Privatization, Free Riding, and Industry-Expanding Lobbying

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Abstract

Critics of privatization have argued that privatization distorts the political system by giving private contractors an incentive to self-interestedly lobby for changes in the substantive law. This paper argues that this charge is not in general true when public-sector actors (for instance, a public-sector union) can also self-interestedly lobby for similar changes.

I show in a formal model that, where the effectiveness of political advocacy in favor of a reform depends on the total amount of political expenditures in support of that position, and where there is no collusion among private contractors and between public and private sectors, privatization decreases political advocacy if the total benefit of the public-sector actors exceeds that of any firm. If there is collusion among private contractors, privatization decreases political advocacy if the total benefit of the public-sector actors exceeds that of the private sector. If all actors collude, privatization decreases political advocacy if the per-unit benefit of the public-sector actors exceeds that of the private sector.

I extend the model to the cases where the effectiveness of political advocacy also depends on the amount of political expenditures by the opponents of the reform, where political expenditures can affect not only the probability of success of a reform but also its content, and where the effectiveness of political expenditures does not merely depend on the sum of all expenditures on a given side.

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I Introduction

The claim that private contractors lobby to influence public policy (see, for instance, Eisenhower 1961, p. 1038; Michaels 2004, pp. 1015–16; Zarate 1998, pp. 87–89) is commonly used as an argument against privatization. For instance, opponents of prison privatization charge that private prison firms support pro-incarceration policy, like longer sentences and less diversion to drug treatment programs, and that their self-interested advocacy is a distortion of the political process (see, for instance, Schlosser 1998, p. 64; Dolovich 2005, p. 542; Shichor 1995, pp. 236, 256; Sarabi and Bender 2000, pp. vii, 21).

I argue in this paper that this claim is not true in general. Under plausible assumptions — chiefly the assumption that the probability that a reform passes is a function of the total amount of expenditures — dividing an industry among
a greater number of providers can lead to less advocacy, because advocacy is a public good and more actors have greater difficulty overcome their collective action problem. Shifting part of the industry to lower-profit providers, as when a monopolistic industry is partly converted to a more competitive one, also leads to less advocacy. The actor that benefits most from the system — weighted both by industry share and by per-unit level of profit — does all the lobbying, and the smaller actors free-ride. For instance, if the public sector remains the dominant actor after some degree of privatization, then privatization decreases total advocacy, because it reduces the advocacy (since it reduces the total benefit) of the public sector while keeping the private sector’s advocacy at zero.

I then discuss how this result changes under different assumptions about collusion among industry actors, and what effect privatization has on the advocacy of industry opponents. I extend the model to the case where expenditures can not only affect the probability that a reform passes but also the substantive content of the reform. Finally, I extend the paper to the case where the probability of success depends not on the total amount of expenditures, but rather on the separate amounts of expenditures by both sectors. The result in this case is not total free-riding, as before, but partial free-riding; the net effect of privatization is ambiguous, though I show that it depends in part on which sector is more effective at political advocacy.

II The Model

A Assumptions

The model has the following assumptions. An industry, whose size is normalized to 1, is divided into small, separate projects that can be run by either the public sector or a private firm. These public or private prison providers operate for infinitely many periods, and because private contracts run out periodically, many projects are put out to bid to the private sector each period. Actors in both the public and private sectors are rational, risk-neutral expected utility maximizers whose utility depends only on their financial well-being, as described below. All workers have reservation wage $W_m$.

**Public sector** The public sector runs a proportion $\alpha_g$ of the projects. The public sector wage is:

$$W_g > W_m$$  \hspace{1cm} (1)

per unit. This is observed empirically (see Section IV.B) and is theoretically plausible; Fraja (1993, p. 466), for instance, argues that public wages are usually higher than those of their private counterparts in part because government employers may be fulfilling social objectives through public employment. I take no view on whether the wage difference is due to unionization or public sector employment (see Robinson and Tomes 1984, pp. 107–08; Freeman 1986,
p. 53; Ehrenberg and Schwartz 1986, pp. 1228–30); all that matters here is that, empirically, public wages are higher.

The public sector employees are presumed to act collectively through their union, membership in which is mandatory. The public sector union is a rent maximizer, so the benefit of provision to the public sector employees is:

$$\Delta W \equiv W_g - W_m$$  \hspace{1cm} (2)

per unit (see, for example, Fung 1995, p. 452; Hirsch and Prasad 1995, p. 64 and n. 5; Dowrick and Spencer 1994, p. 329; Fraja 1993, pp. 459–60; Addison and Hirsch 1989, p. 84; Oswald 1985, p. 162; Pencavel 1985, pp. 201–02; Calvo 1978, p. 68; Menif 1971, p. 22; Rosen 1970, pp. 269–70, using the rent maximization objective).

**Private sector** In the private sector, there is a fixed number of firms $n$, all equally efficient and able to produce at unit cost $C$. Each firm $i$ runs a proportion $\alpha_i$ of the system. The contract price per unit is $P$, so the benefit of provision to the private sector is:

$$\Pi \equiv P - C$$  \hspace{1cm} (3)

per unit.

Appendix A explains why it is harmless to assume that there is only a single form of advocacy and a single type of benefit. I make two assumptions about firm shares under privatization:

- First, I assume that individual firm shares are continuous and differentiable functions of $\alpha_g$; this implies that when privatization increases (in other words, when $\alpha_g$ falls) by a small amount, the individual $\alpha_i$ do not jump discontinuously.

- Second, I interpret privatization as taking certain projects away from the government and awarding them to the private sector according to some allocation method. When privatization increases, I assume that each private firm keeps its original projects, and at least the largest firm acquires some of the formerly government projects. This implies that as $\alpha_g$ falls, the largest $\alpha_i$ increases.\(^1\) Similarly, when privatization decreases ($\alpha_g$ rises), at least the largest $\alpha_i$ falls.

