( CS_d = \frac{1 + \theta}{(2 + \theta - \rho \theta)^2} \). Subtracting the sum of producer and consumer surplus under duopoly from the first best benchmark, we get

\[ \Delta_d = \frac{(1 - \rho \theta)^2}{(1 + \theta)(2 + \theta - \rho \theta)^2}, \]  

whence equation (16) immediately follows.
prices and low variety), now we have:\(^{21}\)

\[
\frac{\Delta_m}{\pi_m} = \frac{5 - 3\theta}{2(1 + \theta)}.
\]  

(17)

For simplicity, let us focus on the case \(\Sigma = 0\). Inserting (16), (17) and \(\Sigma = 0\) into (13), it turns out that the winner-take-all system is preferable when

\[
\rho < \frac{2 - (5 - 3\theta)(1 - p)}{2\theta}.
\]  

(18)

As in the case of homogeneous products, less competition in the product market and a faster race favor the winner-take-all system. What is the effect of a change in the degree of product differentiation? Greater product differentiation affects the comparison between the two regimes in two opposing ways. On the one hand, it makes duplication more valuable – duplication enlarges the variety of products available to consumers. On the other hand, for any given level of the conjectural variations parameter, greater product differentiation relaxes the competition between the first and the second inventor, thereby increasing the ratio \(\frac{\Delta_d}{\pi_d}\). Which effect prevails depends on the intensity of the innovation race, as can be easily confirmed by differentiating the right-hand side of (18):

**Proposition 2** With creative duplication, an increase in the degree of product differentiation militates in favor of the winner-take-all system when the race is not very intense, i.e., \(p < \frac{3}{5}\), otherwise, it militates against the winner-take-all system.

\(^{21}\)More precisely, monopoly profit is still \(\pi_m = \frac{1}{4}\), but now the monopoly deadweight loss is \(\Delta_m = \frac{1}{\pi_m} - \frac{3}{\pi}\).
Figure 2 depicts the region where the winner-take-all system is preferable as a function of $p$ and $\theta$ for values of the intensity of competition $\rho$ corresponding to collusion, Cournot competition, and Bertrand competition, respectively. Remarkably, the winner-take-all system can be desirable even if products are completely independent ($\theta = 0$), provided that the innovation race is sufficiently intense: to be precise, the condition is $p > \frac{3}{5}$. The intuitive reason is that with $\theta = 0$, in a permissive system each firm would conduct the research as a monopolist. The winner-take-all system introduces competition in research among the two firms, speeding up the innovation race. Although our assumptions make the social costs of such a policy especially large (one product is never developed), the overall effect on social welfare can still be positive.

4 Discussion and extensions

In this section we compare our ratio test (13) with that developed in the earlier literature. To better clarify why we arrive at different results, we also analyze several extensions of the baseline model.

4.1 Literature review

There are two main differences between our baseline model and the analysis developed in the earlier literature. First, the earlier literature assumes that there is no duplicative activity in equilibrium. Second, it equates the incen-
The winner-take-all system is preferable above the curves corresponding to various degrees of the intensity of competition.

tive to innovate to industry profits. With these assumptions, the question of which regime is more efficient boils down to the question of whether it is less distorting to raise an euro of profits under oligopoly or under monopoly. Accordingly, the winner-take-all system is preferable only if:

\[
\frac{\Delta_d}{2\pi d} \geq \frac{\Delta_m}{\pi m}.
\]  

(19)

This ratio test is very difficult, if not impossible, to pass. With homogenous products, for instance, inequality (19) is always violated with linear or constant elasticity demand, and can be met only when the demand function is extremely convex (Shapiro, 2006).

To compare our ratio test with the traditional one, let us start from the
case $\Sigma = 0$ since the role of the costliness of duplication is better considered in isolation. Setting $\Sigma = 0$, (13) becomes:

$$\frac{1}{(1 - p)} \frac{\Delta_d}{\pi_d} \geq \frac{\Delta_m}{\pi_m}.$$  

(20)

Comparing (20) with (19) we note two differences; both make our test easier to pass. First, the ratio between duopoly deadweight losses and duopoly profits involves individual, not industry profits. That is, $\Delta_d$ is divided by $\pi_d$, not $2\pi_d$. This effect alone doubles the term representing the social cost of the permissive regime: if product market competition is not too intense, $\frac{\Delta_d}{\pi_d}$ can be greater than $\frac{\Delta_m}{\pi_m}$ even with a linear, or moderately concave, demand curve. Second, in the ratio test (20) the left-hand side is further increased, since $\frac{\Delta_d}{\pi_d}$ is multiplied by a factor greater than one, $\frac{1}{1 - p}$, which increases with the speed of the innovation race $p$.

