Patterns in a Complex System:
An Empirical Study of Valuation in Business Bankruptcy Cases[*]

Bernard Trujillo[**]

Abstract

This Article applies complex systems research methods to explore the characteristics of the bankruptcy legal system, presenting the results of an empirical study of twenty years of bankruptcy court valuation doctrine in business cramdown cases. These data provide solid descriptions of how courts exercise their discretion in valuing firms and assets.

This Article accomplishes two objectives: First, using scientific methodology, this Article explains the content of bankruptcy valuation doctrine. Second, this Article uses doctrine as a variable to explore system dynamics that govern the processes of change over time.

Significant findings include (i) courts tend to "split the difference" in valuations much less frequently than expected; (ii) parties' valuation standards tend to be close together; (iii) bankruptcy courts' valuation approach is substantially influenced by whether the valuation includes a calculation for the time value of money; (iv) there seems to be some geographic distribution of courts' acceptance of valuation models, with Southern states more likely to accept "soft" valuation models, and non-Southern states more likely to accept "hard" valuation models; and (v) the evidence offers preliminary support for the hypothesis that bankruptcy system content may self-organize according to some complex deterministic dynamics.

[**] Assistant Professor, University of Wisconsin Law School. A.B., Princeton University, 1988; J.D., Yale Law School, 1992.

Patterns in a Complex System:
An Empirical Study of Valuation in Business Bankruptcy Cases[*]

Bernard Trujillo[**]

[**] Assistant Professor, University of Wisconsin Law School. A.B., Princeton University, 1988; J.D., Yale Law School, 1992.

Patterns in a Complex System:
An Empirical Study of Valuation in Business Bankruptcy Cases[*]

Bernard Trujillo[**]

List of Figures

Party Success

Valuation Standards

Valuation Models

Self-Organization

Appendix: Methodology

[**] Assistant Professor, University of Wisconsin Law School. A.B., Princeton University, 1988; J.D., Yale Law School, 1992.

Patterns in a Complex System:
An Empirical Study of Valuation in Business Bankruptcy Cases[*]

Bernard Trujillo[**]

Bankruptcy law is a complex, adaptive system.[1] Despite the presence of a complicated statute and a lengthy set of procedural rules, the bankruptcy system leaves much of the most important work up to the discretion of actors "on the ground."[2] Observing how bankruptcy actors operate in a discretion-rich environment may offer important information about the structure and dynamics of legal systems generally.

Valuation in bankruptcy provides a useful vantage point from which we may observe this complex system in action. In a wide variety of circumstances, litigants argue

---

[**] Assistant Professor, University of Wisconsin Law School. A.B., Princeton University, 1988; J.D., Yale Law School, 1992. Thanks to Bill Clune, Howie Erlanger, Bert Kritzer, Bob Lawless, Lynn LoPucki, Stewart Macaulay, Clint Sprott, Martin Shapiro, Bill Whitford, and an anonymous referee. The empirical work in this Article was funded by grants from the National Conference of Bankruptcy Judges, the American Bankruptcy Institute, and the University of Wisconsin at Madison Graduate School. I gratefully acknowledge the research assistance of Mike Morgalla, the statistical assistance of David Merrill and Menesh Patel, and the coding assistance of Chris Rechliz and Jonathan Schuster. Special thanks to Victoria Trujillo.

[1] By "complex", I do not mean that the system has many moving parts, although bankruptcy certainly does. My sense of the term "complex" is borrowed from research in physics and other natural sciences. A system is "complex", in this sense, when it operates far from equilibrium and is characterized by disturbances or through-put. Mathematically, the relationships between variables in a complex system tend to be nonlinear, the arithmetic is discrete (rather than continuous) and the geometry is fractal (rather than Euclidian). Most significantly for this project, the dynamics of complex systems are frequently observed to be "self-organizing" rather than entropic. Complex systems in nature include the ecological (e.g. a river bed) and the biological (e.g. a cell). The literature on complex systems is extensive, and these ideas are spreading from the study of natural systems to the study of social systems. *See, e.g.*, PER BAK, HOW NATURE WORKS: THE SCIENCE OF SELF-ORGANIZED CRITICALITY 183-192 (1996); JULIEN CLINTON SPROTT, CHAOS AND TIME-SERIES ANALYSIS (2003); BENOIT B. MANDELBROT & RICHARD L. HUDSON, THE (MIS)BEHAVIOR OF MARKETS: A FRACTAL VIEW OF RISK, RUIN, AND REWARD (2004); Steven N. Durlauf, *What Should Policymakers Know about Economic Complexity?*, 21 WASH. Q. 157 (1998); J.C. Sprott, *Competition with Evolution in Ecology and Finance*, 325 PHYS. LETT. A 329 (2004); M.E. J. Newman, *Power Laws, Pareto Distributions and Zipf's Law*, arXiv:cond-mat/04122004 v.1 (01Dec2004).

[2] *See* Bernard Trujillo, *Self-Organizing Legal Systems: Precedent and Variation in Bankruptcy*, 2004 UTAH L. REV. 483, 490-500, 509-512 (2004) (discussing institutional conditions fostering bankruptcy court discretion) [hereinafter Trujillo, *Self-Organizing Legal Systems*].

for, and bankruptcy courts assign, monetary values for firms and assets. But the law of valuation in bankruptcy is radically under-determined by hierarchical forces exogenous to the bankruptcy system, such as congressional and appellate court directives. Bankruptcy actors are left substantially on their own.

One significant area of bankruptcy valuation is the business "cramdown" proceeding. In a typical business cramdown, a bankrupt corporation (the "Debtor") has filed a plan of reorganization to which at least one class of creditors has objected. Despite the objection, the Debtor requests that the bankruptcy court confirm the plan anyway, in effect "cramming the plan down" the throats of the objecting creditors.[3] The U.S. Bankruptcy Code[4] (the "Code") allows such a cramdown, but only if judge concludes that the plan gives the objecting creditors full value for their claims.[5]

For example, suppose that Debtor owes Creditor $1 million. Debtor files a reorganization plan promising to pay Creditor 10% equity ownership in the reorganized firm. Creditor, preferring cash, objects to the plan. In the cramdown proceeding, Debtor argues that its plan gives full value for Creditor's claim because the value of the reorganized firm will exceed $10 million (and the Creditor gets 10% of that, or $1 million plus interest, the full value of its claim). Creditor responds that the firm will be worth less than $10 million, and so the plan will not give Creditor full value for its claim. The bankruptcy court must, in effect, find a value for the reorganized firm.

Projecting the value of a not-yet-existing firm is a rather complicated matter. Yet the Bankruptcy Code and appellate court decisions provide very little guidance for this

---

[3] *See generally* Kenneth Klee, *All You Ever Wanted to Know About Cramdown Under the New Bankruptcy Code*, 53 AM. BANKR. L. J. 133 (1979).
[4] 11 U.S.C. § 1129(b) (2005).
[5] *See* 11 U.S.C. § 1129(b)(2). Full value means the value of the creditor's claim "as of the effective date the plan," which amounts to a principal sum plus interest.

crucially important task.[6] Given the difficulty of the task and the lack of hierarchical ordering, one might expect valuation determinations to either (i) show no particular pattern, with decisions all over the map; or (ii) follow a "split the difference" pattern, with judges typically selecting a number part-way between the Debtor's number and the Creditor's. The data, surprisingly, do not confirm these expectations.

I have concluded an empirical study of twenty years of bankruptcy court doctrine, covering all corporate cramdown cases decided from 1979, the first year after the enactment of the Code, through 1998. While lawyers and legal scholars have offered theoretical and normative accounts of valuation in bankruptcy,[7] there remains a need for simple description. What do bankruptcy courts do when asked to value? We find that, despite the lack of exogenous ordering, stable and navigable patterns of valuation have emerged. Moreover, courts have not shown a tendency to "split the difference" between the parties' numbers.

Part I of the Article discusses the study of doctrine and considers some criteria for studying doctrine empirically. Part I also lays out the design of this study. Part II presents the statistical results of this study and some interpretations of those results. Part

[6] *See, e.g.*, 11 U.S.C. § 506(a) ("such value shall be determined in light of the purpose of the valuation and of the proposed disposition or use of such property"); Associates Commercial Corp. V. Rash, 520 U.S. 953 (199&) (failing to provide clear standards for valuing principal); Till v. SCS Credit Corp., 124 S.Ct. 1951 (2004) (failing to provide clear standard for valuing interest rate).

[7] Works treating bankruptcy valuation include John B. Butler, III, *Valuation of Secured Claims under 11 U.S.C. 506(a)*, 89 COMMERCIAL L. J. 342 (1984); Chaim J. Fortgang & Thomas Moers Mayer, *Valuation in Bankruptcy*, 32 U.C.L.A. L. REV. 1061 (1985); James F. Queenan, Jr., *Standards for Valuation of Security Interests in Chapter 11*, 92 COMMERCIAL L. J. 18 (1987); Kaaran E. Thomas, *Valuation of Assets in Bankruptcy Proceedings: Emerging Issues*, 51 MONTANA L. REV. 126 (1990); David Gray Carlson, *Secured Creditors and the Eely Character of Bankruptcy Valuations*, 41 AM. U. L. REV. 63 (1991); Frederick M. Luper & Kenneth M. Richards, *Valuation of Property Issues in Bankruptcy*, 98 COMMERCIAL L. J. 35 (1993); Robert M. Lawless & Stephen P. Ferris, *Economics and the Rhetoric of Valuation*, 5 J. BANKR. L. & PRACT., 3 (1995); John Collen, *Real Estate Valuation Techniques*, 8 J. BANKR. L. & PRACT. 135 (1999). Regarding an overview of valuation in various legal contexts, see Jay W. Eisenhofer & John L. Reed, *Valuation Litigation*, 22 DEL. J. CORP. L. 37 (1997). *See also, infra* Part I.A.

III summarizes the major findings and concludes with some questions and directions for future research.

I. STUDYING DOCTRINE EMPIRICALLY

Three objectives for studying doctrine[8] are: (i) we want to know what the doctrine is; (ii) we want to learn about the behavior of persons or institutions, and we think doctrine offers some explanation of that behavior; and (iii) we want to explain what a legal system is and how it works, and we use doctrine as a variable to test propositions regarding a legal system's structure and dynamics.

In the first type of study ("what is the doctrine?"), researchers regard doctrine as the dependent variable, i.e. the thing-to-be-explained. Researchers consider independent variables, both "formal" (e.g. jurisdiction, time, judge, type of case, party structure, etc.) and "realist" (e.g. politics, economic incentives, race, class, gender, social norms, etc.) [9] in the hope of explaining, and perhaps predicting or ultimately reforming, legal doctrine.

In the second type of research ("what is the behavior of persons and institutions?"), the thing-to-be-explained is behavior or, as the proponents of this type of research like to say, "what is really happening in the world". Doctrine serves as one among many independent variables that might shed light on behavioral hypotheses.

---

[8] Doctrine is just that body of formally defined rules of a legal system. These rules are officially promulgated by the authorized agents of the system, and include statutes, court opinions, and administrative regulations. A common feature of doctrine is that it is written, published, and available to the public. As a simplification, I will be taking the published judicial opinion as the central case of "doctrine" for the purposes of this Article.

[9] *See* Marc Galanter, *The Portable Soc 2; or, What to Do until the Doctrine Comes*, in GENERAL EDUCATION IN THE SOCIAL SCIENCES: CENTENNIAL REFLECTIONS ON THE COLLEGE OF THE UNIVERSITY OF CHICAGO 246, 252 (ed. John J. MacAloon, 1992) [hereinafter Galanter, *Portable Soc 2*] ("...I make no distinction between believers in the model of rules and instrumentalists; nor between formalist believers in autonomous rule development and their realist critics. Thus, where some observers detect a radical break, I see a striking continuity").

In the third type of doctrinal study ("what are the dynamics of a legal system?"), researchers again treat doctrine as an independent variable that may help explain a targeted "thing-to-be-explained." That thing, however, is not some behavioral pattern, but rather the structure and dynamics of the legal system itself.

This project intends no contribution to the second type of doctrinal research. The data in this study (i.e. published bankruptcy court cases) cannot credibly falsify any behavioral hypotheses – we cannot, on the basis of this work, draw conclusions about what is "really happening" in the area of bankruptcy valuation (e.g. what "backroom deals" are struck, what disputes are not brought to formal adjudication, etc.). This project does, however, hope to contribute to both the first and third types of doctrinal study.

A. Explaining Doctrine

One objective of this research is to understand more about the doctrine of bankruptcy valuation. Two tendencies mark my pursuit of this objective: First, this Article suspends discussion of any normative theory[10] of value in favor of straight description. Much of doctrinal research is at least as interested in what the doctrine *should* be as in what the doctrine *is*.[11] By contrast, this Article will try to refrain from

[10] For useful general theorizations of valuation, *see e.g.*, Cass R. Sunstein, Daniel Kahneman, & David Schkade, *Assessing Punitive Damages (with Notes on Cognition and Valuation in Law)*, 107 YALE L.J. 2071 (1998); Margaret Jane Radin, *Compensation and Commensurability*, 43 DUKE L.J. 56 (1993); Cass R. Sunstein, *Incommensurability and Valuation in Law*, 92 MICH. L. REV. 779 (1994). In the bankruptcy area, two of the more useful articles theorizing valuation are Robert M. Lawless & Stephen P. Ferris, *Economics and the Rhetoric of Valuation*, 5 J. BANKR. L. & PRACT. 3 (1995), and David Gray Carlson, *Secured Creditors and the Eely Character of Bankruptcy Valuations*, 41 AM. U. L. REV. 63 (1991).