**Political advocacy** Before service providers operate, actors in the industry can advocate for a reform that would expand their industry by a proportion $\varepsilon$ (that is, from 1 to $1 + \varepsilon$). I first consider a one-sided model, where the advocacy of the opposing side is taken as given. (This assumption is relaxed in Section

\(^1\)If we interpret $\alpha_i$ as the probability that a private firm $i$ gets any project, then $\frac{\alpha_i}{\alpha_m}$ is the conditional probability that it gets the project given that the project goes to the private sector. So, as $\alpha_g$ falls (and thus $\alpha_m$ rises) by $\varepsilon$, $\alpha_i$ rises to $\frac{\alpha_i}{\alpha_m}(\alpha_m + \varepsilon) = \alpha_i + \frac{\alpha_i \varepsilon}{\alpha_m} \forall i$. But I do not need such a strong assumption.
III.A. By spending an amount $e_i$ on industry-expanding advocacy, actor $i$ can affect the probability of this reform. (In the case of prisons, this assumes that “pro-incarceration” policy (see Volokh 2008, p. 20) actually increases the extent of incarceration, rather than decreasing it through deterrence or, say, more lenient behavior by other actors not covered in the reform.)

I make the following assumptions about the probability of the policy change:

- The probability $p(e) \in [0, 1]$ is a continuous and thrice differentiable function of $e \equiv \sum_{i=1}^{n} e_i$; that is, only the total amount of advocacy matters.
- $p' > 0$.
- Decreasing returns to advocacy kick in eventually:

\[
\begin{cases}
    p''' < 0 \\
    \exists e_t = \min\{\forall e > \bar{e}, \ p''(e) < 0\}
\end{cases}
\]

Decreasing returns do not necessarily begin immediately (see Mueller 2003, p. 483 fig. 20.1; Olson 1965, p. 22), though an assumption of decreasing marginal returns everywhere is also common in the literature (see Pecorino 1998, p. 654; Baron 1989, p. 54; Austen-Smith 1987, pp. 128, 130, 135).

- $\exists e'$ such that:

\[
(p(e') - p(0))\alpha_g \epsilon W > e',
\]

and $\exists e''$ such that:

\[
(p(e'') - p(0))\alpha_{i^*} \epsilon \Pi > e'' \text{ for } i^* = \arg \max_i \alpha_i.
\]

That is, for both the public sector and the largest private firm, there is some level of advocacy that makes them better off than no advocacy at all. This rules out the uninteresting case where some sector would be satisfied even if there were no advocacy at all.

B The Effect of Privatization on Political Advocacy

Proposition 1 Under these assumptions:

- No collusion. If the public sector and all private firms act separately: If $\alpha_g \Delta W > \max_i \{\alpha_i\}\Pi$, increasing privatization decreases industry-expanding advocacy. Otherwise, increasing privatization increases industry-expanding advocacy.

- Private sector collusion. If all private firms collude with each other: If $\alpha_g \Delta W > \alpha_m \Pi$, where $\alpha_m \equiv \sum_{i=1}^{n} \alpha_i$, increasing privatization decreases industry-expanding advocacy. Otherwise, increasing privatization decreases industry-expanding advocacy.
• Full collusion. If the public sector and all private firms collude with each other: If $\Delta W > \Pi$, increased privatization decreases industry-expanding advocacy. If $\Delta W < \Pi$, increased privatization increases industry-expanding advocacy. If $\Delta W = \Pi$, increased privatization has no effect.

Proof

No collusion The public employees choose $e_g$ to maximize:

$$ \pi_g(e_g) \equiv \alpha_g(1 + p(e)\varepsilon)\Delta W - e_g. $$

(7)

The first-order condition of this problem is:

$$ p'(e^*)\alpha_g\varepsilon\Delta W \leq 1 \text{ (with equality if } e^*_g > 0) $$

$$ \Rightarrow e^* \geq (p')^{-1}\left(\frac{1}{\alpha_g\varepsilon\Delta W}\right) \text{ (with equality if } e^*_g > 0). $$

(8)

Consider the function $f(e) \equiv [p(e) - p(0)]\alpha_g\varepsilon\Delta W - e$. It is clear that $f(0) = 0$; by assumption, $\exists e' > 0$ such that $f(e') > 0$; and it is likewise clear that $f(\infty) = -\infty$. Therefore, $f(e)$ has an interior maximum, and at that maximum we must have $f'(e) \equiv p'(e)\alpha_g\varepsilon\Delta W - 1 = 0$. The second derivative is $f''(e) \equiv p''(e)\alpha_g\varepsilon\Delta W$, which is negative if $e > e_t$; thus, the maximum of $f(e)$ occurs at some $e > e_t$. These are the same derivatives as those of the public sector’s objective function, so the public sector’s first-order condition (with equality) also has a solution, which is a maximum.

Each private sector firm $i$ chooses $e_i$ to maximize:

$$ \pi_i(e_i) \equiv \alpha_i(1 + p(e)\varepsilon)\Pi - e_i. $$

(9)

The first-order condition of this problem (analogously to the public sector case) is:

$$ p'(e^*)\alpha_i\varepsilon\Pi \leq 1 \text{ (with equality if } e^*_i > 0) $$

$$ \Rightarrow e^* \geq (p')^{-1}\left(\frac{1}{\alpha_i\varepsilon\Pi}\right) \text{ (with equality if } e^*_i > 0). $$

(10)

By an analogous argument, the first-order condition with equality has a unique solution greater than $e_t$, which is a maximum.

Case 1. If $\alpha_g\Delta W > \max_i\{\alpha_i\}\Pi$, then:

$$ \begin{cases} p'(e^*)\alpha_g\varepsilon\Delta W = 1 \\ p'(e^*)\alpha_i\varepsilon\Pi < 1 \forall i \end{cases}. $$

(11)

The public sector does all the advocacy, and $e^*_i = 0 \forall i$. Denote the amount of public sector advocacy, as a function of the public sector share, by:

$$ e^*(\alpha_g) = (p')^{-1}\left(\frac{1}{\alpha_g\varepsilon\Delta W}\right). $$

(12)
Public sector advocacy is increasing in $\alpha_g$, since:

$$\frac{de^*}{d\alpha_g} = -\frac{1}{\alpha_g^2 \varepsilon \Delta W p''(p')^{-1}\left(\frac{1}{\alpha_g \varepsilon \Delta W}\right)} > 0. \tag{13}$$

Thus, increasing privatization (that is, decreasing $\alpha_g$) decreases total advocacy. Conversely, decreasing privatization (or eliminating it entirely) increases total advocacy.