These differences have natural economic explanations. With zero duplication costs, the total reward $P_W + P_L$ equals discounted industry profits in both regimes. The important point, however, is that in our model the incentives to innovate do not depend only on the aggregate prize $P_W + P_L$, but also on its division between the innovator and the duplicator. This has two consequences. First, only the prize to the winner $P_W$ has a positive effect on the incentive to innovate, and in the permissive regime the prize to the winner includes only individual duopoly profits (i.e., those accruing to the first inventor). This explains why the ratio on the left-hand side of
(20) is $\frac{\Delta \pi}{2\pi d}$ rather than $\frac{\Delta \pi}{2\pi d}$. Second, those duopoly profits that accrue to the duplicator affect $P_L$, not $P_W$; and do so negatively. That is, not only duopoly profits accruing to the loser of the race do not contribute positively to the incentive to innovate, but they actually lower such an incentive by increasing the consolation prize. This negative effect on the competitive threat is captured by the factor $\frac{1}{1-p}$ on the left-hand side of (20). This term increases with $p$ because, as we already know, the importance of the competitive threat depends positively on the intensity of the innovation race. Summarizing, duopoly profits provide low-powered incentives to innovate as compared to monopoly profits because only duopoly profits accruing to the first inventor stimulate innovation, whereas those accruing to the duplicator actually slow it down.

Turning to the general case with $\Sigma > 0$, notice that the left hand side of (13), which measures the social cost of stimulating innovative activity in the permissive regime, is augmented by a term proportional to the index of the costliness of duplication, $\Sigma$. Typically the additional term is positive. Thus, duplication costs, which are borne only in the permissive regime, further favors the winner-take-all system.

How are the assumptions underlying the traditional ratio test justified? However, inspection of (13) reveals that the additional term on the left-hand side is actually be negative if $p\Delta_m > \pi_m$. However, this condition can only hold in rather special circumstances when products are homogeneous (although it is not extreme with differentiated products). The intuitive explanation is that while duplication costs are socially wasteful, they also lower the consolation prize and so increase the incentive to innovate for any given level of monopoly and duopoly profits. Therefore, duplication costs have two opposing effects on social welfare in the permissive regime.
There have been various approaches. First, La Manna, MacLeod and De Meza (1989) posit *up-front R&D costs with free entry and constant returns to scale in research.* In their model, firms sink R&D investments at the beginning of the race and do not bear any R&D cost subsequently. This immediately implies that no R&D expenditure is *per se* duplicative in nature. Moreover, it implies that when deciding how much to invest in R&D at the outset, research firms take into account that they might get any prize (first or second) with some positive probability. Since *ex ante* both positions are open to each firm, the expected return from R&D investment equals expected industry profits. Therefore, industry profits uniquely determine the incentives to innovate and hence the probability of success.

Another approach that arrives at the traditional ratio test (19) uses a *timeless, reduced-form model of the innovation race* in which the two firms may succeed simultaneously (a zero-probability event in continuous time): see Shapiro (2006). If both firms succeed, in the permissive system the market becomes a duopoly, whereas in the winner-take-all system Nature chooses randomly which firm is granted a monopoly. In this model, the incentive to innovate is a linear combination of the prize from being the sole innovator (i.e., monopoly profits with probability one in both regimes) and that from inventing *ex equo* (i.e., duopoly profits in the permissive regime and a fifty-fifty chance of getting monopoly profits in the winner-take-all

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23La Manna et al. assume that the timing of the innovation is stochastic, and use more general distributional assumptions than the Poisson process.
It follows that for the incentive to innovate to be the same across regimes, discounted duopoly profits must equal half discounted monopoly profits; or, in other words, discounted industry profits under monopoly and duopoly must be equal.