[11] The identification of hierarchy with the good of order is what John Griffiths would call an "ideological" rather than an "empirical" position. *See* John Griffiths, *What is Legal Pluralism?*, 24 J. LEGAL PLURALISM 1, 3 (1986). Ideology, according to Griffiths, is a "mixture of assertions about how the world ought to be and *a priori* assumptions about how it actually and even necessarily is...." Empirical (or "descriptive", or "scientific") approaches to the study of law, on the other hand, take the phenomenon as the primary object of study, without entertaining questions about how the phenomenon ought to be or what it must "necessarily" be. *See also* Galanter, *Portable Soc 2* at 253 (listing eight propositions of conventional legal

imposing, *ex ante*, any over-arching theory upon the data.[12] This Article revolves around the data, and the data show only what bankruptcy courts have memorialized about their valuation decisions.[13]

Second, this Article attempts to apply scientific methodology to doctrinal study by treating *doctrine as a quantitative unit*. This methodological approach works particularly well in studying bankruptcy, which is an almost ideal laboratory for the empirical study of legal systems. In part because of the existence of over 330 specialty bankruptcy courts, the U.S. bankruptcy system emits a tremendous amount of visible data relative to many other legal systems. A system that emits large amounts of easily detectible data can be studied scientifically and quantitatively, without resort to anecdote or idiosyncrasy, and without the biases of expectation or experience.[14] The Article tries to

---

studies, and noting that the listed propositions "have a dual, composite character, fusing both descriptive and normative. They are thought to state what is normal and typical in legal systems – to reflect the inherent and proper shape of legal reality. This fusion of factual and normative assertion...establishes them as ideological statements – statements about what a legal system ... ought to be like").

[12] *See Commissioner v. Marshall*, 125 F.2d 943, 946 (2d Cir. 1942, Frank, J.) ("The fallacy in that argument stems largely from lack of recognition of the eely character of the word 'value.' It is a bewitching word which, for years, has disturbed mental peace and caused numerous useless debates. Perhaps it would be better for the peace of men's minds if the word were abolished. Reams of good paper and volumes of good ink have been wasted by those who have tried to give it a constant and precise meaning."). Heeding the pleas of Judge Jerome Frank, this Article attempts description rather than theorization.

[13] Specifically, these data show the variation of forms that exist in the doctrine of cramdown valuations. We see, for example, the frequency of different types of valuation evidence (*see infra* Part II.B) and the significance that the presence of financial elements have on valuation (*see infra* Part II.E.4). More generally, these data shed light on the entry and proliferation of certain forms in the bankruptcy legal system (*see infra* Part II.D). *See infra* n. 19 (discussing the definition of "forms" for purposes of this research).

[14] The methodological approach of this research follows the inspiration of the "evidence-based medicine" ("EBM") approach to the practice of medicine. EBM promotes the scientific study of the efficacy of medical treatments. Such research allows medical professionals to prescribe treatment, not because of tradition or path-dependence, but because of a well-informed belief that the treatment will be effective. See generally DAVID L. SACKETT ET AL., EVIDENCE-BASED MEDICINE: HOW TO PRACTICE AND TEACH EBM (1998). A frequently updated database of evidence is available for practitioners of EBM at http://www.clinicalevidence.com/ceweb/conditions/index.jsp I believe that an "evidence-based law" approach to doctrine can move us past anecdote and unexamined path dependence, and perhaps begin a systematization and verification of knowledge about legal doctrine.

make transparent all choices regarding selection and interpretation of data,[15] and uses statistics to interpret the sample data drawn from this complex system-in-motion.[16]

B. Using Doctrine to Explain Legal Systems

The second objective of this research is to use doctrine to explain the dynamics of the bankruptcy legal system. At the threshold, we must establish whether doctrine is actually probative as evidence of a legal system's dynamics.

Doctrine (understood here as the published opinions of bankruptcy courts) offers only uncertain support for some types of propositions. If, for example, we want to describe features of the entire population of bankruptcy disputes, then studying doctrine gives little or no help.[17]

Suppose, on the other hand, that we are interested in describing how the bankruptcy system *works* – how networks composed of various actors (e.g. judges,

---

[15] Often, doctrinal research bases its conclusions on a set of data that is both highly selective and rather small relative to the total amount of available data (e.g. limiting the data set to a few appellate court opinions), typically without explaining its criteria for data selection and interpretation. By scientific standards, both the selection and review of data are idiosyncratic, and the idiosyncrasies are not expressly revealed. As a consequence, such research cannot be replicated by other scholars.

[16] There is an interesting similarity between the rise of statistical method in the physical sciences and the (roughly contemporaneous) rise of impressionist method in the visual arts. Both are ways of representing systems in motion, of knowing those systems by sampling them. Henri Matisse wrote an essay, entitled "Exactitude is Not Truth", to accompany four sketches of his own face in a mirror. Matisse makes the point that the four renditions are all very different, but unmistakably represent the same subject. They are, if you will, four samples of the same complex system. *See* Henri Matisse, *Exactitude is Not Truth, 1947*, in THEORIES OF MODERN ART at 137-39 (ed. Herschel B. Chipp) (U. Cal. Press: 1968) ("These drawings seem to me to sum up observations that I have been making for many years on the characteristics of a drawing, characteristics that do not depend on the exact copying of natural forms, nor on the patient assembling of exact details, but on the profound feeling of the artist before the objects which he has chosen....The leaves of a tree – of a fig tree particularly – the great difference of form that exists among them does not keep them from being united by a common quality. Fig leaves, whatever fantastic shapes they assume, are always unmistakably fig leaves....Thus there is an inherent truth which must be disengaged from the outward appearance of the object to be represented. This is the only truth that matters....*L'exactitude n'est pas la vèrité*"). I thank Bill Clune for bringing this remarkable source to my attention.

[17] *See, e.g.*, Peter Siegelman & John J. Donohue III, *Studying the Iceberg from its Tip: A Comparison of Published and Unpublished Employment Discrimination Cases*, 24 LAW & SOC'Y REV. 1133 (1990).

litigators, clients, potential clients, etc.)[18] receive and transmit forms,[19] and how some forms acquire normative status while others do not. If we want to describe the dynamics of a legal system, then published opinions are singularly probative as data, because published opinions are an important "communications device"[20] that travel among the elements of the system, like proteins in a cell.[21] Judges intend their published opinions not only as a communication to the parties in the particular case that gave rise to the

---

[18] See Trujillo, *Self-Organizing Legal Systems* at 520-21 (presenting players list for U.S. bankruptcy legal system).

[19] All forms are simply data that make up a given legal system. See, e.g., Trujillo, *Self-Organizing Legal Systems* at 486, n. 1 (defining "forms" for the purposes of this research, as ranging from the formal and institutional, to the informal and ephemeral).

[20] Martin Shapiro's work is fundamental in establishing the communications aspect of judicial opinions. Shapiro observed:

> In recent years political science has focused not on judicial opinions but on judicial decisions (who won and lost) as keys to understanding judicial attitudes. In legal theory...there has been much attention to judicial opinions as justifications or explanations...[b]ut nearly all the commentators...treat the opinion *in vacuo*, asking whether it meets certain general standards and thus turning the problem into one of logic or philosophy. Somehow we ignored the fact that appellate courts and the lawyers that serve them spend an overwhelming proportion of their energies in communicating with one another, and that the judicial opinion ... is the principal mode of communication.... It would seem appropriate, therefore, to examine the opinion-writing activity of courts in the context of communication.

Martin Shapiro, *Toward a Theory of Stare Decisis*, 1 J. LEGAL STUD. 125, 134 (1972). *See generally* MALCOLM M. FEELEY & EDWARD L. RUBIN, JUDICIAL POLICY MAKING AND THE MODERN STATE: HOW THE COURTS REFORMED AMERICA'S PRISONS 211-52 (1998). Feeley and Rubin write that:

> Probably the most common, and certainly the most studied, means of [judges communicating ideas to one another] is a judicial opinion....In writing the opinion, the judge sets down her public account of the way that her personal beliefs can be integrated with her perception of existing legal doctrine. That is a thought process of its own, born of the need to act consistently with one's role expectations. It represents the judge's real feelings in her institutional role, and the question of whether it represents her real feelings about the integrative effort, apart from the personal attitudes that have already been incorporated in the integration process, is too metaphysical to be productive.

Edward Rubin & Malcolm Feeley, *Creating Legal Doctrine*, 69 S. CAL. L. REV. 1989, 2021 [hereinafter Rubin & Feeley, *Creating Legal Doctrine*]. *See also* MARC GALANTER, LAW AND SOCIETY IN MODERN INDIA 3-11 (1989) (on using doctrine in the field of India studies).

[21] Other communications devices in legal systems would include contracts, *see* Stewart Macaulay, *Non-Contractual Relations in Business: A Preliminary Study*, 28 AM. SOC. REV. 55, 65 ("a fairly detailed contract can serve as a communication device within a large corporation"), speeches and articles, *see* Rubin & Feeley, *Creating Legal Doctrine*, at 2011-12 ("other ways for judges to communicate...including attending judicial conferences, making personal contacts, serving on multimember panels, and stating their views in law reviews and similar publications. In a concrete, anti-formalist account of judging, such means of communication count just as much as the traditional means of written opinions.").

opinion, but also as a communication to other judges,[22] other lawyers, other litigants, and other actual and potential participants in the legal system.

The selection process that precedes publication makes doctrine even more probative of system dynamics. Judges determine when to write an opinion down instead of delivering it orally from the bench. Judges also determine which, from among their written opinions, they will send to the publisher.[23] Published opinions thus represent a sub-set of all written opinions, which in turn represents a sub-set of all formal adjudications, which in turn represents a sub-set of all dispute resolutions. Behavioral scholars point to this winnowing process to bolster their claim that published opinions are freighted with judicial bias and are thus very poor evidence of what is "really happening." This claim is correct if the target of research is the entire pool of disputes to be resolved. But if the purpose of the research is to study inter-network and intra-network communications signals, then this winnowing process improves the sample rather than biases it. Bankruptcy valuation doctrine is useful evidence for studying system dynamics precisely because it is not an unvarnished report of an objective event ("what was the found value of the asset?"), but rather a subjective account of an objective event ("what did the judge say about finding the value of the asset?").[24]

---

[22] Rubin & Feeley, *Creating Legal Doctrine*, at 2016 ("Judges do not meet together very often....For the most part, they communicate [with each other] through their written opinions....").

[23] Criteria for publishing opinions varies with the individual judge. A judge may publish opinions on issues that are particularly important, difficult, or novel. A judge may publish opinions on pedestrian, oft-repeated claims as a way of signaling her tendencies to the bar and encouraging settlement. A judge may publish (and write with extreme care) opinions in cases she feels will be appealed (perhaps because of the parties' makeup). All of these reasons "improve the sample" in that they represent the judges' *ex officio* speech to a particular audience (i.e. future litigators in her own district, fellow judges in other districts, appellate judges).

[24] Studying doctrine to understand system dynamics is a "content-neutral" use of doctrine, in the sense that we will learn about the system's dynamics regardless of the doctrine's content (that is, regardless of whether a given court, say, adopted or rejected a particular discounted cash flow model). Such research is designed not necessarily to predict future system content, but rather to describe the processes that produce future system content. *See, e.g.*, RAYMOND T. NIMMER, THE NATURE OF SYSTEM CHANGE: REFORM

If doctrinal data can explain something about system dynamics, then the data of this study may illuminate at least two features of the bankruptcy legal system: (i) the effects of discretion; and (ii) the tendency to self-organization.

Bankruptcy law contains a high amount of judicial discretion.[25] Scholars and policymakers often paint judicial discretion as a problem, in part because the outcomes of a discretion-rich system are (at least in theory) uncontrolled and subject to caprice. Yet despite the theoretical possibility that "anything can happen" in a discretion-rich space, the data in this Article show that, in fact, just "anything" does not happen. Rather, very constrained and navigable patterns have emerged in the area of bankruptcy valuation.[26] This project hopes to make transparent how bankruptcy courts use their discretion in the area of valuation. As a more general aspiration, one may hope that by measuring the uses and effects of judicial discretion in a specific area, legal actors and commentators may begin to understand and become comfortable with discretion as an omnipresent (and unremarkable) feature of complex legal systems.[27]

Another characteristic of bankruptcy system dynamics is self-organization, i.e. the emergence of patterns that are not attributable to exogenous, extra-systemic, events. The data in this Article, echoing other empirical bankruptcy work,[28] show that the bankruptcy

---

IMPACT IN THE CRIMINAL COURTS 3 (1978) ("My premise is that the reform process is an independently significant phenomenon, typified by common patterns and delimiting characteristics. This phenomenon can and should be studied quite apart from the specific goals of specific reforms.").

[25] *See* Trujillo, *Self-Organizing Legal Systems*, at 490-500, 509-512.

[26] *See infra* Part II.D.