**Case 2.** If $\alpha_g \Delta W < \alpha_i \Pi$ for some $i$: Let $I$ denote the set of $i$ such that $i = \arg \max_j \{\alpha_j\}$. Then all firms in $I$, as the dominant actor(s), do all the advocacy, and $e^*_g = e^*_i = 0 \forall i \notin I$. (If there is more than one $i \in I$, those firms advocate as much as they would if they were a single firm; the division of advocacy among those firms is arbitrary. Case 3 below explains the mechanism.) Denote $\hat{\alpha} = \max_i \{\alpha_i\}$. Total private sector advocacy, $e^*_m(\hat{\alpha}) = (p')^{-1}\left(\frac{1}{\alpha \varepsilon \Pi}\right)$, is increasing in $\hat{\alpha}$, by an analogous argument to Case 1. By assumption, as privatization increases (up to privatization of the entire industry), $\hat{\alpha}$ increases, so the private sector’s advocacy increases, and thus total advocacy increases.

**Case 3.** If $\alpha_g \Delta W = \alpha_i \Pi = K$ for some $i \in I$, then the first-order conditions of the public sector and of the firms in $I$ hold with equality simultaneously, and $p'(e^*) = \frac{1}{\varepsilon K}$.

Any division of advocacy expenses between the public sector and the firms in $I$ can be sustained as a Nash equilibrium. Let $(e_g, e_1, ..., e_n)$ be any division of advocacy expenses such that, $\forall i, e_i = 0$ if $\alpha_i \Pi < K$, and:

$$e_g + \sum_{i \in I} e_i = (p')^{-1}\left(\frac{1}{\varepsilon K}\right). \tag{14}$$

Then the first-order conditions of the public sector and of the firms in $I$ are satisfied, and all other first-order conditions hold with strict inequality. Therefore, this division is individually rational for each firm, so no firm would benefit from deviating. This division is thus a Nash equilibrium.

If $\alpha_g$ decreases, by assumption, all the $\alpha_i$ increase. Then we are back in Case 2, and total advocacy increases. If $\alpha_g$ increases, the largest $\alpha_i$ falls; then we are in Case 1, and advocacy also increases.

**Private sector collusion** As before, the public sector’s first-order condition is:

$$p'(e^*)\alpha_g \varepsilon \Delta W \leq 1 \text{ (with equality if $e^*_g > 0$)}$$

$$\Rightarrow e^* \geq (p')^{-1}\left(\frac{1}{\alpha_g \varepsilon \Delta W}\right) \text{ (with equality if $e^*_g > 0$).} \tag{15}$$

For the same reasons as in subsection a above, the first-order condition with equality has a unique solution $e^*_g > e_i$ for any $\alpha_g$, which is a maximum.

The private sector chooses $e_m$ to maximize:

$$\pi_m(e_m) \equiv \alpha_m (1 + p(e) \varepsilon) \Pi - e_m, \quad \text{ (16)}$$
where \( \alpha_m = \sum_{i=1}^{n} \alpha_i \) over all private firms. The first-order condition of this problem is:

\[
p' \left( e^* \right) \alpha_m \varepsilon \Pi \leq 1 \quad \text{(with equality if } e_m^* > 0)\]

\[
\Rightarrow e^* \geq \left( p' \right)^{-1} \left( \frac{1}{\alpha_m \varepsilon \Pi} \right) \quad \text{(with equality if } e_m^* > 0). \tag{17}
\]

For the same reasons as above, the first-order condition with equality has a unique solution \( e_m^* > e_t \) for any \( \alpha_m \), which is a maximum.

**Case 1.** If \( \alpha_g W > \alpha_m \Pi \), as before, we have:

\[
\begin{cases}
  p'(e^*) \alpha_g \varepsilon \Delta W = 1 \\
  p'(e^*) \alpha_m \varepsilon \Pi < 1
\end{cases} \tag{18}
\]

The public sector does all the advocacy, and \( e_m^* = 0 \). The amount of public sector advocacy, \( (p')^{-1} \left( \frac{1}{\alpha_g \varepsilon \Delta W} \right) \), is increasing in \( \alpha_g \); thus, increasing privatization decreases total advocacy, and decreasing privatization (or eliminating it entirely) increases total advocacy.

**Case 2.** If \( \alpha_g W < \alpha_m \Pi \), then the private sector, as the dominant sector, does all the advocacy, and \( e_g^* = 0 \). The private sector’s advocacy, \( (p')^{-1} \left( \frac{1}{\alpha_m \varepsilon \Pi} \right) \), is increasing in \( \alpha_m \); thus, increasing privatization (up to privatizing the entire industry) increases advocacy.

**Case 3.** If \( \alpha_g W = \alpha_m \Pi = K \), then both first-order conditions hold with equality simultaneously, \( p'(e^*) = \frac{1}{K} \), and again any division of advocacy expenses between the public and private sectors can be sustained as a Nash equilibrium. If \( \alpha_m \) increases, then we are back in Case 2; the private sector takes over all the advocacy, which increases, and the public sector falls to 0, so the total amount of advocacy increases.

**Full collusion** The colluding public and private sectors choose \( e \) (and divide that contribution among themselves in some way) to maximize:

\[
\pi(e) = (1 + p(e) \varepsilon)(\alpha_g \Delta W + \alpha_m \Pi) - e. \tag{19}
\]

This has an interior maximum, by a reasoning analogous to that given above. The first-order condition is:

\[
p'(e^*) = \frac{1}{\varepsilon (\alpha_g \Delta W + \alpha_m \Pi)}. \tag{20}
\]

Differentiating, we obtain:

\[
p''(e^*) \frac{de^*}{d\alpha_m} = -\frac{\varepsilon (\Pi - \Delta W)}{(\alpha_g \Delta W + \alpha_m \Pi)^2}
\]

\[
\Rightarrow \frac{de^*}{d\alpha_m} = -p''(e^*) \frac{\varepsilon (\Pi - \Delta W)}{(\alpha_g \Delta W + \alpha_m \Pi)^2}. \tag{21}
\]
which is negative if \( \Pi < \Delta W \), positive if \( \Pi > \Delta W \), and 0 if \( \Pi = \Delta W \). This effect holds for all values of \( \alpha_m \). Thus, if \( \Pi < \Delta W \), increasing privatization (up to privatizing the entire industry) decreases advocacy, and decreasing privatization (up to eliminating it entirely) increases advocacy. The opposite holds if \( \Pi > \Delta W \); and if \( \Pi = \Delta W \), privatization never has any effect.