Still another approach assumes that in the permissive regime the first inventor licenses preemptively the innovation to potential duplicators (Maurer and Scotchmer, 2002). If there is free entry of potential duplicators and the innovator has all the bargaining power, in the permissive regime the first inventor can appropriate industry profits fully. It follows that the incentive to innovate is again determined by aggregate industry profits. Moreover, even though duplication is potentially profitable, no duplicative activity will occur in equilibrium and so duplication costs will not be actually borne.

That different sets of specific assumptions may lead to the traditional ratio test (19) is remarkable, but does not necessarily mean that the conclusion is a robust one. In the remainder of this section we extend our baseline model to account for some of the effects analyzed in the earlier literature, such as licensing, free entry, and the case in which R&D expenditures are an up-front payment rather than flow expenditures. In some extensions the permissive regime fares better than in the baseline model, but the dominance result of the earlier literature is re-obtained only in extreme cases.

\(^{24}\)In a timeless framework, the weights in the linear combination depend only on the rival’s effort, not on own effort. In other words, own R&D expenditures affect the probability of success, but not the relative chances of being the sole winner or succeeding \textit{ex equo}.

\(^{25}\)Cugno and Ottoz (2004) extend the Maurer and Scotchmer model to the case in which there is only one potential duplicator.
4.2 Licensing

As we have seen above, Maurer and Scotchmer (2002) argue that the permissive regime is especially attractive in the presence of licensing. If the first inventor can immediately license the innovation to its competitor, this saves duplication costs and increases the reward to the winner of the race. Both effects favor the permissive regime: the former increases social welfare directly, the latter raises the incentive to innovate for any given level of monopoly and duopoly deadweight losses.26

This suggests that the appropriate ratio test will become more difficult to pass when licensing is feasible. While it confirms this intuition, the analysis below highlights two caveats. First, both additional effects are related to the magnitude of duplication costs, and hence both vanish when duplication costs are negligible. When $\Sigma = 0$, that is to say, the appropriate test remains (20) even with licensing. Second, the traditional test is obtained only in the extreme case in which duplication costs absorb expected duopoly profits fully and the first inventor has all the bargaining power in the licensing negotiation.

As is well known, licensing agreements can often be crafted anti-competitively. To proceed, let us therefore consider which type of agreements should be permitted in our framework. Since monopoly profits generally exceed aggregate duopoly profits, the first inventor has an incentive to pay the laggard simply

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26 Generally speaking, however, various obstacles may impede technology licensing, especially when the innovative technological knowledge is a trade secret.
to stay out of the market. Similarly, the first inventor has and incentive to let the licensee practice the innovation only after some distant future date, or to set an exorbitantly large royalty rate, compensating the potential duplicator with a side payment (i.e., a negative fixed fee). However, these licensing clauses would be anti-competitive and as such could be struck down by antitrust authorities. Here, we assume that antitrust authorities perfectly enforce the principle stated in the 1995 Antitrust Guidelines for the Licensing of IP, which define as potentially anti-competitive “those restraints that have adverse effects on competition that would have occurred in the absence of the license”. As a result, licensing contracts must be drafted so that the licensee can start to compete not later than the expected time of duplication, and duopoly prices under licensing cannot be greater than those that would prevail under duplication. In other words, licensing takes place against an up-front payment (no running royalties), and the licensee will enter the market at a date \( \ell \) such that

\[
\frac{1-e^{-\tau \ell}}{r} = \frac{r}{y^*+y^*+r} = 1 - q.
\]

In view of these assumptions, the potential duplicator will have to choose between parallel development and licensing, knowing that his decision will neither affect the expected starting date of production – which in normalized time units equals \( 1 - q \) – nor the magnitude of duopoly profits – \( \pi_d \). The only effect of licensing is to save duplication costs \((1 - q) \frac{\delta(y^*)}{r}\), which therefore represent the bargaining surplus. In the negotiation between the first inventor and the potential duplicator, the first inventor will get a share

\[27\text{See Gilbert (1995) for an excellent discussion of the guidelines.}\]
of the bargaining surplus and the laggard the remaining share \((1 - \gamma)\), where \(\gamma\) reflects the winner’s bargaining power. Under these assumptions, we get:

**Proposition 3** Under competition-neutral licensing, the winner-take-all regime is preferable to the permissive regime in terms of social welfare if

\[
\frac{\Delta_d}{\pi_d} \geq [(1 - p) + \gamma (1 + p) \Sigma] \frac{\Delta_m}{\pi_m}.
\]