[27] *See* Edward L. Rubin, *Discretion and Its Discontents*, 72 CHI.-KENT L. REV. 1299, 1299 (1997) (suggesting that judicial discretion is presented as "problem," when it is actually ubiquitous and unremarkable feature of modern law); Rubin & Feeley, *Creating Legal Doctrine*, at 2037 ("[Judicial creation of law] can be described, understood, and justified. It is one of the basic, quotidian elements of our legal system.").

[28] *See, e.g.*, Teresa A. Sullivan, Elizabeth Warren, and Jay Lawrence Westbrook, *The Persistence of Local Legal Culture: Twenty Years of Evidence from the Federal Bankruptcy Courts*, 17 HARV. J. L. & PUB. POL'Y 801, 804 (1994); Lynn M. LoPucki, *Legal Culture, Legal Strategy, and the Law in Lawyers' Heads*, 90 NORTHWESTERN L. REV. 1498, 1506-07 (1996); Jean Braucher, *Lawyers and Consumer Bankruptcy:*

legal system produces stable and navigable patterns of behavior, despite the absence of rigorous hierarchical control by appellate courts and Congress.

C. Design of this Study

The data for this Article are drawn from reported opinions of U.S. Bankruptcy Courts in Chapter 11 cramdown proceedings that contain a judicial valuation of at least one asset.[29] The database consists of 180 observations drawn from 145 published opinions reported in the Westlaw computer database, and decided from 1979 through 1998.[30] A law student, working closely with me, coded each observation according to a substantial coding instrument.[31] Following the initial coding, a substantial portion of the observations was re-coded by a different law student to test for reliability.[32]

Besides the fact that the data are drawn from published cases,[33] other potential limitations of the database include the possibility that the computer search failed to identify relevant cases, that Westlaw misclassified some relevant cases, and that the coding process registered some "false negatives" (i.e. failing to classify an item as an

---

One Code, Many Cultures, 67 AM. BANKR. L. J. 501, 532 (1993); Lynn M. LoPucki, *The Demographics of Bankruptcy*, 63 AM. BANKR. L.J. 289 (1989).

[29] In this cramdown database, judicial valuations typically occurred in one of two contexts: First, a dissenting class of creditors (typically a single secured creditor) seeks "cramdown rights" under § 1129(b)(2)(A) through a judicial valuation of the collateral securing its claim. Second, the plan proposes to extinguish a creditor's lien in a piece of collateral in exchange for compensation, and the court values the item to determine if the compensation is appropriate and the plan confirmable. The parties to these valuation proceedings are corporations.

[30] The Westlaw search term used to acquire the cases was "51K3563 51K3564 51K3565 & DA(AFT 1978 & BEF 1999)". This term yields a list of 388 cases. This list generated 180 observations, reported in 145 cases. Observations consist of a reported numerical valuation of an asset by a court. Where a single case reported valuations of multiple assets, each asset was coded as a separate observation. The earliest observation is from December 5, 1980. The latest observation is from November 20, 1998.

[31] *See infra* Appendix (describing how reader can access a copy of the Coding Instrument). *See generally* HERBERT JACOB, USING PUBLISHED DATA: ERRORS AND REMEDIES (1984, vol. 42, Sage series on Quantitative Applications in the Social Sciences) (discussing methodology generally applicable to this study); ROBERT PHILLIP WEBER, BASIC CONTENT ANALYSIS (1990, vol. 49, Sage series on Quantitative Applications in the Social Sciences) (same).

[32] *See infra* Appendix, Figure 19 (presenting reliability statistics).

[33] *See supra* Part I.B (discussing relationship between data's publication and its probative value).

"observation" despite the presence of a reported judicial valuation). Thus, while these data support statistical conclusions about the population of all reported cases from 1979 through 1998, the present database is properly understood as a only sample of that population.

Each observation was coded for several variables. The meanings of most of the variables are obvious (e.g. date of adjudication, federal circuit, etc.), but four variables merit some initial explanation: (i) party success; (ii) finance element; (iii) valuation standard; and (iv) valuation model.

(i) "Party success" is a numerical definition of the degree to which the parties in the valuation proceeding won or lost.[34]

(ii) "Finance element" is a variable for whether the valuation contained a calculation or discussion of the time value of money. Of the 180 observations, 74 (i.e. 41% of the total) did contain a finance element, and 106 (59%) did not.[35]

(iii) "Valuation standard" is a variable that attempts to represent the internal criteria used by a particular valuer (court, debtor, or creditor) when it assigned value to an item (i.e. whether the valuer based the valuation on the item's anticipated use, or re-sale, or some mix between use and re-sale).[36]

(iv) "Valuation model" tracks the way parties argue for a particular valuation, for example by stating what an asset would be worth if it were liquidated (i.e. a

[34] *See infra* Part II.A for the mathematical construction of the "success" variable and analysis.
[35] A case was coded as containing a "finance element" if the court, as part of the valuation, also determined an interest rate or discount rate.
[36] *See infra* Part II.B for the mathematical construction of the "valuation standards" variable and analysis.

liquidation model), or by constructing a prediction of what cash flows an asset would earn over time (i.e. a discounted cash flow model).[37]

II. STUDY RESULTS AND INTERPRETATIONS

This Part presents the conclusions of statistical analysis of the data, and offers some interpretations of the results.

A. Party Success

The database illuminates some patterns of party success in valuation proceedings. We define "success" quantitatively, by comparing the valuation proposed by the parties with the valuation found by the court. Using two simple mathematical formulae,[38] we describe the range of party success from "1" (i.e. Debtor received all that it sought) to "0" (i.e. Creditor received all it sought).

---

[37] *See infra* Part II.C for further analysis of valuation models.

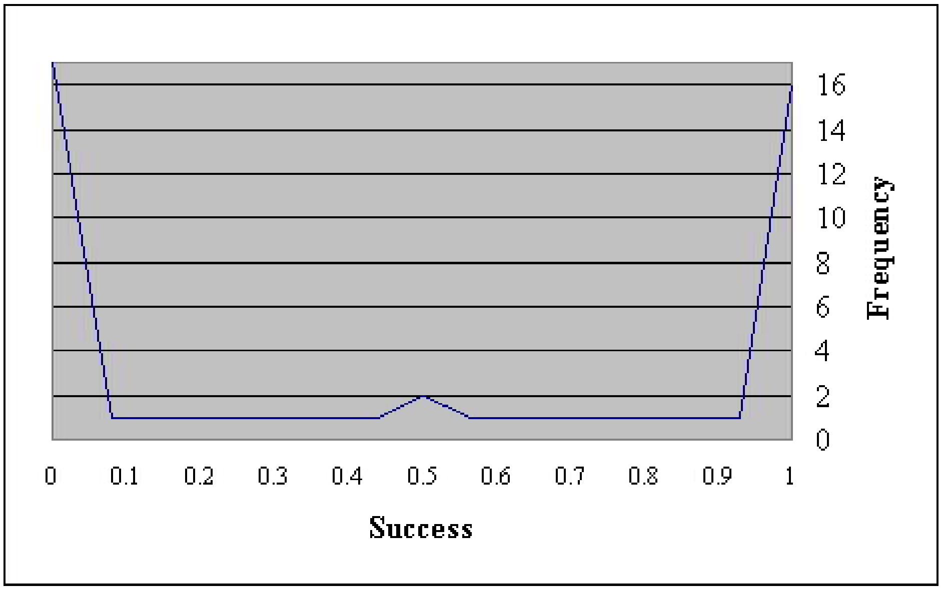
[38] To calculate the "success" variable (denoted "S" in the following equations), we took observations that contained numerical expressions of (i) the Debtor's requested value ("D"); (ii) the Creditor's requested value ("C"); and (iii) the Court's found value ("Ct"). Values of D and C must be unique. n = 62. Where C > D, S = C - Ct / C – D. To illustrate, suppose C = 10, D = 6, Ct = 9. So 10 - 9 / 10 - 6 = 0.25 (a number closer to 0 than one, indicating C prevailed). On the other hand, suppose C = 10, D = 6, Ct = 7. So 10 - 7 / 10 - 6 = 0.75 (indicating D prevailed). Where C < D, S = Ct-C / D – C. To illustrate, suppose C = 6, D = 10, Ct = 9. So 9-6 / 10 - 6 = 0.75 (D prevails). On the other hand, suppose C = 6, D = 10, Ct = 7. So 7 - 6 / 10 - 6 = 0.25 (C prevails). Thanks to Menesh Patel for help in developing this version of the "Success" variable.

Figure 1 shows the frequency of values for the “success” variable.

Figure 1

Party Success – Frequency Distribution

Success	Freq.	Percent
0	17	27.42
0.081008	1	1.61
0.101567	1	1.61
0.102564	1	1.61
0.166667	1	1.61
0.1766	1	1.61
0.28	1	1.61
0.285714	1	1.61
0.297189	1	1.61
0.301887	1	1.61
0.368421	1	1.61
0.408263	1	1.61
0.425865	1	1.61
0.428571	1	1.61
0.438356	1	1.61
0.5	2	3.23
0.565217	1	1.61
0.575071	1	1.61
0.579832	1	1.61
0.622419	1	1.61
0.627707	1	1.61
0.666667	1	1.61
0.733753	1	1.61
0.767123	1	1.61
0.775001	1	1.61
0.781818	1	1.61
0.789474	1	1.61
0.930233	1	1.61



This figure shows a “one-shot” aggregation of success patterns, without breaking down the variable by time.[39] The curve in Figure 1 looks a bit like a “W”, describing a pattern in which the Court picks either the Debtors’ number or the Creditors’ number (i.e. complete success for the Debtor[40] or complete success for the Creditor[41]) about equally.

[39] For ease of presentation, one value of 0.003546 was omitted.
[40] Represented in the chart by the 16 “ones.”
[41] Represented in the chart by the 17 “zeroes.”

Remarkably, Courts very rarely “split the difference” between the Debtors’ and Creditors’ numbers (i.e. the small hump in the middle of the W-curve).

These findings are somewhat surprising, in that they upset the common wisdom that Courts frequently resolve valuation disputes by splitting the difference between the offered numbers.[42] If the “common wisdom” were true, then the curve in Figure 1 would resemble the letter “A” (i.e. high observations in the middle, with lower observations on either end)[43] rather than the letter “W” (i.e. high observations on either end, with only a little bump in the middle). The “W-curve” of Figure 1 is unexpected. It indicates that judges engage in simple baby-splitting much less frequently than had been believed.[44]

B. Valuation Standards

i. Definitions and descriptive statistics

The “valuation standard” is a variable constructed to represent the criteria used by a given valuer (i.e. the Court, Debtor, or Creditor) when it assigned value to an item. The variable is constituted across three categories, reflecting whether the valuer based the valuation on the item’s anticipated use (“independent” value), re-sale (“common” value), or some mix between use and re-sale (“mixed” value). These three categories attempt to

[42] *See, e.g.*, Louis W. Levit, *Secured Claims*, in 2 CHAPTER 11 THEORY & PRACTICE § 15.04, at 15:12 (James F. Queenan, et al., eds. 1994) (“Unless the parties can agree [on a valuation], they must resort to the modern version of ‘trial by battle’ in which the contenders are not gladiators or knights in armor but expert (and expensive) appraisers. Each appraiser will trot out his credentials and carefully prepared report showing the usual charts and illustrations designed to bolster the conclusions which will give his side what it wants. When the trial is concluded, the court will probably ‘cut the baby in half’ and end up somewhere in the middle.”).

[43] Or a “head and shoulders” curve, with slightly higher observations at the poles and much higher observation in the middle.

[44] Figure 1 resembles the decision pattern that would have been predicted under the (short-lived) rule announced by Chancellor Allen in *Cede* when he proclaimed that henceforth he would accept only one party’s valuation model. Cede & Co. v. Technicolor, 1990 WL 161084, *reversed by* Cede & Co. v. Technicolor, Inc., 684 A.2d 289, 300 (Del. 1996).

represent the range of possible valuation approaches. Taken together, the three categories of the dependent variable express a theory of value – an account of what a valuer is doing when it assigns value to an item.

We take our polar definitions of common and independent value from the "auction" literature of economics:[45] A valuer's valuation is "independent" when it is not influenced by other valuers' valuation of the same item.[46] For example, suppose a valuer bids on a painting she intends for her private use. Her valuation of the painting depends on her personal tastes, her intended use of the painting, and other considerations that are unique to her.[47] If she learned that other valuers placed a different value on the same painting, it might change her bidding strategy, but it would not change her valuation.[48]

A valuer is using "common" value when her valuation of an item is a best guess of how others will value the item in some common market.[49] Common value models

---

[45] *See, e.g.*, PAUL MILGROM, PUTTING AUCTIONS TO WORK 157-162 (2004); R. Preston McAfee & John McMillan, *Auctions and Bidding*, 25 J. ECON. LIT. 699, 705 (1987). *See also* Peter Cramton & Alan Schwartz, *Using Auction Theory to Inform Takeover Regulation*, 7 J. LAW, ECON. & ORG. 27 (1991); Orley Ashenfelter & Kathryn Graddy, *Auctions and the Price of Art*, 41 J. ECON. LIT. 763 (2003).