C Discussion

As an application of this model to prisons, Section IV gives some back-of-the-envelope estimates of the parameters involved. These informal estimates suggest that, in the prison industry, \( \alpha_p \Delta W > \max_i \{ \alpha_i \} \Pi \), that \( \alpha_p \Delta W > \alpha_m \Pi \), and that \( \Delta W > \Pi \) — that is, at current levels of privatization, wages, and profits, the public prison sector is the dominant actor, regardless what assumptions we make about the form of collusion.

This general result is familiar from the literature on public goods: Any degree of fragmentation in the industry reduces expenditures on public goods that benefit the whole industry, because each actor receives only a portion of the benefit from advocacy attributable to his contribution to the public good. The stark free-riding result presented here occurs when (as here) utility is quasi-linear in income — that is, when the public good doesn’t affect the marginal utility of income — which is a reasonable assumption with business firms, which are unlikely to enjoy their consumption more if their industry is larger.

The result here depends on the form of collusion among industry actors. Under privatization, projects are often distributed by auction (see Klemperer [2004, pp. 28–29 and nn. 75–77] on collusion in auctions). But collusion may not always be practically significant. In the prison context, for instance, if it is true that \( \alpha_p \Delta W > \max_i \{ \alpha_i \} \Pi \), that \( \alpha_p \Delta W > \alpha_m \Pi \), and that \( \Delta W > \Pi \) (see Section IV), then whether there is collusion, and if so, among whom, does not matter much; at current levels, privatization is expected to decrease political advocacy regardless of the collusion assumption.

III Extensions of the Model

A A Two-Sided Model

Now I relax the assumption that advocacy by those who are opposed to the expansion of the industry is fixed. I alter the assumptions of Proposition 1, so that the probability of expanding the industry is \( p(\epsilon, y) \), where \( \epsilon \) is the expenditure of service providers and \( y \) is the expenditure of their opponents (for instance, anti-incarceration forces such as the ACLU or drug treatment providers (see Volokh 2008, pp. 44–46)) to prevent the increase \( \epsilon \). The disutility of these forces from the expansion of the industry is \( B < 0 \).

The assumptions about \( p \) are the same as in the basic model, except as amended by the following:
• \( p \) is a continuous and thrice differentiable function of \( e \equiv \sum_{i=1}^{n} e_i \) and \( y \); that is, only the total amount of advocacy by each side matters.

• \( p_1 > 0 \) and \( p_2 < 0 \) — more advocacy by the pro-incarceration side increases the probability and more advocacy by the anti-incarceration side decreases it.

• Decreasing returns to each type of advocacy kick in eventually:

\[
\begin{align*}
\begin{cases}
\begin{align*}
p_{111} &< 0 \\
p_{222} &> 0
\end{align*}
\end{cases},
\forall y, \exists e_t(y) = \min\{\tilde{e}(y) | \forall e > \tilde{e}(y), p_{111}(e) < 0\}
\end{align*}
\]

\[
\begin{align*}
\forall e, \exists y_t(e) = \min\{\tilde{y}(e) | \forall y > \tilde{y}(e), p_{222}(e, y) > 0\}
\end{align*}
\]

\[
\text{\( \forall y, \exists e' \) such that:}
\]

\[
(p(e', y) - p(0, y))\alpha_g\varepsilon\Delta W > e',
\] (23)

and \( \exists e'' \) such that:

\[
(p(e'', y) - p(0, y))\alpha_m\varepsilon\Pi > e''.
\] (24)

Similarly, \( \forall e, \exists y' \) such that:

\[
(p(e, y') - p(e, 0))\varepsilon B > y'.
\] (25)

That is, for the public sector, private sector, and anti-incarceration forces, there is some level of advocacy that makes them better off than no advocacy at all; this rules out the uninteresting case where some actor would be satisfied even if there were no advocacy at all.

The private and public sectors’ objective functions remain the same, with \( p(e) \) replaced by \( p(e, y) \). The objective of the anti-incarceration forces (assumed to be a unitary black box), taking advocacy into account, is:

\[
\pi_y(y) \equiv (1 + p(e, y)\varepsilon)B - y.
\] (26)

**Proposition 2** Under these assumptions and if all private firms collude with each other: Whether privatization decreases industry-expanding advocacy depends on whether \( \alpha_g\Delta W > \alpha_m\Pi \). Otherwise, increasing privatization increases industry-expanding advocacy. In either case, increased privatization has an ambiguous effect on anti-industry-expanding advocacy.

**Proof.** See Appendix B. □

It is easy to show that the results are analogous using the other collusion assumptions from Proposition 1. If \( \alpha_g\Delta W > \alpha_m\Pi \), then, as in the previous
model, the amount of industry-expanding advocacy goes down, because $e_m$ remains zero and $e_g$ falls with privatization (because of the same public goods problem). The “total amount” of advocacy $e + y$ is probably not meaningful in this model; the more relevant quantity is the probability $p(e, y) - p(0, 0)$, which is the total effect of all advocacy. We can interpret $p(0, 0)$ as the probability that the policy change happens anyway under “fair deliberative conditions” (see Dolovich 2005, p. 515). In any event, the effect of privatization on $p(e, y)$ is ambiguous without further assumptions.

B  Expenditures and Substantive Influence on Policy

Suppose that $\varepsilon$ is not exogenous but can be influenced by expenditures. Then the probability of success (in a one-sided model) can be expressed not as $p(e)$ but as $p(\varepsilon(e), e)$, where $p_\varepsilon < 0$, $p_e > 0$, and $\varepsilon' > 0$.

Proposition 3 If the substantive content of the reform can be influenced by expenditures (i.e., the reform is to increase the industry by $\varepsilon(e)$, and the probability that this reform prevails is $p(\varepsilon(e), e)$, where $p_\varepsilon < 0$, $p_e > 0$, and $\varepsilon' > 0$), the comparative static results of Proposition 1 remain unchanged.

Proof. See Appendix C. ■

Thus, even in a model where advocacy alters the substantive content of an industry-expanding reform — and not just the probability that the reform will succeed — privatization still decreases the advocacy of the dominant sector, which is the public sector if $\alpha_g \Delta W > \max_i \{\alpha_i\} \Pi$, $\alpha_g \Delta W > \alpha_m \Pi$, or $\Delta W > \Pi$ (depending on which collusion case we are in). Since private-sector advocacy remains 0, privatization thus still decreases total advocacy.