(21)

When \(\Sigma = 0\) or \(\gamma = 0\), the ratio test is unaffected by licensing and, more generally, the effect of licensing is small if \(\gamma \Sigma\) is small. The main difference with respect to the baseline model is that an increase in the index of costliness of duplication now unambiguously militates against the winner-take-all system. However, the traditional test is obtained only under the extreme assumptions \(\Sigma = 1\) (expected duopoly profits are entirely absorbed by duplication costs) and \(\gamma = 1\) (the first inventor has all the bargaining power in the licensing game). The intuition is that with \(\gamma \Sigma = 1\), the consolation prize vanishes and the first innovator obtains aggregate duopoly profits; therefore, the incentives to innovate is given by industry profits and so (19) applies.

With linear demand, homogeneous products, and symmetric bargaining power (\(\gamma = \frac{1}{2}\)), inequality (21) reduces to:

\[
\rho \leq p - \frac{1}{2} \Sigma (1 + p).
\]

(22)

Following Maurer and Scotchmer (2002), let us focus on the case of Cournot competition. In this case, the independent-invention defense regime is opti-
mal only if $\Sigma > \frac{2p}{1+p}$. With $p = 0.5$, for instance, this inequality is met only if $\Sigma > \frac{2}{3}$. This means that even in the presence of licensing, in highly innovative industries duplication costs must be relatively large for an independent invention defense to be socially desirable.\cite{28}

### 4.3 Free entry

There are two main differences between our baseline model and that of La Manna et al. (1989). First, so far we have assumed that only two firms are engaged in research, while they assume that there is free entry. Second, we have assumed that R&D expenditures are a flow cost sustained until the innovation is achieved, whereas they assume that R&D expenditures are an up-front payment sunk at the beginning of the race. In what follows we argue that the traditional ratio test (19) is appropriate only when both of the assumptions made by La Manna et al. (1989) hold. With either assumption, but not both, the appropriate ratio test is easier to pass than the traditional one, and possibly even than the ratio test obtained in the baseline model, i.e., inequality (13).

We first relax the assumption that only two firms can conduct the research. Instead, we analyze the case where the equilibrium number of firms is determined by a free entry condition both in the innovation and the du-

\footnotetext[28]{In our model, the innovator licenses the technology to a unique firm. With multiple would-be licensees, an additional cost of a permissive regime is that it may render exclusive technology transfers infeasible, since the licensor could secretly breach the exclusive deal and sell the technology to a different firm. In turn, the latter could claim that it developed the innovation independently. See Bhattacharya and Guriev (2006) for an analysis of this problem.}
plication stage. Although the assumption of free entry may seem extreme, the analysis of this case may shed light on the likely effects of an increase in the number of firms participating in the race.\footnote{For simplicity, we assume that there is only one duplication stage. The analysis, however, can be easily extended to the case with an arbitrarily large number of duplication stages (i.e., where the innovation can be duplicated an arbitrarily large number of times).}

**Proposition 4** With free entry in innovation and duplication, the winner-take-all regime is preferable to the permissive regime in terms of social welfare if

\[
\frac{\Delta_d}{\pi_d} + (1-q)\frac{\pi_m}{\pi_d} + q \geq \frac{\Delta_m}{\pi_m}
\]

(23)

The ratio test (23) can actually be even easier to pass than (13). With homogeneous products and a linear demand curve, for instance, inequality (23) is always satisfied, irrespective of the magnitude of duplication costs and the intensity of the innovation race.

The intuition behind (23) is that under free entry expected profits are brought down to zero at all stages. This implies that there is no second prize, and so the competitive threat equals the profit incentive not only in the winner-take-all regime but also in the permissive regime. However, duplication costs are now largest since they absorb all expected profits upon duplication, and the effect of these duplication costs on the welfare comparison is unambiguously adverse, since they no longer have any positive effect on the competitive threat.\footnote{See footnote 22 above.}
4.4 R&D costs committed up front

Finally, we consider a model where R&D expenditures are committed up front by the racing firms. As before, such R&D expenditures determine the hazard rates of the Poisson processes describing the arrival of the innovation.\footnote{This model is similar to that developed by Henry (2007), who analyzes a “marginal” move from the winner-take-all regime to the independent-invention defense regime. Starting from a pure winner-take-all regime, such a marginal move is socially desirable under weaker conditions than those under which a drastic move, like that considered here, is.}