[46] Represented mathematically, "independent" valuation describes a situation in which each valuer's valuation of an item is an independent draw from some probability distribution $F(v_\iota)$, where $v_\iota = v^1, v^2, v^3, \ldots v^n$ (i.e. v_ι represents the range of valuations held by valuers in the probability distribution). The holder of v^1 will not change her valuation if she learns that another valuer holds v^2. In such a market, there will be no change in the valuation of an item, even if all valuers revealed everything they knew (this, incidentally, is one definition of a perfectly "efficient" market, see M. Rubinstein, *Securities Market Efficiency in an Arrow-Debreu Economy*, 65 AM. ECON. REV. 812, 824 (1975). A market is thus "non-adjusting" when all valuers of the item hold independent valuations.

[47] *See generally* Louis Kaplow & Steven Shavell, *Property Rules versus Liability Rules: An Economic Analysis*, 109 HARV. L. REV. 713, 760 (discussing "idiosyncratic" value).

[48] *See generally* BENJAMIN GRAHAM, THE INTELLIGENT INVESTOR 109 (1973) (discussing "parable" of "Mr. Market", in which market information does not affect holder's valuation).

[49] Represented mathematically, a "common" valuation is a draw from some probability distribution $H(v_\iota|\mathrm{V})$ where $v_\iota = v^1, v^2, v^3, \ldots v^n$ The only difference between probability distribution H (common valuation) and probability distribution F (independent valuation) is that the H quantity contains a term "V" representing the reference point by which all components of vi adjust their valuations. On this theory of value, each component of v_ι is an estimate on the content of "V", i.e. the item's "true value"; these estimates stand to improve as each valuer learns the valuations of others. This "true worth" theory of value is the one employed by Bob Lawless & Steve Ferris. *See* Robert M. Lawless & Stephen P. Ferris, *Economics and the Rhetoric of Valuation*, 5 J. BANKR. L. & PRACT., 3, 11-12 (1995)

assume that the item has “exactly the same value to each bidder”[50], and the variation in bids is explained by bidders’ incomplete information regarding that value. Suppose, for example, an art dealer bidding for a painting she intends to re-sell. Her valuation is based on her information about what the painting will fetch in the re-sale market. Her valuation of the item will almost certainly change as she learns more about the valuations of others.

Using these categories of valuation, we assigned a code to each instance of valuation in our database. The possible codes ranged on a continuum from extreme “common” value to extreme “independent” value. Initial coding of the data assigned a number from 1 to 8 (with “1” representing extreme “common” and “8” representing extreme “independent”) for each valuation by Courts, Debtors, and Creditors. Subsequent statistical analysis[51] showed that the valuation standard could be collapsed into three categories: Common (initial codes 1 and 2), Mixed (initial codes 3, 4, and 5), and Independent (initial codes 6, 7, and 8). The fact that the research project went fishing for eight categories and ended up with three is, itself, an interesting finding. This finding may verify, with statistical analysis of real-world bankruptcy data, the robustness of auction theory’s conceptual categorization of valuation.

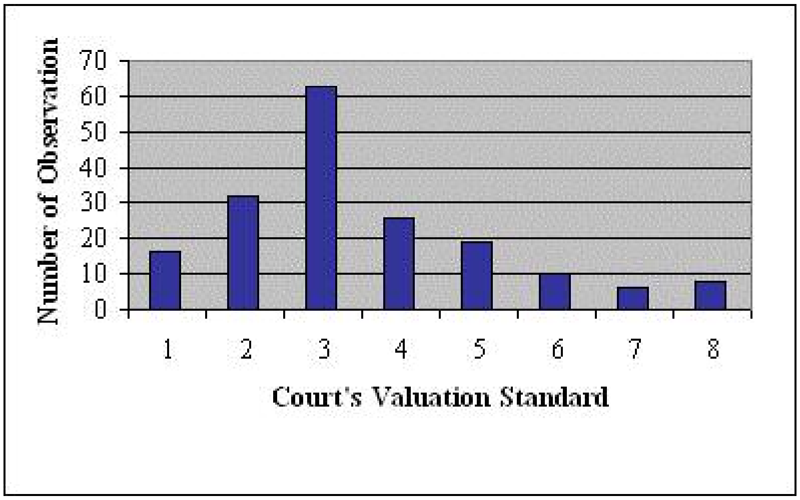
Figure 2 shows the Courts’ valuation standard (“CtVS”) in the 180 observations of the database, as coded on an eight-part continuum ranging from 1 (extreme common) to 8 (extreme independent).

[50] MILGROM at 162

[51] Specifically, a multinomial logistic analysis, a likliehood ratio test, and a Wald test all showed that the conditional distribution of the valuation standard could be collapsed from eight categories to three. *See infra* APPENDIX.

Figure 2
Court Valuation Standard – Frequency

CtVS	*# of Obs*	%
1	16	8.9
2	32	17.8
3	63	35
4	26	14.4
5	19	10.6
6	10	5.6
7	6	3.3
8	8	4.4
TOTAL	180	100



We see that the valuations performed by bankruptcy courts in the database were skewed toward the "common" pole of the continuum, with most of the valuations coded at "3" (i.e. common, but not extreme common).[52]

Figure 3 shows the distribution of CtVS, broken out by whether the valuation contained a finance element.

Figure 3
Court Valuation Standard by Finance Element

	Frequency	Mean	Standard Deviation
Finance element	74	3.20	1.48
No Finance Element	106	3.75	1.86

The presence of a finance element in the valuation tends to drive the CtVS closer to the "common." We will see that the observed correlation between "common" valuation and

[52] These data tend to support a general impression that the cases in the database showed a rebuttable presumption in favor of more common-based valuation, which presumption had to be overcome by the party seeking a more independent-based valuation.

the presence of a finance element is not only a feature of the sample (i.e. our database), but is also strong enough to allow an inference that the same correlation is true across the entire population.[53]

Figure 4 gives the mean and standard deviation of the CtVS for each Circuit.

Figure 4

Court Valuation Standard by Circuit

Circuit	Mean	Standard Deviation	Frequency
1st	3.38	1.30	8
2nd	3.96	2.01	27
3rd	2.88	1.36	17
4th	3.40	1.96	15
5th	4.80	1.37	15
6th	3.17	1.63	24
7th	2.91	1.04	11
8th	3.32	1.95	19
9th	3.23	1.24	13
10th	3.50	2.43	6
11th	3.79	1.77	24
DC	3	n/a	1
All Circuits	3.52	1.73	180

We see that the bankruptcy courts of the 3rd and 7th Circuits have standards closest to common, and that the bankruptcy courts of the 5th Circuit have standards closest to independent.[54]

2. The need for an "ordered" model

Our statistical model attempts to (i) explain how bankruptcy courts approach their task of valuing assets in cramdown proceedings, and (ii) predict how bankruptcy courts will approach valuation in future cases. Accordingly, the dependent variable for these

[53] *See infra* Part II.B.4.

[54] The relatively low standard deviation in the 3rd, 5th, and 7th Circuits contributes to the reliability of the findings because it shows that the mean (low in the 3rd and 7th Circuits, high in the 5th Circuit) was achieved by observations that were consistently low (in the case of the 3rd and 7th Circuits) or high (in the case of the 5th Circuit).

inferential statistics (i.e. the thing we try to explain) is the "valuation standard" of the bankruptcy court, represented as one of three categories: common, independent, or mixed. Taken together, the three categories of the dependent variable attempt to express a theory of value – an account of what a valuer is doing when she assigns value to an item.

We explain the court's valuation standard by reference to three independent variables: (i) Debtors' valuation standard ("DVS")[55]; (ii) Creditors' valuation standard ("CVS"); and (iii) Finance element ("FINANCE"). This is the variable for whether there is a finance element (i.e. a calculation for the time-value of money) involved in the valuation.[56]

How one measures the dependent variable determines the selection of the appropriate statistical model. A very common statistical model is the "least-squares" regression. Least-squares regressions, however, require that the dependent variable be presented in "interval measures": cardinal numbers that specify the distance between each value. For example, the dependent variable "height" is expressed in interval measures because differences between values (say between 5'8" and 5'10") are fixed and meaningful.

The present study tries to explain how Bankruptcy Courts approach valuation. This dependent variable cannot be expressed with interval measures. We measure standards of valuation based on a set of conceptual categories. The values of our

[55] The three variables representing valuation standards (i.e. CtVS, DVS, and CVS) were originally coded on same continuum of 1 to 8. We subsequently converted that continuum to a three-part code for the Court's valuation standard. We preserved the eight-part coding for the DVS and CVS because the larger number of distinctions among the explanatory variables made the estimation process more reliable and provided a better fit for the model.

[56] The Finance variable is coded as a binary unit: a code of "zero" means there is no finance element in the valuation, and "one" means there is a finance element in the valuation.

dependent variable are thus expressed not <u>cardinally</u> (with numbers such as 1 and 2, or 5’8” and 5’10”) but rather <u>ordinally</u> (with numbers such as 1st and 2nd). While the values of cardinal numbers are known absolutely, the values of ordinal numbers are known only by the relationships among the values (e.g. “the second largest amount”), such that the distances between ordinal categories are not quantified.

Ordinal categories present a special problem for statistical model-building. Where data are presented in interval measures, the difference between one category and another is quantified by definition. But the statistician must take special steps to quantify the difference between ordinal categories.

Because the data in this study are ordinally measured, we need a statistical model equipped to estimate the “intercepts” for each category, i.e. where one category ends and the next begins.[57] This study uses an “ordered logit” model, which estimates the intercepts for each category of the dependent variable.

---

[57] With cardinally measured categories, the function of the regression coefficients will be unique (i.e. there will be only one set of solutions to the regression’s equations). But with ordinally measured categories, the function of the regression coefficients will be non-unique. In order for the statistical model to generate results, the problem of non-uniqueness must be resolved by estimating the relationship among the ordinal categories. The “ordered logit” statistical model performs these estimates. *See generally*, J. SCOTT LONG, REGRESSION MODELS FOR CATEGORICAL AND LIMITED DEPENDENT VARIABLES 115 (1997).

3. The model

For an ordinal variable with *J* categories, let the odds that the dependent variable (the "x" term in the equation below) will have a value less than or equal to a given intercept *m*, versus the odds that the dependent variable will have a value greater than a given intercept *m*, be:

$$\Omega_{\leq m_k | > m_k}(y) = \Pr(x \leq m_k | y) / \Pr(x > m_k | y)$$

where **y** is a vector of explanatory variables, and we estimate for *J*-1 thresholds.[58] This equation merely defines the odds of an event (i.e. that the dependent variable will have a certain value relative to the independent variables) as a ratio of the probabilities for that event.

The log of the odds as defined in the first equation is thus equal to:

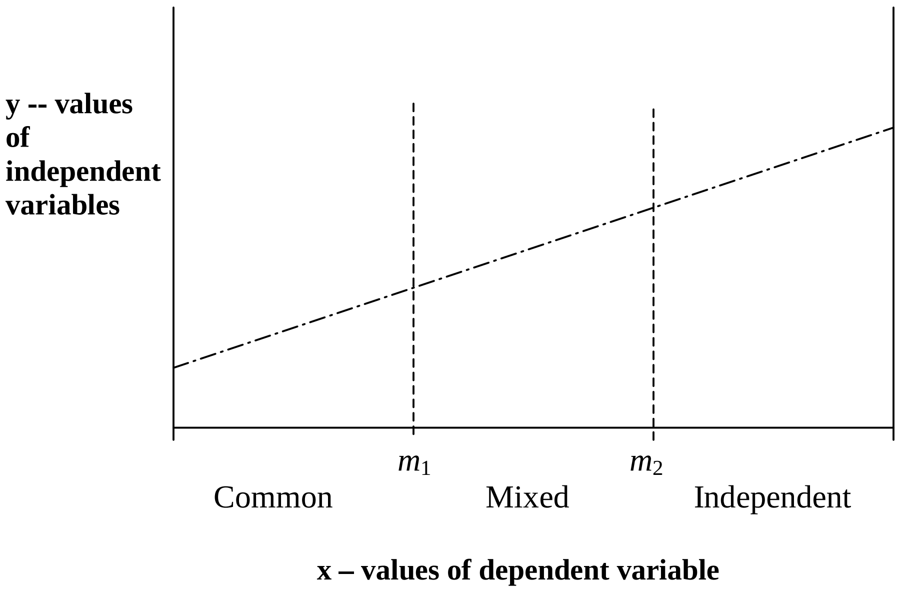
$$1n\ \Omega_{\leq m_k | > m_k}(y) = \tau_{m_k} + y\beta$$

where τ_{m_k} states the estimated thresholds, y estimates the intercepts along the y axes, and β (as a vector of the coefficients) states the slope of the dependent variable as it moves along the x axis. This equation predicts the value of the dependent variable given certain independent variables. Figure 5 graphically represents the parameters and predictions of the model.

[58] In our application of this model, J = 3 (i.e. we have three categories of the dependent variable – common, mixed, and independent) and so we estimate for two thresholds, giving us two values of "m". The "k" subscript to the "m" terms simply allows "m" to take an infinite number of forms. Note that the total of the probability of an outcome being "less than or equal to" plus the probability of an outcome being "greater than" for any given *m* will equal 1.