This expanded model is still fairly easy because the opponent of the initiative is the status quo, which, being the status quo, doesn’t act strategically. Things get more difficult if we instead make this a race between two candidates. The substantive influence effect — an actor’s desire to make the ultimate policy favor him, whoever wins — could make him contribute to both candidates simultaneously. Moreover, his contributions to one candidate can influence not only that candidate’s position but also that of his opponent. As a result, the marginal benefit of advocacy expenditures becomes quite a bit more complicated (see Mueller 2003, pp. 479–80, esp. p. 480 eq. (20.9)). Nonetheless, the qualitative result should be the same.

C  Partial Free-Riding

Instead of assuming that the probability of getting the change in policy is $p(e)$, where $e = \sum_{i=1}^n e_i$, let us assume that the probability of the policy change is $p(e_g) + q(e_m)$, where $e_g$ and $e_m$ are the respective contributions of the public and private sectors. The assumptions of this model are the same as those in Proposition 1, with $q$ behaving like $p$. The only exception is that, so that the
probabilities make sense, we also have \( p(\infty) + q(\infty) \leq 1 \). As before, \( p(0) + q(0) \), the world without self-interested advocacy, can be interpreted as the probability that the reform occurs under conditions of “fair deliberation.”

If the public and private sectors were colluding with each other, they would, given any total advocacy amount, allocate \( e_g \) and \( e_m \) optimally, and would then choose an optimal total advocacy amount (see Appendix A). But let us continue supposing that the two sectors are not colluding with each other (though the private firms are colluding among themselves), and each has a share \( \alpha_g \) and \( \alpha_m \).

**Proposition 4** Under these assumptions and if all private firms collude with each other, increasing privatization has an ambiguous effect on industry-expanding advocacy.

**Proof.** See Appendix D. □

In this model, both sectors can advocate, and no sector totally free rides off the other, as the smaller sector did in the previous model.

The result of this proposition makes sense: Privatization increases the advocacy of the private sector but decreases the advocacy of the public sector. Of course, these models are all polar cases; in principle, there can be other intermediate advocacy effectiveness functions \( p(e_g, e_m) \) or \( p(e_g, e_m, y) \) (or, more generally, \( p(e_x, e_g, e_m, y) \), where \( e_x \) is the industry-expanding advocacy from outside the industry). But they do show that concerns that privatization will increase the amount or effect of advocacy, or even that they run a risk of doing so, are unfounded unless one is more specific about the effectiveness and interaction of advocacy by the different sectors. The next proposition shows one way of being more specific:

For instance, suppose \( p(e) = \beta q(e) \); that is, for \( \beta < 1 \), the public sector is less effective at political advocacy, and becomes infinitely less effective as \( \beta \to 0 \).

**Proposition 5** Suppose \( p(e) = \beta q(e) \), where \( \beta < 1 \). Then there exists \( \tilde{\beta} \in (0, 1) \) such that, for all \( \beta < \tilde{\beta} \), privatization increases political advocacy. Similarly, suppose \( q(e) = \gamma p(e) \), where \( \gamma < 1 \). Then there exists \( \tilde{\gamma} \in (0, 1) \) such that, for all \( \gamma < \tilde{\gamma} \), privatization decreases political advocacy.

**Proof.** See Appendix E. □

It makes sense that, if \( \beta \) is low enough, privatization increases political advocacy, because then the industry is being shifted to a private sector that is relatively more competent. By symmetry, if \( q(e) = \gamma p(e) \) where \( \gamma < 1 \), the private sector is less effective, so if \( \gamma \) is low enough, the industry is being shifted to a relatively politically “slick” public sector.

Obviously, if the effect of privatization is ambiguous in this one-sided model, it remains ambiguous if we add anti-industry-expanding advocacy. Thus, there is no need to look into the two-sided model.
IV An Informal Application to Prisons

A Estimating $\alpha_m$

Of the 1.5 million prisoners under the jurisdiction of federal or state adult correctional authorities at the end of 2004, 6.6% were held in private facilities; this includes 13.7% of federal prisoners and 5.6% of state prisoners. Of the 34 states with at least some prisoners in private facilities, the percentages ranged from near 0.0% (seven states had percentages below 1.0%) to 42.1% (five states had percentages above 25.0%). Among these 34 states, the median percentage in private facilities was between 7.9% (Louisiana) and 9.2% (Georgia). (See U.S. Department of Justice 2004.)

One may be interested not in the proportion of total prisoners in private prisons, but rather in the proportion of the flow of prisoners that go to private prisons; that is, the proportion of marginal prisoners. Unfortunately, from year to year, this approach yields widely varying numbers because of small state-by-state numbers and temporary blips in prison populations.

For instance, Wyoming added 4 total prisoners from Dec. 31, 2000 to Dec. 31, 2001, but the private prison population increased by 191. This yields a marginal private share of 47.75% for Wyoming over that period. On the other hand, North Dakota’s prison population stayed constant at 1168 between June 30, 2002 and June 30, 2003, but its private prison population dropped from 40 to 1 during this period, which seems to yield a negative infinite marginal private share for North Dakota over that period. Taking longer periods doesn’t help much: Mississippi added 184 total prisoners between June 30, 2001 and June 30, 2005, but added 1394 private prisoners, for a marginal private share of 75.8%. All numbers here and in this portion of the text are taken from the spreadsheets associated with the Bureau of Justice Statistics’ Prisoners in 2004 report (U.S. Department of Justice 2004), and its predecessors, and the Prison and Jail Inmates at Midyear 2005 report (U.S. Department of Justice 2005), and its predecessors. Where numbers differ between reports, I have used the numbers from the latest report.

As my best stab at this problem, I offer the following: Over the period from June 30, 2000 to June 30, 2005, state systems added 88,500 total prisoners and 5703 private prisoners, for a marginal private share of 6.4%. (It makes sense that the marginal private share in state prisoners is about the same as the total private share in state prisoners, since the total private share has stayed about constant over the last five years.) Similarly, over this same five-year period, the federal system added 41,954 total prisoners and 22,615 private prisoners, for a marginal private share of 53.9%. (It likewise makes sense that the marginal private share in federal prisoners is so much larger than the total private share in federal prisoners, since the total private share has increased substantially over the last five years, from 2.8% at the end of 1999 to 14.4% in mid-year 2005.)