Following La Manna et al. (1989), assume that R&D expenditure is a linear function of R&D efforts. With up-front R&D costs the expected profit of firm $i = A, B$ is:

$$
\Pi_i = \frac{x_i P_W + x_j P_L}{x_i + x_j + r} - cx_i,
$$

where $c$ is the unit R&D cost. The comparison between the winner-take-all and the permissive regime is now modified in two ways. First, there are no duplication costs, since duplication occurs as a result of the up-front R&D expenditures borne by the loser. Second, the consolation prize effect loses some of its strength, since now even the second prize stimulates the up-front investment in research: the more a firm invests at the outset, the less it will have to wait to become duopolist upon losing the race. We get:

**Proposition 5** When R&D costs are committed up front, the winner-take-all regime is preferable to the permissive regime in terms of social welfare.

31 This model is similar to that developed by Henry (2007), who analyzes a “marginal” move from the winner-take-all regime to the independent-invention defense regime. Starting from a pure winner-take-all regime, such a marginal move is socially desirable under weaker conditions than those under which a drastic move, like that considered here, is.
if
\[
\frac{\Delta_d}{\pi_d} \geq (1 - p)(2 + p) \frac{\Delta_m}{\pi_m}.
\] (25)

The function \((1 - p)(2 + p)\) decreases monotonically from 2 \((p = 0)\) to 0 \((p = 1)\). This confirms that the winner-take-all system is more desirable when the innovation race is intense and so \(p\) is large. The traditional test (19) is obtained only for \(p = 0\), however. As soon as the industry does not stagnate indefinitely in an equilibrium with no research, the winner-take-all system is more desirable than inequality (19) would suggest.

5 Conclusion

In this paper, we have argued that the winner-take-all system is optimal in a broad set of circumstances. Our conclusion challenges the claim, which is gaining popularity among economists and law scholars, that a more permissive system is generally preferable. This emerging consensus rests upon the analysis of a series of rather specific models, all of which have in common the property that the incentive to innovate is proportional to industry profits and there are no duplication costs. We have argued that in an innovation race the incentive to innovate depends crucially on the division of industry profits among early and late innovators, and duplication is socially costly. With these additional effects at work, the winner-take-all system becomes relatively more desirable. In particular, our analysis points out that the winner-take-all system is preferable in highly innovative industries, whereas
the permissive system is more likely to be optimal in mature industries, where product market competition is strong and innovation is a relatively rare occurrence.
References


Appendix

The proof of Lemma 1 and Propositions 2, 4, 5, and 6 follows.

Proof of Lemma 1. Implicit differentiation of (2) gives

\[
\frac{dx_i}{dx_j} = - \frac{P_W - P_L - c'(x_i)}{c''(x_i)(x_i + x_j + r)}
\]

and so the stability condition \(|\frac{dx_i}{dx_j}| < 1\) becomes

\[
P_W - P_L - c'(x_i) < c''(x_i)(x_i + x_j + r)
\]

Implicit differentiation of (3) gives

\[
\frac{dx^*}{dP_W} = - \frac{x^* + r}{P_W - P_L - c'(x^*) - c''(x^*)(2x^* + r)}
\]

and

\[
\frac{dx^*}{dP_L} = \frac{x^*}{P_W - P_L - c'(x^*) - c''(x^*)(2x^* + r)}
\]

By the stability condition, the denominator of these expressions is negative, whence the result follows.

Proof of Proposition 2. From the equilibrium condition (3), we immediately see that for the two regimes to yield the same innovation efforts we must have:

\[
P_W^{WTA}(r + x^*) = P_W^{IID}r + (P_W^{IID} - P_L^{IID})x^*,
\]

that is:

\[
P_W^{WTA} = P_W^{IID} - pP_L^{IID},
\]