Figure 5

Ordered Logit Model of CtVS: Graph and Equation



The equation

$$1n\ \Omega_{\leq m_k|>m_k}(y) = \tau_{m_k} + y\beta$$

predicts the changing value of x (i.e. the Courts' valuation standard) by

1. estimating the m_1 and m_2 cutpoints along the horizontal axis (modeled by the τ_{m_k} term);

2. estimating the points at which the dependent variable intercepts the vertical axes (modeled by the y term); and

3. estimating the slope of the dependent variable as it moves along the horizontal axis (modeled by the β term).

This is an ordered logit model. The natural logarithm of the odds (known as the "logit") is a linear function of the independent variables. In order to account for the ordinality of the dependent variable, $J - 1$ equations are estimated and the coefficients of the independent variables are constrained to be equal across equations.

4. Findings

Figure 6 presents results from the ordered logit model.

Figure 6

Results: Ordered Logit Models of Court Valuation Standard

Variable	***Estimates***
DVS	0.933 (0.172)** [2.541]
CVS	0.853 (0.186)** [2.347]
FINANCE	-1.613 (0.561)** [0.199]
Thresholds	
cut 1	3.552 (0.782)**
cut 2	9.890 (1.366)**
Log Likelihood	-54.724
BIC	-84.613
R^2	0.475
Observations	119

Standard errors in parentheses
Odds Ratios in brackets
** significant at 1% level

Figure 6 reports the effect that each independent variable has on the court's valuation standard, while holding constant the effect of all other independent variables. Figure 6 lists three quantities for each independent variable: the top number is the coefficient for the natural log (or "logit"); the second number (in parentheses) is the standard error, showing the statistical significance of each result;[59] and the third number (in brackets) gives the "odds ratio" for each independent variable. The Figure also reports estimates for the cutpoints.[60]

The model shows that the explanatory variables DVS, CVS, and FINANCE predict CtVS variance at a statistically significant level. The "r-squared" is remarkably good for social science research, showing that the independent variables explain 47.5% of the dependent variable's observed variance. Overall, it appears that CVS, DVS, and FINANCE very usefully explain and predict the bankruptcy courts' valuation standard.

The "odds ratios" is a quantity that helps explain the model's results. An odds ratio[61] gives the odds that the dependent variable will be in one category versus another for every one-unit change in the independent variable.[62] For example, if CVS increases

[59] The stars represent significance at the 1% level: i.e., the two cutpoints and three independent variables (DVS, CVS, FINANCE) are all "statistically significant" such that the results are very unlikely to have been obtained by chance. Statistical significance "at the 1% level" means that our Model capture the entire range of the data's distribution (assuming a bell-shaped distribution) except for the two extreme tails of 0.5% each.

[60] Ordered logit models assume proportional odds, i.e. the effect of the independent variable on the dependent variable is assumed to be proportional for all comparisons of categories. In other words, the slopes coefficients are identical across the estimated equations. In this case, two equations are estimated for each of the binary logistic regressions, yielding the coefficients denoted in Figure 6 as "cut 1" and "cut 2". Though the slopes are the same, the probability curve is shifted to the left or right depending on the categorical comparison.

[61] The "odds ratios" are the bracketed quantities in Figure 6. We get the odds ratio for a given independent variable by exponentiating its coefficient. For example, we exponentiate the FINANCE coefficient by taking "base e" (i.e. roughly 2.718) and raising it to the power of –1.613 (i.e. FINANCE's coefficient), yielding an odds ratio of 0.199.

[62] The "odds" of X happening is the probability of X happening divided into the probability of X not happening. An odds of "1" indicates that X stands a 50/50 chances of happening. An odds of greater than 1 indicates that the probability of X happening is greater than the probability of X not happening. And an

by one unit, the odds of CtVS being "mixed" becomes 2.347 times higher than the odds of CtVS being "common". Likewise, when CVS increases by 1 unit, the odds of CtVS being "independent" becomes 2.347 times higher than the odds of CtVS being either "common" or "mixed."

The results of the ordered logit models (as represented in Figure 6) support three interpretations:

1. Courts are much more likely to use a "common" valuation standard when the valuation contains a "finance element" (i.e. a calculation for the time value of money).[63] This correlation is the strongest finding of our study.[64] But why would the presence of a finance element tend to drive valuations toward common standards? One possible explanation is that judges deploy common valuation standards in high-complexity cases,[65] while using more independent valuations in lower complexity cases.[66]

---

odds of between 0 and 1 indicates that the probability of X not happening is greater than the probability of X happening.

[63] This finding can be discerned from Figure 10, which reports an "odds ratio" of 0.199 for the "finance" variable. The odds ratio of 0.199 means that, holding all other variables constant and looking just at the relationship between CtVS & FINANCE, if the observed valuation possessed a "finance element", then (i) the odds of CtVS being "mixed" is 0.199 times the odds of CtVS being "common"; and (ii) the odds of CtVS being "independent" is 0.199 times the odds of CtVS being either "mixed" or "common". The data show that the presence of a "finance" element in the valuation pushes the Courts' valuation standard sharply towards the "common" pole. This finding is remarkably strong.

[64] This finding might be explained away as a meaningless "coding error" if cases containing a finance calculation had been automatically coded closer to "common" than "independent". The codes, however, demonstrate no such tendency. Observations with finance elements show codes that span the range of valuation standards. *See* APPENDIX, (web access to spreadsheet of coding results, column BZ). Generally, a statistical model cannot return sensible results unless there is meaningful variation in the data (which variation would be foreclosed by a coding error classifying finance cases as common). Note also that many cases that were coded closer to the "independent" pole contained a finance element. *See infra* note 77.

[65] *See generally* Elizabeth Warren, *Vanishing Trials: The Bankruptcy Experience*, 1 J. EMPIRICAL LEGAL STUD. 913 (2004) (contrasting "high-complexity/low-volume" bankruptcy cases with "low-complexity/high-volume" cases). In our database, cases using common valuation standards tended to involve competing and complicated models of the asset's valuation, constructed from the "ground up" by dueling experts.

[66] In our database, cases using independent valuations tended to utilize book value and owner testimony to determine the value of underlying assets. For finance cases, the court tends to rely on rates stated in pre-petition contracts or statutory rates.

2. The Courts' valuation standard correlates strongly with the Debtors'[67] and Creditors'[68] valuation standard. From this statistical result we can verify the unsurprising proposition that lawyers frame the parameters of the valuation dispute, and that judges typically work within those parameters.

3. The valuation standards of the opposing parties tend to be close together.[69] This finding is somewhat surprising, in that it upsets the expectation that parties in litigation pursue a "highball/lowball" strategy[70] when submitting their valuation arguments to the court.[71]

C. Valuation Models

A valuation model is a particular argument or showing about the value of an item. The most common valuation models in the database were:

---

[67] Figure 6 shows an odds ratio of 2.541 for DVS. This means that, holding all other variables constant and looking just at the relationship between CtVS and DVS, (i) as DVS increases by one unit, the odds of CtVS being "mixed" is 2.541 times higher than the odds of CtVS being "common"; and (ii) a one unit increase in DVS makes the odds of CtVS being "independent" 2.541 times higher than the odds of CtVS being either "common" or "mixed".

[68] Figure 6 shows an odds ratio of 2.347 for CVS. This means that, holding all other variables constant, and looking just at the relationship between CtVS & CVS; (i) as CVS increases by one unit, the odds of CtVS being "mixed" is 2.347 times higher than the odds of CtVS being "common"; and (ii) a one unit increase in CVS makes the odds of CtVS being "independent" 2.347 times higher than the odds of CtVS being either "common" or "mixed".

[69] The inferential statistics showed that CtVS was equally influenced by DVS and CVS, i.e. the Parties' valuation positions were close to each other, and the Courts' valuation position were close to the Parties' valuation positions.

[70] A strategy of "highball/lowball" occurs when the party seeking a low number presents a valuation model arguing for the lowest number that can possibly be pronounced with a straight face, and the party seeking a high number presents valuation evidence requesting the highest number that can be requested without incurring a Rule 11 violation. In this way, the Parties' numbers are pushed to the extremes, and the usefulness of their valuation arguments for the court is diminished. *See* Cede & Co. v. Technicolor, 1990 WL 161084 (Chancellor Allen attempting to defuse parties' highball/lowball strategy), *reversed by* Cede & Co. v. Technicolor, Inc., 684 A.2d 289, 300 (Del. 1996).

[71] *See also* Part II.A (presenting descriptive statistics regarding the "success" variable, and upsetting the expectation that Courts tend to "split the difference" between Parties' valuation positions). It is possible that courts' rejection of "split-the-difference" adjudication (*supra* Figure 1) is the cause of litigants' strategy to avoid overclaiming. This finding would verify Martin Shapiro's hypothesis that mediation, where it promotes "split-the-difference" outcomes, will tend to destabilize adjudication by driving the parties' claims farther apart. *See generally* MARTIN SHAPIRO, COURTS (1981) (chapters 1 and 5). My thanks to Martin Shapiro for drawing this to my attention.

- Comparables, in which the modeler estimates an asset's value by looking to the known values of other, ostensibly similar, assets.[72] Comparables are a common model for valuing residential real estate, in which a house put up for sale is compared to the recent sale prices of other houses of a similar location, size, style, etc.
- Discounted cash flow ("DCF") models, which estimates the present value of future expected cash receipts and expenditures.[73] A DCF model is typically generated by a financial professional who is introduced as an expert witness. The financial expert will estimate the firm's future cash flows and then discount those expected future returns to present value through the use of a discount rate.
- Cost or liquidation, in which the modeler estimates an asset's value by looking to what price the asset would fetch if sold.[74]
- Testimony by a current or prospective user of the asset regarding the valuation aspects of the intended use.
- A pre-Petition contract rate agreed upon by the parties (used in the finance, or time-value, aspect of the valuation).
- An accounting made for the "risk profile" of the party acquiring the asset (again used for the finance aspect of the valuation – higher risk yields an increased interest rate).

[72] *See, e.g.*, MARK GRINBLATT & SHERIDAN TITMAN, FINANCIAL MARKETS AND CORPORATE STRATEGY 368 (1998) [hereinafter GRINBLATT & TITMAN].
[73] *See, e.g.*, RICHARD A. BREALEY & STEWART C. MYERS, PRINCIPLES OF CORPORATE FINANCE 73 (1996).
[74] *See, e.g.*, GRINBLATT & TITMAN 560.

- General observations about market and economic conditions affecting the asset's value.

In addition to these categories, we also coded as "Unknown" those observations where the language in the opinion could not support the assigning of a code.

Figure 7 shows the valuation models offered by Debtors and Creditors, and accepted by the Courts.

Figure 7

Valuation Model Offered by Debtor, Creditor; Accepted by Court

Valuation Model	Debtor	Creditor	Court
Comparables	24.4%	17.8%	23.3%
Discounted Cash Flow ("DCF")	10.5%	8.9%	10.6%
Cost or liquidation approach ("Cost")	4.4%	2.2%	3.9%
Testimony	6.7%	4.4%	3.9%
Preexisting contract between debtor and creditor ("Contract")	5.0%	2.8%	3.3%
General market conditions	0.6%	0.6%	1.7%
Comparables & risk profile	8.9%	10.6%	16.1%
Comparables & Cost	5.0%	0.6%	4.4%
DCF & Comparables	7.8%	8.9%	9.4%
DCF & Comparables & Cost	3.3%	1.7%	2.2%
General market conditions & Contract	0.6%	0.0%	0.6%
DCF & General market conditions	0.0%	0.0%	0.6%
DCF & Cost	0.6%	1.1%	1.1%
Comparables & General market conditions	0.6%	0.0%	1.7%
Comparables & Contract	1.1%	0.6%	1.1%
Comparable AND testimony	0.0%	0.0%	0.6%
Other	6.1%	3.9%	7.2%
DCF & Comparables & other	0.0%	0.0%	0.6%
Comparable AND other	0.0%	1.1%	1.1%
Other AND testimony	0.0%	0.0%	0.6%
Unknown	14.4%	35.0%	5.6%
TOTAL	100%	100.0%	99.6%[75]

[75] Column total does not equal 100% due to rounding.

One way of organizing the data on Court acceptance is to break the valuation models into two groups, roughly generalizing between "hard" (includes DCF, Cost, Comps, calculation of a risk profile, and pre-existing contract rate between the parties) and "soft" (includes owner testimony, consideration of general market conditions, other, and unknown) valuation models. Figure 8 shows the distribution of Court-acceptance of valuation model, broken out into "hard" and "soft" groups.