Adding this all up, total prisoners increased by 130,454, and private prisoners increased by 28,318, for a marginal private share of 21.7%.
B Estimating $\Delta W$

Public corrections officers’ wages are substantially higher than those of their private counterparts.\footnote{Some have claimed that, in particular cases, private wages have been competitive with public wages (Ring 1987, p. 29; Joel 1993, p. 65). But this isn’t the rule. Several sources indicate that private wages are competitive with market wages, which makes sense (David 2005, p. W6; Nelson 2003).} The 2000 Corrections Yearbook reported that, at private prisons responding to their survey, corrections officers faced an average entry-level salary of $17,628 and an average maximum salary of $22,082. By contrast, corrections officers at public prisons faced an average entry-level salary of $23,002 (30% more than at private prisons) and an average maximum salary of $36,328 (65% more). (See Criminal Justice Institute 2000a, p. 98; Criminal Justice Institute 2000b, 150–51. This is apparently the same data cited in AFSCME [2000] and Nelson [2003].)

So public-private salary differences span quite a big range, and these national averages conceal significant state-level variation. In Pennsylvania, the differences were somewhat higher — entry-level salaries were 39% higher in public prisons and maximum salaries were 125% higher — while in Texas, the differences were somewhat lower — entry-level salaries were 9% higher in public prisons and maximum salaries were 21% higher (compare Criminal Justice Institute 2000a, p. 98, with Criminal Justice Institute 2000b, pp. 150–51).

Other states are harder to compare: Three private prisons in California responded to the survey, but the public system didn’t submit its numbers. Nonetheless, we can get an idea of the differences by consulting a different source. A 2001 survey by the Corrections Compendium reports the annual starting salary for corrections officers in California as “$33,708/$38,988” (American Correctional Association 2001, p. 8). Even the lower one of these numbers is more than twice the average starting salary for corrections officers at the three reporting private prisons in California, which is $16,310 (see Criminal Justice Institute 2000a, p. 98). (But keep in mind that the numbers may have been gathered differently in the Yearbook and in the Compendium surveys. Promisingly, the two sources overlap for starting salaries in Arizona and Oklahoma. They’re different for other states, but then again, salaries may also have changed between 2000 and 2001.) Another source reports that California public guards’ average base salary was boosted to $65,000 a year in 2002 from about $50,000 (see Los Angeles Times 2002, California Metro Section, p. 10). This $50,000 number (presumably for 2001) is presumably less than the average maximum salary, but even that is more than twice the average 2000 maximum salary for corrections officers at the three reporting private prisons in California, which is $22,174 (Criminal Justice Institute 2000a, p. 98).

One limitation of the Corrections Yearbook numbers is that only some private prisons responded to their survey. For a few states, only one private prison responded to the survey (but the public numbers are reported for the entire system). With this caveat in mind, the corresponding differences in Arizona were 33% and 73%, the differences in Georgia were 16% and 90%, the differences
in Oklahoma were 9% and 110%, the differences in Ohio were 22% and 48%, and the differences in Utah were -6% and 43%. (Compare Criminal Justice Institute 2000a, p. 98, with Criminal Justice Institute 2000b, pp. 150–51.) That the Utah public average of starting-level salaries was 6% lower than at the reporting private prison probably illustrates, more than anything else, the pitfalls of relying on a single data point. The Arizona numbers may also not be representative, since one source (admittedly from the popular press) reports that the public-private divide in Arizona is on the low side (Nelson 2003). (For impressionistic reports from other states, see Miniclier [1999, p. B4], Harmon [1999, p. 1A], and Schlosser [1998, p. 58].)

Other sources give qualitatively similar results. For instance, an AFSCME chart comparing public to private hourly salaries in selected cities in the occupational category of “Guard I,” using 1993 data, shows that public sector hourly salaries ranged from 26% higher in Kansas City to 87% higher in San Francisco. In Chicago, the median city included on AFSCME’s chart, salaries were 57% higher in the public than in the private sector. (See AFSCME n.d.)

Thus, one can take 30–65% as a reasonable range for the public wage premium, with (somewhat arbitrarily) 9–125% as an outer range. Properly speaking, these numbers are estimates not of $\Delta W = W_g - W_m$, but of $\frac{W_g}{W_m} - 1$. But if $\frac{W_g}{W_m} - 1 = \omega$, then $\Delta W = W_g - W_m = \omega W_m$, or, equivalently:

$$\Delta W = W_g - \frac{W_g}{\omega + 1} = \frac{\omega}{\omega + 1} W_g.$$  \hspace{1cm} (27)

The next step is to estimate $\frac{W_g}{C_g}$, so that we can put $\Delta W$ in comparable terms with $\Pi$ (since we will know $\frac{\Pi}{C_m}$ and can estimate $\frac{C_g}{C_m}$). Salaries are generally reported to be 60–80% of most prisons’ operating expenses (see Shichor 1995, p. 149; Logan, p. 81; Donahue 1989, p. 163; Schlosser 1998, p. 65; Dolovich 2005, p. 475 n. 134). (Similarly, in Georgia in fiscal year 2004, “personal services” were $546$ million out of total costs of $944$ million, which makes 58% (see Georgia Department of Corrections 2004, p. 28); and in Virginia in fiscal year 2005, “personal services” were $544$ million out of total costs of $859$ million, which makes 63% (see Virginia Department of Corrections 2005).) I take this to mean that $\frac{W_g}{C_g} \in (0.6, 0.8)$ (since most prisons, from which the 60–80% figure derives, have been public).

Thus, denoting $\frac{W_g}{C_g}$ as $\eta$:

$$\Delta W = \frac{\omega}{\omega + 1} W_g = \frac{\omega \eta}{\omega + 1} C_g.$$  \hspace{1cm} (28)

This fraction is increasing in both $\omega$ and $\eta$, so its low end should correspond to $\omega = 0.3$ and $\eta = 0.6$, and its high end should correspond to $\omega = 0.65$ and $\eta = 0.8$. Thus, we have, approximately:

$$\Delta W \in (0.14 C_g, 0.32 C_g).$$  \hspace{1cm} (29)
C Estimating $\Pi$

<table>
<thead>
<tr>
<th></th>
<th>CCA</th>
<th>GEO</th>
<th>Industry median</th>
<th>Market median</th>
</tr>
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<tbody>
<tr>
<td>Gross profit margin</td>
<td>26.80%</td>
<td>84.30%</td>
<td>40.60%</td>
<td>51.50%</td>
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<tr>
<td>Pretax profit margin</td>
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<td>1.60%</td>
<td>1.70%</td>
<td>6.40%</td>
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<tr>
<td>Net profit margin</td>
<td>7.90%</td>
<td>2.40%</td>
<td>1.20%</td>
<td>5.00%</td>
</tr>
<tr>
<td>Return on equity</td>
<td>10.7%</td>
<td>10.9%</td>
<td>6.5%</td>
<td>9.7%</td>
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<tr>
<td>Return on assets</td>
<td>4.9%</td>
<td>3.1%</td>
<td>1.3%</td>
<td>1.6%</td>
</tr>
<tr>
<td>Return on invested capital</td>
<td>5.2%</td>
<td>3.9%</td>
<td>3.7%</td>
<td>4.3%</td>
</tr>
</tbody>
</table>