where \(p = \frac{x^*}{x^* + r}\) is the stand-alone, discounting-adjusted probability of success. The prize to the winner has to be lower in the winner-take-all regime than in the permissive regime because in the latter the consolation prize reduces the incentives to innovate. The difference between the first prizes in the two regimes, \(P_W^{WTA}\) and \(P_W^{IID}\), increases with the size of the consolation prize and also with \(p\). Intuitively, the more intense is the race to innovate, the stronger is the adverse incentive effect of the consolation prize, and
hence the more important is the competitive threat. Using (5), (6) and (8),
expression (A1) can be re-written as follows:
\[
\tau \frac{\pi_m}{r} = (1 - q) \frac{\pi_m}{r} + q \frac{\pi_d}{r} - p \left[ q \frac{\pi_d}{r} - (1 - q) \frac{s(y^*)}{r} \right].
\]
Thus, in order to have the same incentives to innovate, we must have
\[
\tau = (1 - q) + q(1 - p) \frac{\pi_d}{\pi_m} + (1 - q) p \frac{s(y^*)}{\pi_m}.
\]
Note that \( \tau \) must exceed the lead time \((1 - q)\) in the permissive regime by an amount that increases with duopoly profits and duplication costs.

Using (9) and (10), one immediately sees that when the R&D effort is the same across regimes, social welfare is greater under the winner-take-all regime if
\[
\frac{v}{r} - \frac{\Delta m}{r} + (1 - \tau) \frac{v}{r} \geq (1 - q) \frac{v - \Delta m - s(y^*)}{r} + q \frac{v - \Delta d}{r}.
\]
Inserting into this expression the value of \( \tau \) that makes the incentives to innovate equal across regimes, one gets that the winner-take-all regime dominates in terms of social welfare if:
\[
v - \left[ (1 - q) + q(1 - p) \frac{\pi_d}{\pi_m} + (1 - q) p \frac{s(y^*)}{\pi_m} \right] \Delta m \geq (1 - q) [v - \Delta m - s(y^*)] + q (v - \Delta d).
\]
Simplifying and rearranging we get
\[
\frac{\Delta d}{\pi_d} + \frac{(1 - q) s(y^*)}{q \pi_d} \geq \frac{\Delta m}{\pi_m} \left[ (1 - p) + p \frac{(1 - q) s(y^*)}{q \pi_d} \right].
\]
Noting that \( \Sigma = \frac{(1 - q) s(y^*)}{q \pi_d} \) and rearranging we get
\[
\frac{\Delta d}{\pi_d} + \Sigma \left( 1 - p \frac{\Delta m}{\pi_m} \right) \geq (1 - p) \frac{\Delta m}{\pi_m}
\]
Proof of Proposition 4. The winner-take-all regime is unaffected by the possibility of licensing, since licensing will never take place anyway. In the permissive regime, instead, we now have
\[
P_{W}^{WID} = (1 - q) \frac{\pi_m}{r} + q \frac{\pi_d}{r} + \gamma (1 - q) \frac{s(y^*)}{r}, \quad \text{(A2)}
\]
40
and

\[ P_{LD}^{IIID} = q \frac{\pi_d}{r} - \gamma (1 - q) \frac{s(y^*)}{r} \]

\[ = q \frac{\pi_d}{r} (1 - \gamma \Sigma). \quad \text{(A3)} \]

Now, a rise in duplication costs increases both the profit incentive and the competitive threat. As a consequence, the greater are duplication costs, the greater is the value of the duration of exclusivity \( \tau \) necessary to induce the same incentives to innovate in the two regimes.

Using (8), (A2) and (A3), expression (A1) can be re-written as follows:

\[ \tau \frac{\pi_m}{r} = (1 - q) \frac{\pi_m}{r} + q \frac{\pi_d}{r} + \gamma (1 - q) \frac{s(y^*)}{r} - p \left[ q \frac{\pi_d}{r} - \gamma (1 - q) \frac{s(y^*)}{r} \right]. \]

Thus, in order to have the same incentives to innovate, we must have

\[ \tau = (1 - q) + q (1 - p) \frac{\pi_d}{\pi_m} + \gamma (1 - q) (1 + p) \frac{s(y^*)}{\pi_m}. \quad \text{(A4)} \]

Turning to the welfare comparison, notice that duplication costs are no longer borne in the permissive regime. Social welfare therefore becomes:

\[ W_{IIID} = \frac{2x^*}{2x^* + r} \left[ (1 - q) \frac{v - \Delta_m}{r} + q \frac{v - \Delta_d}{r} \right] - \frac{2c(x^*)}{2x^* + r}. \]