Figure 8

Court Acceptance of "Hard" Valuation Models: Distribution by Circuit

Circuit	Hard	Soft	Total obs
1st	8 (100%)	0	8
2nd	24 (88.89%)	3 (11.11%)	27
3rd	15 (88.24%)	2 (11.76%)	17
4th	10 (66.67%)	5 (33.33%)	15
5th	8 (53.33%)	7 (46.67%)	15
6th	21 (87.5%)	3 (12.5%)	24
7th	11 (100%)	0	11
8th	18 (94.74%)	1 (5.26%)	19
9th	11 (84.62%)	2 (15.38%)	13
10th	4 (66.67%)	2 (33.33%)	6
11th	15 (62.5%)	9 (37.5%)	24
DC	1 (100%)	0	1
TOTAL	146 (81.11%)	34 (18.89%)	180

The data seem to offer some support for the proposition that courts in the Southern states of the U.S. are more inclined to accept "soft" valuation models than non-Southern states. Looking just at those Circuits with more than 10 observations, we see that the 7th Circuit (Illinois, Wisconsin, and Indiana) most often accepts hard models, followed by the 8th (North Dakota, South Dakota, Minnesota, Nebraska, Iowa, Missouri, and Arkansas), the 2nd (New York, Connecticut, and Vermont), the 3rd (Pennsylvania, New Jersey, Delaware, and the Virgin Islands), the 6th (Michigan, Ohio, Kentucky, and Tennessee), and then the 9th (Alaska, Hawaii, Washington, Oregon, California, Arizona, Nevada, Idaho, Montana, and Guam). Courts from the 5th Circuit (Texas, Mississippi, and Louisiana) are most likely to accept soft models, followed by the 11th Circuit (Alabama, Georgia, and Florida), and then the 4th Circuit (Maryland, West Virginia, Virginia, North Carolina, South Carolina).[76]

F. Self-Organization

These data are useful for testing the proposition that the U.S. bankruptcy legal system might be a self-organizing system, i.e. a system whose content contains patterns not fully explained by exogenous dynamics.[77] This section presents the evidence for self-organization as observed in three categories of variables: (i) the use of valuation standards by Courts, Debtors, and Creditors; (ii) the use of valuation models by Debtors and Creditors; and (iii) Courts' acceptance of experts.

1. Valuation standards

If a system's content self-organizes, we should be able to measure a decline over time in the variability of some of the system's variables. That is, we treat variation itself as a structural variable to be measured & observed.[78]

---

[76] For empirical work using the Federal Circuit as a unit of analysis, *see generally* Tracey E. George, *The Dynamics and Determinants of the Decision to Grant En Banc Review*, 74 WASH. L. REV. 213 (1999); Tracey E. George & Michael E. Solimine, *Supreme Court Monitoring of the United States Courts of Appeals En Banc*, 9 SUP. CT. ECON. REV. 171 (2001).

[77] *See* Trujillo, *Self-Organizing Legal Systems*, at 519-43 (explaining process of self-organization).

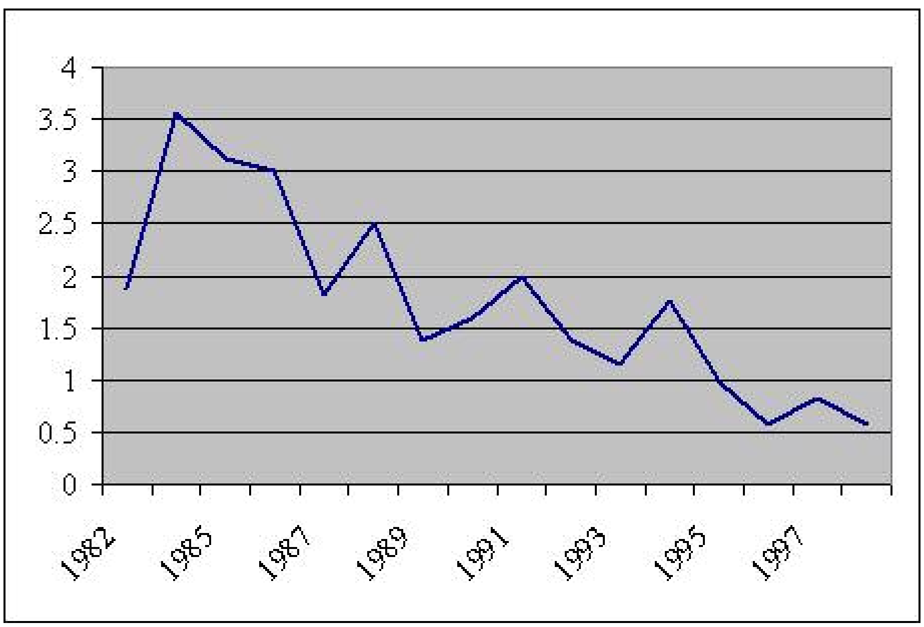
[78] The field of "sociolinguistics" has contributed much to my understanding of the construction variation as a variable in order to study the change of a social artifice over time. *See generally*, ALESSANDRO DURANTI, LINGUISTIC ANTHROPOLOGY 79 (1997) ("Whereas the realization of such variability convinced formal grammarians to ignore it by establishing an idealized homogeneity, sociolinguists decided to *face variability* and make it the subject matter of their investigation."); J.K. CHAMBERS, SOCIOLINGUISTIC THEORY: LINGUISTIC VARIATION AND ITS SOCIAL SIGNIFICANCE 11 (1995) ("the admission of the variable as a structural unit in linguistic analysis...represents a breakthrough of considerable magnitude in linguistic theory").

The measure of a variable's variation is its standard deviation. Just as the mean, median, and mode say something about the central tendency of a distribution, the standard deviation shows how that central tendency was achieved (with relatively larger standard deviations signifying a wide range of variation, and relatively smaller standard deviations signifying a smaller range of variation). If the standard deviation of a variable declines over time, this may indicate a sort of "learning" or "patterning" of system content over time. Such patterns, unless explained by extra-system forces (such as control by appellate courts or Congress), may constitute evidence of self-organization.

Figure 9 shows the standard deviation of the Creditors' Valuation Standard over time.

Figure 9
Creditors' Valuation Standard – Descriptive

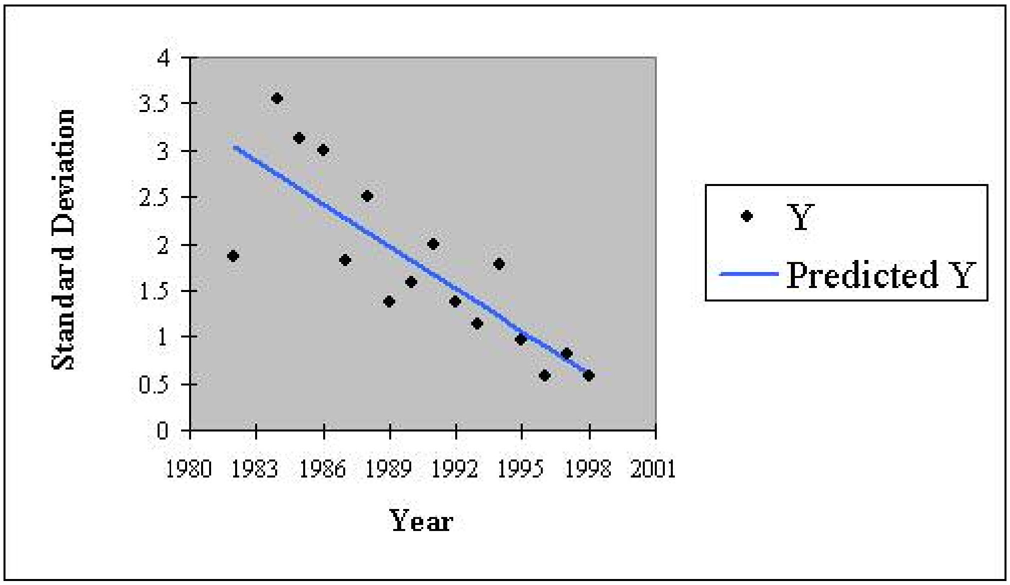
Year	**SD**
1982	1.8708
1984	3.559
1985	3.12517
1986	3
1987	1.8172
1988	2.5
1989	1.37032
1990	1.58114
1991	1.98206
1992	1.37069
1993	1.14354
1994	1.76777
1995	0.97183
1996	0.57735
1997	0.8165
1998	0.57735



As a matter of descriptive statistics, it appears that the variability in the CVS declined markedly over time.

We can test whether the decline, observed in the sample, is statistically significant by performing a standard linear regression.[79] We test the hypothesis that the population variance decreases with time by linearly regressing the sample variance over time.[80] The results of the regression are displayed in Figure 10.

<u>Figure 10</u>
Creditors' Valuation Standard – Inferential



The line estimated by the linear regression model explains the behavior of the sample data quite well,[81] allowing us to conclude that the CVS variable does organize over time.

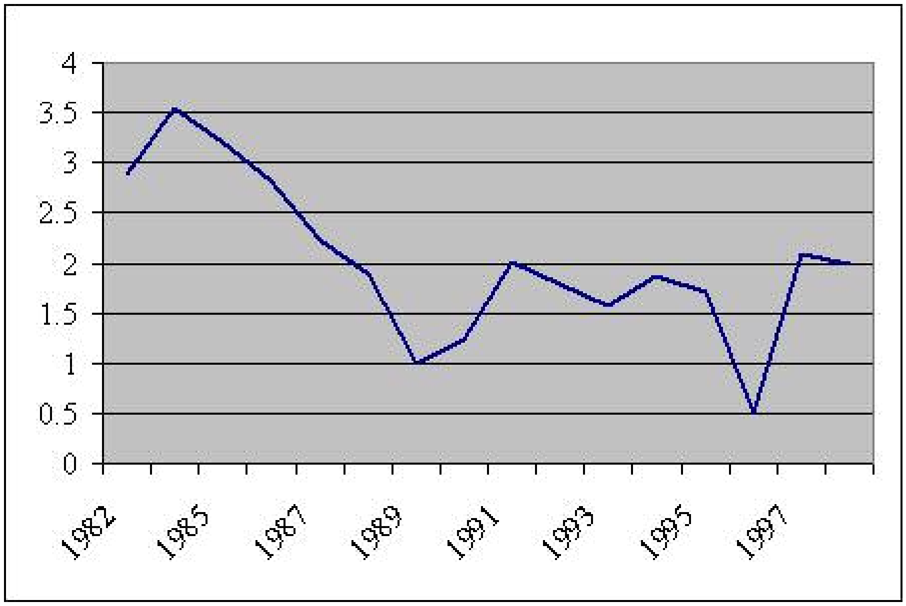
---

[79] We can use a linear regression here, but we could not for the model of valuation standards presented in Figure 6, because here our dependent variable is the sample variance (i.e. a variable that is measured out in cardinal numbers). It was the ordinal measure of the dependent variable in Figure 6 that required the use of an ordered model. *See supra* Part II.B.2.
[80] The formula for this standard regression is x = A + By, where x is the dependent variable (i.e. the variance of CVS), A is the intercept of the line, B is the slope of the line, and y is the independent variable (i.e. the year). This model estimates a line describing the relationship between the dependent and independent variables in the population. A negative slope provides evidence of a negative relationship between variation and time.
[81] r2 = 0.6719, which is quite strong given the limitations of social science data.

Figure 11 shows the standard deviation of the Debtors' Valuation Standard over time.

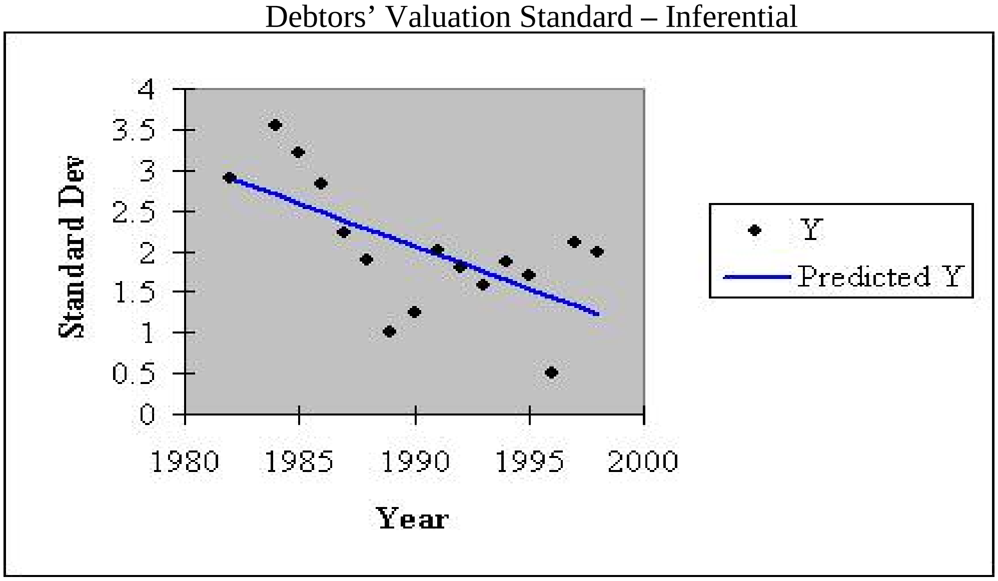
Figure 11
Debtors' Valuation Standard – Descriptive

Year	SD
1982	2.886751
1984	3.544949
1985	3.207135
1986	2.828427
1987	2.219933
1988	1.889822
1989	0.996205
1990	1.242118
1991	2.00454
1992	1.785165
1993	1.57181
1994	1.874755
1995	1.705947
1996	0.5
1997	2.097618
1998	2



This figure, while not as obvious as Figure 9, does indicate some decline in the sample variance over time. We can test the statistical significance of that decline by applying the same standard linear regression as above. Regression results are shown in Figure 12.

Figure 12
Debtors' Valuation Standard – Inferential



The line estimated by the linear regression model explains the behavior of the sample data,[82] allowing us to conclude that the DVS variable organizes over time.