The above table (constructed from numbers available at hoovers.com as of March 2007) shows different profitability numbers for the two largest private prison firms, CCA and GEO—which account for about three quarters of the industry—as well as medians for the private prison industry as a whole and the market as a whole. The industry medians are lower than the market medians for all measures—this is not a terribly profitable industry. The last four lines seem to be the most relevant measures of profit, and since economic profit is only the excess of profit over the next-best option, 10% seems like an overestimate of private prison industry profits. To be conservative, 0–10% can be taken as a reasonable range for private firms’ economic profitability, i.e., $\Pi = \theta C_m$.

Denoting this proportion as $\theta$, we thus have:

$$\Pi = \theta C_m.$$  \hspace{1cm} (30)

To make this comparable to equation (28), we need to estimate $\frac{C_g}{C_m}$. It is known that private prisons save money both on the wage and the non-wage components of costs. For instance, savings at the design and construction cost stage have been estimated at 15–25% (see Harvard Law Review 2002, pp. 1878–79; Moore 1998, pp. 4–5). On the one hand, this seems somewhat less than the wage savings that one might estimate from the public-sector wage premia (a wage premium of 30–65% means that the private sector saves 23–39% on the wage bill, even ignoring any incentives to use labor more efficiently by, for instance, reducing overtime). On the other hand, few comparative studies of the cost-effectiveness of public and private prisons have found cost savings greater than 20% (see Segal and Moore 2002, p. 3).

If private prison savings were equal for the wage and non-wage components, then we would have:

$$\frac{C_g}{C_m} = \frac{W_g}{W_m} = 1 + \omega,$$ \hspace{1cm} (31)

so we have approximately:

$$C_m = \frac{1}{1 + \omega} C_g \in (0.61C_g, 0.77C_g),$$ \hspace{1cm} (32)

and thus approximately:

$$\Pi = \theta C_m \in (0, 0.077C_g).$$ \hspace{1cm} (33)
Or we can make another extreme assumption, that all private prison savings come from the wage bill. Define:

\[ C_g = C_x + W_g, \]
\[ C_m = C_x + W_m, \]

where \( C_x \) is the non-wage cost of a prison project (scaled up to the size of the whole system), assumed equal for both sectors, and \( W_g \) and \( W_m \) are the public and private wage bill, respectively. Then we have:

\[ W_g = (1 + \omega)W_m, \]
\[ \frac{W_g}{C_x + W_g} = \eta. \]

Algebra yields:

\[ \frac{W_g}{C_x + W_g} = \frac{(1 + \omega)W_m}{C_x + (1 + \omega)W_m} = \eta \]
\[ \Rightarrow (1 + \omega)W_m = \eta(C_x + (1 + \omega)W_m) \]
\[ \Rightarrow (1 + \omega)(1 - \eta)W_m = \eta C_x \]
\[ \Rightarrow W_m = \frac{\eta C_x}{(1 + \omega)(1 - \eta)}. \]

Thus, the ratio of public and private costs is:

\[ \frac{C_g}{C_m} = \frac{C_x + W_g}{C_x + W_m} = \frac{C_x + (1 + \omega)\eta C_x}{\eta C_x} = \frac{1 + \omega}{(1 + \omega)(1 - \eta) + \eta} = \frac{1 + \omega}{1 + \omega(1 - \eta)}. \]  

which is increasing in both \( \omega \) and \( \eta \). Since \( \omega \in (0.3, 0.65) \) and \( \eta \in (0.6, 0.8) \), we find the lowest value of this fraction by setting \( \omega = 0.3 \) and \( \eta = 0.6 \), and the highest value by setting \( \omega = 0.65 \) and \( \eta = 0.8 \). This means:

\[ \frac{C_g}{C_m} \in \left( \frac{1.3}{1 + (0.3)(0.4)}, \frac{1.65}{1 + (0.65)(0.2)} \right) = \left( \frac{1.3}{1.12}, \frac{1.65}{1.13} \right) \approx (1.16, 1.46), \]

which gives us approximately:

\[ C_m \in (0.68C_g, 0.86C_g), \]

and thus approximately:

\[ \Pi = \theta C_m \in (0, 0.086C_g). \]

To be on the safe side, we can combine the two possible ranges of \( \Pi \) from equations (33) and (42) (which in this case comes out to adopting the wider range), which gives us:

\[ \Pi \leq 0.086C_g. \]
We thus have $\Pi < \Delta W$, provided the parameters are within the specified ranges. Because $\alpha_m < \alpha_g$ in most cases (and even if one focuses on the marginal private share of federal prisoners, which is slightly above 50%), this also means that $\alpha_m \Pi < \alpha_g \Delta W$.

V Further extensions

I have treated $W_g$ as a constant — not dependent on $\alpha_g$ — for simplicity. In fact, $W_g$ probably decreases with privatization, since the public sector union then has less bargaining power. I leave the complications this would add to the analysis for further research, perhaps along the lines of Fraja (1993, pp. 461–66), who explicitly considers wage determination in a “mixed duopoly” consisting of a public firm and a private firm with unions.

I have treated $P$ as a constant — not dependent on $\alpha_i$ — because it’s not clear what effect different industry shares will have on $P$. In the first place, these industry shares are not “market shares” as the phrase is used in antitrust; that is, the companies may not be directly competing against each other, for instance if they just cover different geographic areas. In the second place, even if they were competing against each other, it’s not clear whether privatization would lead to the entry of more firms, the growth of all existing firms, or the consolidation of existing firms into fewer firms. Finally, suppose privatization led to the entry of more firms. Our intuitions suggest that collusion would be more difficult in this case, and so $P$ should drop; but Pecorino (1998) points out that, under standard models, this intuition may not necessarily be true (which might either rebut our intuitions or indict the standard models; see also Kühn [2006]). Further research would explore the effect of $\alpha_i$ on $P$.