In the winner-take-all regime, social welfare is still given by (9). Inserting (A4) into (9) and comparing to \( W_{IIID} \), one obtains that the winner-take-all regime dominates in terms of social welfare if:

\[ v - \left[ (1 - q) + q (1 - p) \frac{\pi_d}{\pi_m} + \gamma (1 - q) (1 + p) \frac{s(y^*)}{\pi_m} \right] \Delta_m \geq (1 - q) [v - \Delta_m] + q (v - \Delta_d). \]

Simplifying and rearranging the result follows.\( \blacksquare \)

**Proof of Proposition 5.** Suppose that there are \( n \) identical active firms. The profit function of a representative firm \( i \) is therefore

\[ \Pi_i (x_i, X_{-i}) = \frac{x_i}{x_i + X_{-i}} P_W + X_{-i} P_{e} - c (x_i), \]

where \( X_{-i} = \sum_{j \neq i} x_j. \)
With free entry into the innovation race, the equilibrium number of active firms \( n^* \) and the individual research efforts \( x^* \) are determined simultaneously as the solution to the system comprising the first-order condition

\[
WP \cdot r + (PW - PL) X_{-i} - c'(x_i) (x_i + X_{-i} + r) + c(x_i) = 0
\]

and the zero-profit condition

\[
x_i \cdot PW + X_{-i}PL - c(x_i) = 0.
\]

As we have seen above, the consolation prize \( PL \) vanishes in the winner-take-all system. Free entry at the duplication stage, however, make the consolation prize vanish in the permissive regime as well: the firm that loses the race gets zero reward, as at the duplication stage free entry brings the expected profit down to zero. As a consequence, the equilibrium conditions become:

\[
P_{W}r - c'(x_i) (nx_i + r) + c(x_i) = 0, \quad \text{(A5)}
\]

and

\[
x_i \cdot PW - c(x_i) = 0 \quad \text{(A6)}
\]

Clearly, for the two systems to yield the same incentives to innovate, they must guarantee the same prize to the winner of the race.

In the winner-take-all system the prize to the winner is again given by equation (8). In the permissive regime, however, things are now slightly different. The aggregate duplication effort is determined by conditions similar to (A5) and (A6), i.e.,

\[
P_{W}r - s'(y_i) (n_dy_i + r) + s(y_i) = 0, \quad \text{(A7)}
\]

where \( n_d \) is the number of firms that participate in the duplication race, and

\[
y_i \cdot PW - s(y_i) = 0, \quad \text{(A8)}
\]

where \( Y^* = n_d^*y^* \) denotes aggregate duplication effort, where \( n_d^* \) and \( y^* \) are the solution to the above system. Then, the prize to the winner in the permissive regime is again given by (6), i.e.,

\[
P_{W}^{III} = (1 - q) \frac{\Sigma_{i} \pi_i}{r} + q \frac{\Sigma_{i} \pi_i}{r}, \quad \text{where}
\]
now \( q = \frac{Y^*}{1 + \tau} \). By the zero-profit condition (A8), aggregate duplication costs are
\[
\frac{n_s^* [s (y^*) + F_d]}{Y^* + r} = q \left[ (1 - q) \frac{\pi_m}{r} + q \frac{\pi_d}{r} \right].
\]

Thus, social welfare in the permissive regime is
\[
W^{IID} = \frac{2x^*}{2x^* + r} \left\{ (1 - q) \frac{v - \Delta_m}{r} + q \left[ \frac{v - \Delta_d}{r} - (1 - q) \frac{\pi_m}{r} - q \frac{\pi_d}{r} \right] \right\} - \frac{2c(x^*)}{2x^* + r}
\]
whereas in the winner-take-all regime it continues to be given by (9). Condition \( P_{W}^{WTA} = P_{W}^{IID} \) can now be re-written as follows:
\[
\tau \frac{\pi_m}{r} = (1 - q) \frac{\pi_m}{r} + q \frac{\pi_d}{r}.
\]
Thus, in order to have the same incentives to innovate, we must have
\[
\tau = (1 - q) + q \frac{\pi_d}{\pi_m}.
\]