Note that we do not see the parties' valuation standard converge on a single numeric value, or mean, within a given year. Such a finding would have indicated that the parties gradually came to agree on a particular valuation theory, something quite unlikely to happen given that the facts of every case and the litigation goals of every party are different. What we do see, however, is a year-to-year shrinkage of the range of variation within the Debtors' network and also within the Creditors' network. It is as if there is increasingly perfect communication among the actors comprising the Debtors' network, and among the actors within the Creditors' network, regarding the range of legitimate valuation approaches to be utilized in a given market environment.[83] Put another way, the nature of the organization over time that we observe in the parties' valuation standard demonstrates not "uniformity" (i.e. parties agreeing on one valuation theory), but rather "unity" (i.e. parties within a given practice network become diachronically more coherent in the range of approaches they attempt).[84]

While the valuation standards for the Debtors and Creditors seems to show some patterning over time, the Courts' valuation standard show no such organization. Figure 13 shows the standard deviation of the CtVS over time.

---

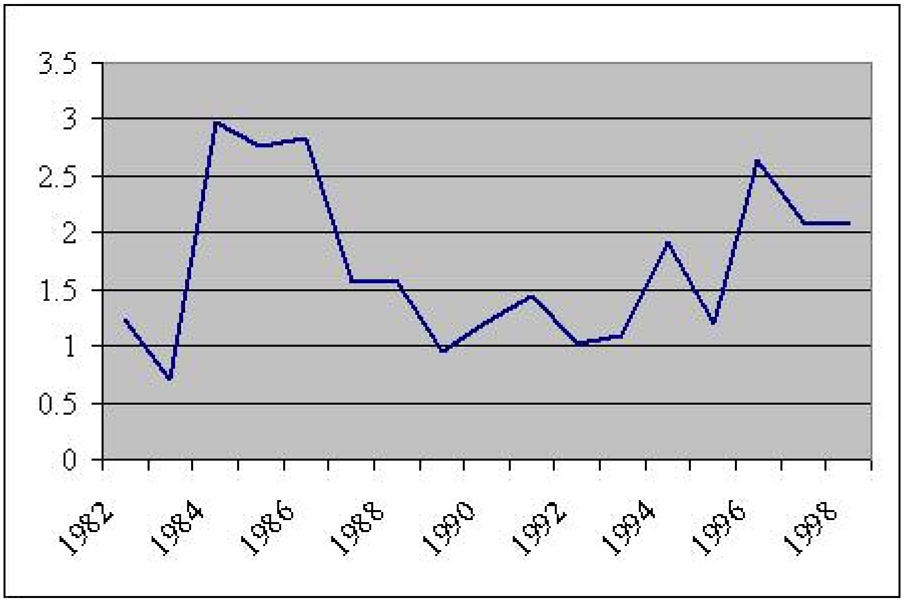
[82] r2 = 0.4097.

[83] Within the "valuation standard" variable, we do not see a convergence of the valuers upon a given form, such that a certain valuation standard becomes the norm. Rather, we see a shrinkage of variation, itself understood as a variable.

[84] *See* HERBERT KAUFMAN, THE FOREST RANGER 203 (1960) (finding "unity without uniformity" in organizational analysis of U.S. Forest Service).

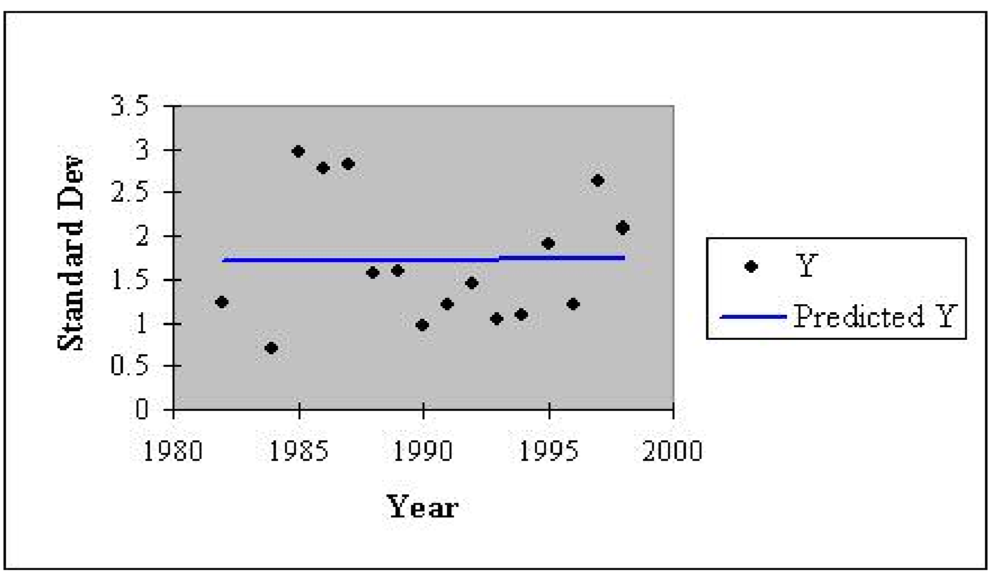
Figure 13
Courts' Valuation Standard – Descriptive

Year	SD
1982	1.2247
1983	0.7071
1984	2.9761
1985	2.7646
1986	2.8284
1987	1.5579
1988	1.5811
1989	0.9535
1990	1.2071
1991	1.4367
1992	1.0271
1993	1.088
1994	1.9183
1995	1.1983
1996	2.63
1997	2.0976
1998	2.0736



The eye detects no systematic decline in the sample variance, and this conclusion is born out by the regression shown in Figure 14.

Figure 14
Courts' Valuation Standard – Inferential



We can safely conclude that the Courts' valuation standard shows no organization or patterning over time.[85] Thus, while we observe some ordering phenomenon in the "valuation standard" variable among Debtors and Creditors, we observe no such phenomenon among the Courts.[86]

2. Parties' valuation models

In a self-organizing system, we would also expect to see forms introduced from outside the system, as well as a process of competition by which some of these forms achieve normative status.[87] We perform time-series analyses of the parties' use of particular valuation models to test for the presence of such form innovation and norm emergence.

Figure 15 shows the Debtors' use of DCF models during the years of the sample.

[85] The slope of the regression line is not negative, and the r2 is 0.0003.

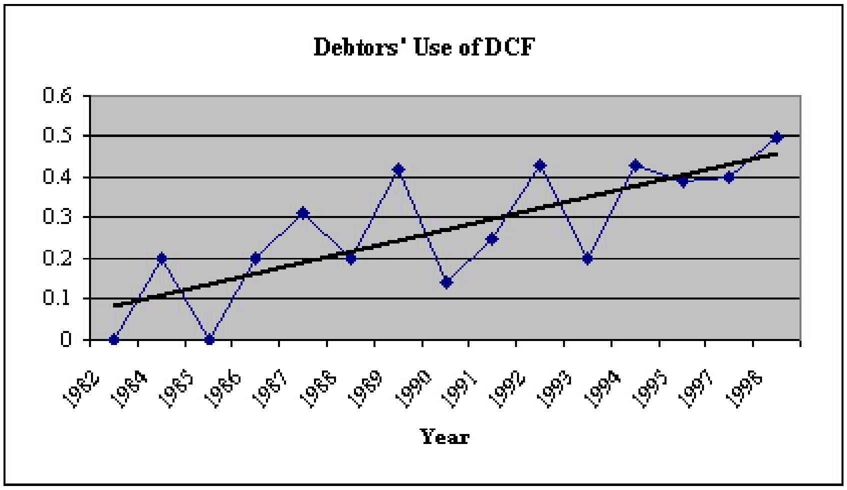
[86] This is, in itself, an intriguing finding. Using the language of physics, we can say that we have observed an "attractor", or pattern, among the CVS and DVS variables. This attractor is explained by parameters of the system, which must themselves be "off the attractor." Here, the attractor (i.e. the pattern we observe) is reported litigant behavior. Courts (more specifically: court allocation of the scarce resource of official recognition) are (part of? most of?) the condition-set that explains the attractor. An example from natural systems: it would be bizarre to ask, "How is magnetism affected by magnetism?" We ask how tides, or electrons, or the needle on my compass, are affected by magnetism. Each of those explananda shows an attractor that can be explained by the dynamical conditions we call "magnetism." But if we treat magnetism as the explanandum, the explanations have to be "off" the phenomenon we are trying to explain. We would have to explain magnetism in some other terms (e.g. dark matter).

[87] *See* J.C.Sprott, *Competition with Evolution in Ecology and Finance*, 325 PHYS. LETT. A 329 (2004); J.C. Sprott, et al., *Coexistence and Chaos in Complex Ecologies* (forthcoming, 2005) (on file with author).

<u>Figure 15</u>

Debtors' Use of DCF Valuation Models

Year	Mean
1982	0
1984	0.2
1985	0
1986	0.2
1987	0.31
1988	0.2
1989	0.42
1990	0.14
1991	0.25
1992	0.43
1993	0.2
1994	0.43
1995	0.39
1997	0.4
1998	0.5



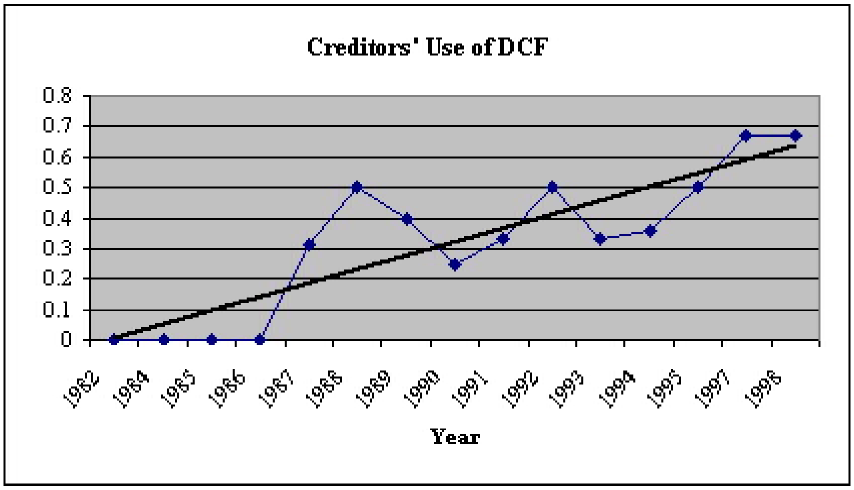
This figure shows simple descriptive evidence that Debtors' increased their use of DCF models during the years of this sample.[88] A mean of "zero" indicates that Debtors' did not use DCFs at all in the given year, while a mean of "one" would indicate that Debtors' used DCFs in every instance of a given year. The data tell a story of the DCF steadily gaining usage among Debtors' networks, until the point where it becomes a regular "arrow in the quiver" of Debtors' valuation arguments.

Figure 16 charts the Creditors' use of the DCF model over the same time period, showing an even stronger upwards trend than the Debtors' data.

[88] A mean of "zero" indicates that Debtors' did not use DCFs at all in the given year, while a mean of "one" would indicate that Debtors' used DCFs in every instance of a given year. In Figures 15-18, the variable is a binary and so only the mean is presented. Where the variable is a binary, the standard deviation is simply a transformation of the mean, and so adds no information to the means description of the distribution.

Figure 16
Creditors' Use of DCF Valuation Models

Year	Mean
1982	0
1984	0
1985	0
1986	0
1987	0.31
1988	0.5
1989	0.4
1990	0.25
1991	0.33
1992	0.5
1993	0.33
1994	0.36
1995	0.5
1997	0.67
1998	0.67



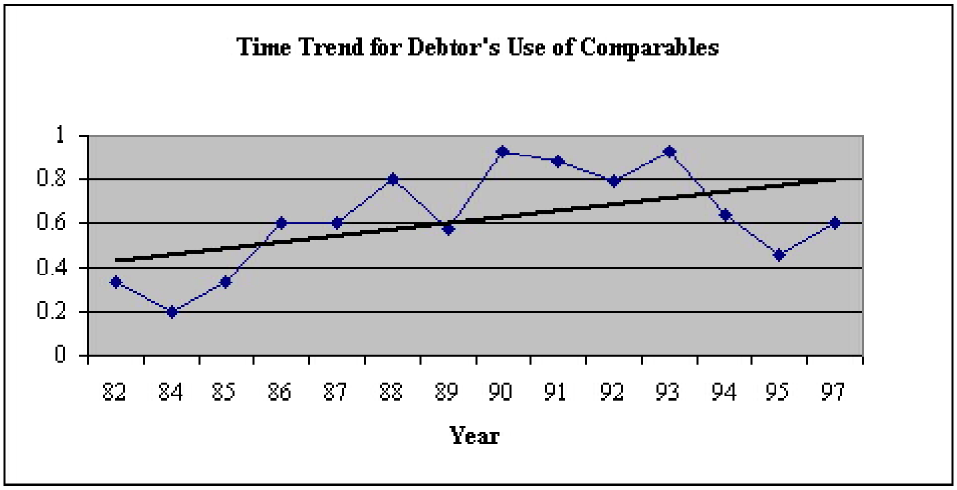
The data support an interpretation that the DCF valuation model entered the legal system in the early 1980s[89] and gradually rose to a level during the sample period where it became a common way of arguing about value.

Figure 17 shows the Debtors' use of "Comparables" valuation models over time.