Using (9) and (A9), one immediately sees that when the R&D effort is the same across regimes, social welfare is greater under the winner-take-all regime if
\[
\tau \frac{v - \Delta_m}{r} + (1 - \tau) \frac{v}{r} \geq (1 - q) \frac{v - \Delta_m}{r} + q \left[ \frac{v - \Delta_d}{r} - (1 - q) \frac{\pi_m}{r} - q \frac{\pi_d}{r} \right].
\]
Inserting into this expression the value of \( \tau \) that makes the incentives to innovate indeed equal across regimes, one gets that the winner-take-all regime dominates in terms of social welfare if:
\[
v - \left[ (1 - q) + q \frac{\pi_d}{\pi_m} \right] \Delta_m \geq (1 - q) (v - \Delta_m) + q [v - \Delta_d - (1 - q) \pi_m - q \pi_d].
\]
Simplifying and rearranging we get the result. \( \blacksquare \)

Proof of Proposition 6. With up-front R&D costs, \( P_{W}^{WTA} \) continues to be given by (8) and \( P_{L}^{WTA} = 0 \) by definition. In the permissive regime, however, the prizes to the winner and to the loser are now different than in the baseline model, as the probability of duplication no longer depends on imitation effort. Instead, the instantaneous probability that the innovation
is duplicated is now the same as the instantaneous probability that a firm innovates for the first time. It follows that:

\[ P_{IID}^{i,W} = \frac{r}{x_j + r} \frac{\pi_m}{x_j + r} + \frac{x_j}{x_j + r} \frac{\pi_d}{r} \]

and

\[ P_{IID}^{i,L} = \frac{x_i}{x_i + r} \frac{\pi_d}{r}. \]

Note that \( P_{IID}^{i,W} \) does not depend on \( i \)'s own effort, but \( P_{IID}^{i,L} \) does.

Accordingly, the equation of firm \( i \)'s reaction curve in the permissive regime is now

\[
\left( P_{IID}^{i,W} + \frac{P_{IID}^{i,W} - P_{PID}^{i,L}}{x_j + r} \right) \frac{r}{x_i(x_i + r)} - c(x_i + x_j + r)^2 = 0.
\]

(A10)

Comparing (A10) to (2), the additional term on the left-hand side reflects the fact that by increasing its research effort, a firm now increases the value of the second prize to it, as it gets sooner to duopoly profits in case it does not win the race.

In a symmetric equilibrium, we have:

\[
\pi_m + \frac{x^*(2r + 3x^*)}{(r + x^*)^2} \pi_d = c(2x^* + r)^2
\]

in the permissive regime, and

\[
\tau \frac{x^* + r}{r} \pi_m = c(2x^* + r)^2
\]

in the winner-take-all regime. In order to get the same incentive to innovate, it must be:

\[
\tau \frac{x^* + r}{r} \pi_m = \pi_m + \frac{x^*(2r + 3x^*)}{(r + x^*)^2} \pi_d,
\]

which gives:

\[
\tau = \frac{r}{x + r} + \frac{rx(2r + 3x)}{(x + r)^3} \frac{\pi_d}{\pi_m} = (1 - p) \left[ 1 + p(2 + p) \frac{\pi_d}{\pi_m} \right].
\]
Social welfare in the permissive regime is

\[ W^{IID} = \frac{2x^*}{2x^* + r} \left[ \frac{v - \Delta_m + x^* \frac{v - \Delta_d}{r}}{x^* + r} \right] - 2cx^* \]

whereas under the winner-take-all regime it is

\[ W^{WTA} = \frac{2x^*}{2x^* + r} \left[ \frac{\tau v - \Delta_m}{r} + (1 - \tau) \frac{v}{r} \right] - \frac{2cx^*}{2x^* + r}. \]

Thus, social welfare will be greater in the winner-take-all regime iff

\[ \frac{\tau v - \Delta_m}{r} + (1 - \tau) \frac{v}{r} \geq \frac{v - \Delta_m + x^* \frac{v - \Delta_d}{r}}{x^* + r} \]

or

\[ \frac{\Delta_m + x^* \frac{\Delta_d}{r}}{x^* + r} \geq \frac{\tau \Delta_m}{r} \]

If the two regimes provide the same incentives to innovate, this inequality reduces to

\[ (1 - p)\Delta_m + p\Delta_d \geq (1 - p) \left[ 1 + p(2 + p) \frac{\pi_d}{\pi_m} \right] \Delta_m \]

which simplifies to

\[ \frac{\Delta_d}{\pi_d} \geq (1 - p)(2 + p) \frac{\Delta_m}{\pi_m}. \]