[89] By the 1980s, DCF models were, of course, widely used in financial networks. Elsewhere I have argued that the DCF presents an example of inter-system form transportation, in which a form that is well established in one network (here, a financial-based network) is received into another network (here, a legal network). *See* Trujillo, *Self-Organizing Legal Systems*, 555, 558.

Figure 17
Debtors' Use of Comparables Valuation Models

Year	Mean
82	0.33
84	0.2
85	0.33
86	0.6
87	0.6
88	0.8
89	0.58
90	0.93
91	0.88
92	0.79
93	0.93
94	0.64
95	0.46
97	0.6

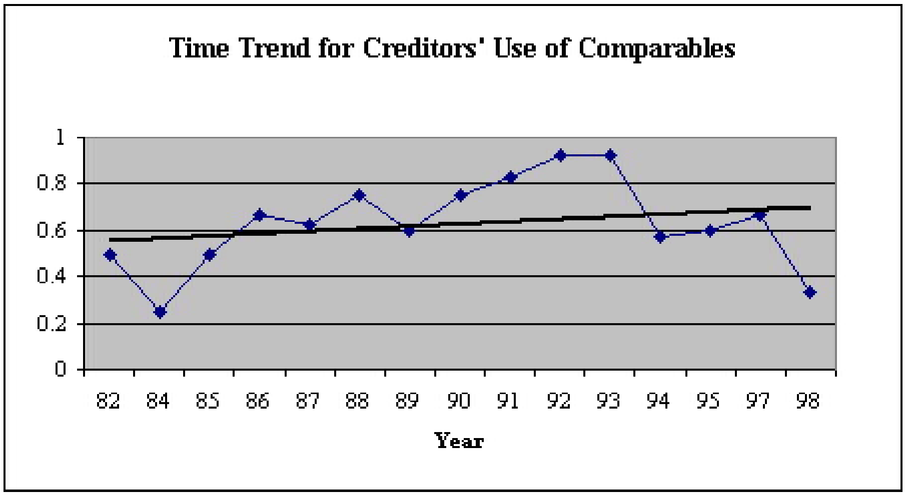


Note the period from 1990-1993) during which the mean is over .8. During this period, we can conclude that Debtors' use of "comps" had become normative.

Figure 18 shows the data for Creditors' use of comps.

Figure 18
Creditors' Use of Comparables Valuation Models

Year	Mean
82	0.5
84	0.25
85	0.5
86	0.67
87	0.62
88	0.75
89	0.6
90	0.75
91	0.83
92	0.92
93	0.92
94	0.57
95	0.6
97	0.67
98	0.33



Creditors' data show a trendline that is essentially flat, indicating that Comps was part of the Creditors' arsenal throughout the sample period. Note the same period from 1990-1993 that we saw in the Debtors' data. We can conclude that Parties' use of "comps" rose to a normative level, before tapering off somewhat during the mid-1990s.

Generally speaking, we can say that the introduction of such forms as the DCF, and Comps support our subjective impression of the database that, following some initial unfamiliarity, bankruptcy actors became more sophisticated in their handling of valuation disputes over time.[90]

Such a process of adaptation or learning is a form of self-organization. Adaptation, including the emergence of new systemic features, occurs as system actors acquire expertise by importing valuation forms from outside the legal system, and gradually coming to master those forms.[91] From a courts-based perspective, one could say that the judges' exercise of their discretion gradually became more transparent. From a clients-based perspective, one could say that the "risk" (defined as the degree of dispersion of possible outcomes[92]) of submitting a dispute to adjudication declined over time.

Note that an increase in the valuation sophistication of bankruptcy actors does not mean that those actors move closer to a "common" valuation standard. The phenomenon

---

[90] *See, e.g.*, John B. Butler, III, *Valuation of Secured Claims under 11 U.S.C. 506(a)*, 89 COMMERCIAL L. J. 342, 342 (1984) (noting increase in bankruptcy valuation proceedings as "bankruptcy practitioners become more practiced in their art"); Chaim J. Fortgang & Thomas Moers Mayer, *Valuation in Bankruptcy*, 32 U.C.L.A. L. REV. 1061 (1985) (sketching basic finance principles for bankruptcy valuation proceedings). The articles from Butler (a practitioner in a South Carolina law firm) and Fortgang & Mayer (practitioners at Wachtell, Lipton in New York City) are themselves data indicating the bar's gradually increasing financial sophistication in trying valuation disputes.

[91] This learning occurs, first as judges become educated by experts about financial valuation methods, and then as similar issues recur multiple times, deepening the courts' experience and increasing their confidence to criticize and depart from experts' models. *See* Trujillo, *Self-Organizing Legal Systems*, 536-39, 547-53 (discussing inter-system "transportation-cum-distortion" of forms).

[92] *See e.g.* WILBUR G. LEWELLEN, THE COST OF CAPITAL (1969).

of "rising sophistication" in conducting valuation determinations, e.g. the importation of "managerial norms" into the courts,[93] seems largely independent of whether the valuer uses a common or independent theory of value.[94]

4. Summary of Findings

The data offer preliminary support for the conclusion that some aspects of the U.S. bankruptcy legal system show a tendency to self-organize. Conclusive evidence of self-organizing dynamics in a legal system could have substantial jurisprudential significance.

We know that simple deterministic dynamics do not explain the data we observe in legal systems. Since the decline of legal formalism, the dominant mode of explanation has been to attribute a randomness, or nondeterminism, to legal system dynamics and to suggest that any observable patterns are due to exogenous ordering (such as decisionmaker bias) that affect legal ordering inter-systemically.[95] Evidence of self-organizing dynamics would suggest the possibility that at least some of the patterns we observe are generated by deterministic dynamics operating intra-systemically.

Legal realism and critical legal studies moved legal explanation from simple determinism to nondeterminism plus exogenous ordering. Now, evidence of self-organization may move legal explanation back in the direction of a *complex determinism*.

---

[93] *See, e.g.*, Oliver E. Williamson, *Chester Barnard and the Incipient Science of Organization*, in ORGANIZATION THEORY 172 (Oliver E. Williamson ed., expanded ed. 1995).
[94] For example, the DCF is a sophisticated valuation technique, yet is based entirely upon projections of an individual firm's peculiar use of the property.
[95] The "Efficient Capital Markets Hypothesis" ("ECMH"), as a form of financial explanation, follows a similar course. The ECMH attributes a randomness to the trade-to-trade movements of stock prices and argues that only exogenous forces (i.e. the "surprise" of new information) explain price movements. *See, e.g.*, Eugene Fama, *Efficient Capital Markets: A Review of Theory and Empirical Work*, 24 J. FIN. 383 (1970).

Self-organization would suggest that at least some patterns in legal data are generated by a complex and nonlinear deterministic dynamics, i.e. chaos.[96] Put another way, the data in this study may falsify quantitatively the jurisprudential hypothesis that legal dynamics are random rather than chaotic.

Any such argument based on these data is, however, far from conclusive because of the low number of observations (i.e. 180, with some variables possessing fewer than 180 observations). These data are simply not sufficient to take advantage of all that time series analysis has to offer in substantiating robust conclusions about the dynamics of this legal system.[97]

This research on legal systems' self-organization can take at least three possible future directions. First, we can go about collecting much more data, in order to get the number of observations up to the point that will support robust conclusions. Second, we can pursue a number of methodologies that have been developed to assist in the analysis of low-observation data-sets.[98] Finally, we can construct a mathematical model of the system dynamics we want to explain, and that model will generate sufficient observations to support full time series analysis.

---

[96] *See* JULIEN CLINTON SPROTT, CHAOS AND TIME-SERIES ANALYSIS 20 (2003) ("chaotic systems…have a number of characteristics: 1. They are aperiodic (they never repeat); 2. They exhibit sensitive dependence on initial conditions (and hence they are unpredictable in the long term); 3. They are governed by one or more control parameters, a small change in which can cause the chaos to appear or disappear; and 4. Their governing equations are nonlinear.").

[97] *See* JULIEN CLINTON SPROTT, CHAOS AND TIME-SERIES ANALYSIS 236-38 (2003) (discussing time-delayed space embedding).

[98] *See, e.g.*, Alberto Malinverno, *A Simple Method to Estimate the Fractal Dimension of a Self-Affine Series*, 17 GEOPHYSICAL RESEARCH LET. 1953 (1990); J. C. Gallant, et al., *Estimating Fractal Dimension of Profiles: Comparison of Methods*, 26 MATHEMATICAL GEOLOGY VOL. 455 (1994).

III. CONCLUSIONS

This Article has presented some initial results from the bankruptcy cramdown valuation database. I have been interested in explaining both the doctrine of bankruptcy valuation, and what that doctrine might tell us about the dynamics of the bankruptcy legal system.

Major findings of this study include:

1. Courts tend to "split the difference" in valuations much less frequently than we had expected (Figure 1).
2. Parties' valuation standards tend to be close together. We observe less "highball/lowball" litigation strategy than we had expected (Figure 6).
3. Bankruptcy Courts are much more likely to use a "common" valuation standard when the valuation includes a calculation for the time value of money (Figure 6).
4. There seems to be some geographic distribution of Courts' acceptance of valuation models, with Southern states more likely to accept "soft" valuation models (e.g. owner testimony, evidence of general market conditions), and non-Southern states more likely to accept "hard" valuation models (e.g. discounted cash flow models, comparables, cost) (Figure 8).
5. There is evidence that bankruptcy system content tends to self-organize. Specifically, variation decreases over time (Figures 9, 10, 11, 12, 13, 14), and we see evidence of form transportation and norm emergence (Figures 15, 16, 17, 18).

Areas of future research based on these data, include an exploration of the relationship of "valuation models" to "success," to find out whether certain arguments about valuation achieve client victory more often than others. Another area of future research involves the sub-set of observations classified as “finance” cases. These observations are rich in detail, and perhaps time-series analysis could yield important information about how the bankruptcy legal system has received and utilized quantification practices from outside the legal system.

More generally, these data coupled with a methodology based on quantification and statistical inquiry may put us in a position to make progress on a fundamental question in legal studies: whether legal system dynamics are essentially nondeterministic.

APPENDIX: METHODOLOGY

Reliability

Most of the variables coded in this study are objective (e.g. date of case, presence of finance element, amount of parties' and court's valuation). The "valuation standard" variable, however, involves a subjective judgment made by the coder; deciding, for example, to code one observation as a "4" and another observation as a "3". Standard social science methodology directs that, when there is a subjective element involved in the coding, the research take special steps to test the reliability of the coding.

Accordingly, after the completion of the initial codes upon which the statistics are based, a subset of the data for the dependent variable was independently re-coded by different coders. The results of that "reliability re-code" are presented in the Figure 19.

Figure 19
Reliability Re-code Statistics

Difference of +/- 1	
Total Number of Cases Recoded	150
Total Number of Cases with a Difference of +/- 1	6
Percentage Difference	4.00%
Percentage Same	96.00%
Difference of +/- 2	
Total Number of Cases Recoded	150
Total Number of Cases with a Difference of +/- 2	0
Percentage Difference	0.00%
Percentage Same	100.00%
Difference of +/- 3	
Total Number of Cases Recoded	150
Total Number of Cases with a Difference of +/- 3	2
Percentage Difference	1.33%
Percentage Same	98.67%

Difference of +/- 4

Total Number of Cases Recoded	150
Total Number of Cases with a Difference of +/- 4	0
Percentage Difference	0.00%
Percentage Same	100.00%

Difference of +/- 5

Total Number of Cases Recoded	150
Total Number of Cases with a Difference of +/- 5	0
Percentage Difference	0.00%
Percentage Same	100.00%

Difference of +/- 6

Total Number of Cases Recoded	150
Total Number of Cases with a Difference of +/- 6	0
Percentage Difference	0.00%
Percentage Same	100.00%

Difference of +/- 7

Total Number of Cases Recoded	150
Total Number of Cases with a Difference of +/- 7	0
Percentage Difference	0.00%
Percentage Same	100.00%

Difference of +/- 8

Total Number of Cases Recoded	150
Total Number of Cases with a Difference of +/- 8	0
Percentage Difference	0.00%
Percentage Same	100.00%

Difference When Code Assigned No Value and Recode Assigned Value

Total Number of Cases Recoded	150
Total Number of Cases with a Difference	7
Percentage Difference	4.67%
Percentage Same	95.33%

Total Difference

Total Number of Cases Recoded	150
Total Number of Cases with a Difference	15
Percentage Difference	10.00%
Percentage Same	90.00%

The original coding process yielded 180 observations. The re-code took 50 observations (i.e. 27.8% of the observations in the database) and independently assigned codes for the three “valuation standard” variables: CtVS, DVS, and CVS. This process thus yielded 150 re-codes.

Figure 19 shows that, of the 150 re-codes, 135 received the same code as in the initial coding process, resulting in a “re-code reliability rate” of .9, which is well within the boundaries of reliability for social science research.

Access to Documents of Potential Interest

Documents useful for the verification and replication of this study are maintained at http://www.law.wisc.edu/facstaff/pubs.asp?ID=429

These documents include: (i) a list of the 388 cases that constituted the initial database; (ii) the coding instrument; (iii) the spreadsheet of coding results; (iv) an explanation of the statistical tests (multinomial regression, Wald, and likliehood ratio) that explain the reconfiguration of the “valuation standard” variable from an eight-point continuum to a three-point continuum.