My restriction here to industry-expanding advocacy, rather than pro-industry advocacy generally, is important. In general, splitting up an industry can have an ambiguous effect on pro-industry advocacy. Consider, for instance, another type of reform—an “anti-competitive” reform that would increase the contract price $P$ toward the monopoly price. A monopolistic industry has no need for such a reform, since it can charge the monopoly price directly. A duopoly may not find such a reform terribly useful if it can collude easily. If a more competitive industry finds it harder to collude in the product market (which is possible, but see Pecorino [1998]), a price-increasing reform will be more useful. So if an industry with a few firms can coordinate advocacy expenditures more easily than it can collude in the product market, one might find that splitting up the industry increases such “anti-competitive” advocacy. (Technically, allowing for such advocacy would fundamentally change the model, as the benefit of a reform would no longer simply be multiplied by $\alpha$ when a firm’s proportion of the industry is $\alpha_i$.) However, such an effect is not present for purely industry-expanding advocacy. (See also Peltzman 1976, pp. 223–24.) Further research could take these other forms of advocacy into account.

Finally, this discussion has taken the effectiveness of the different sectors at advocacy (the precise change of the $p$ function and the $\beta$ or $\gamma$ multipliers)
as given, and evaluated marginal privatizations (changes in $\alpha_g$) from an arbitrary baseline. But in the real political system, jurisdictions privatize or don’t privatize because of their own political realities. A state may have low levels of privatization because its public-sector unions are powerful, or high levels of privatization because its public-sector unions are weak. The level of $\alpha_g$ in a jurisdiction may thus give us information about the level of $\beta$ or $\gamma$, which would affect both the effect and the feasibility of privatization. For instance, given a state where $\alpha_g$ is very low because $\gamma$ is very low—a strong union, low-privatization state—privatization would decrease industry-expanding political advocacy but would also be politically unlikely.

VI Conclusion

The claim that privatization increases self-interested industry-expanding political advocacy is thus not in general true. If the probability of success of a reform is a function of total political advocacy in support of the reform, then privatization decreases political advocacy as long as the private sector does not become too big. This depends somewhat on the precise assumptions about collusion, though empirically the assumptions may not matter much in particular cases, such as prisons. The effect on anti-industry political advocacy is ambiguous. The result is qualitatively similar if political advocacy can also affect the substantive content of a reform. If we relax the assumption that the success of the reform depends on total advocacy, and allow each sector’s advocacy to affect success independently, the effect of privatization is ambiguous, though it depends to some degree on which sector is more effective at political advocacy.
Appendices

A The Harmlessness of Homogeneous Advocacy

This section shows that it is harmless to assume that there is a single type of advocacy expenditure that goes to obtain a single type of benefit.

Suppose, instead, that there are two types of expenditure, $e$ and $i$, used to obtain two types of benefit, $X$ and $Y$. Instead of merely having a benefit $B(e) \equiv \alpha_i p(e) X$, one then has a benefit:

$$B^+(e, i) \equiv \alpha_i [p(e) X + q(i) Y], \quad (A1)$$

where both $p$ and $q$ satisfy the assumptions of Proposition 1, and instead of choosing $e$ to maximize $U(e) \equiv B(e) - e$, one chooses $e$ and $i$ to maximize:

$$U(e, i) \equiv B^+(e, i) - e - i. \quad (A2)$$

But this is equivalent to defining $M \equiv e + i$, and then choosing $e$ and $M$ to maximize:

$$V(e, M) \equiv B^+(e, M - e) - M. \quad (A3)$$

And this, in turn, is equivalent to the two-step problem of:

- first choosing $e^*$ to maximize $V(e, M)$, denoting the solution $e^*(M)$ and the maximized value:

  $$V^*(M) \equiv V(e^*(M), M) \equiv B^+(e^*(M), M - e^*(M)) - M = B^*(M) - M, \quad (A4)$$

- then choosing $M$ to maximize $V^*(M)$.

Consider the “new” benefit function $B^*(M)$, which is a function of total advocacy expenditures $M$. Taking derivatives, we have (by the Envelope Theorem):

$$\begin{align*}
\frac{dB^*}{dM} &= B^+_2(e^*(M), M - e^*(M)) = \alpha_i q'(M - e^*(M))Y > 0 \\
\frac{d^2B^*}{dM^2} &= B^+_{22}(e^*(M), M - e^*(M)) = \alpha_i q''(M - e^*(M))Y \\
\frac{d^3B^*}{dM^3} &= B^+_{222}(e^*(M), M - e^*(M)) = \alpha_i q'''(M - e^*(M))Y < 0 \quad (A5)
\end{align*}$$

So the first and third derivatives of $B^*$ behave like the first and third derivatives of $B$. As for the second derivative, we need to check whether it eventually becomes negative, for which a sufficient condition is that $\lim_{M \to \infty}(M - e^*(M)) = \infty$, for which in turn a sufficient condition is that $\frac{de^*(M)}{dM} < 1$.

Note that at the first stage of choosing $e^*$ to maximize $V(e, M)$, the first-order condition is:

$$\alpha_i [p'(e^*)X - q'(M - e^*)Y] = 1, \quad (A6)$$
and the second-order condition is
\[ p''(e^*)X + q''(M - e^*)Y < 0. \]  
(A7)

Differentiating equation (A6) with respect to \( M \) (and assuming, for simplicity, an interior solution), we obtain:

\[ [p''(e^*)X + q''(M - e^*)Y] \frac{de^*}{dM} = q''(M - e^*)Y \]

\[ \Rightarrow \frac{de^*}{dM} = \frac{q''(M - e^*)Y}{p''(e^*)X + q''(M - e^*)Y}. \]  
(A8)

We know from equation (A7) that the denominator of \( \frac{de^*}{dM} \) is negative, and we know by assumption that \( p''(e^*)X < 0 \). If \( q''(M - e^*)Y > 0 \), it is easy to see that \( \frac{de^*}{dM} < 1 \). If \( q''(M - e^*)Y < 0 \), then the numerator is positive, which, with equation (A7), again implies \( \frac{de^*}{dM} < 1 \).

Therefore, the second derivative of \( B^* \) likewise acts like the second derivative of \( B \). So \( U(e) \equiv B(e) - e \) can thus be interpreted as though it were a more generalized value function \( V^*(M) \equiv B^*(M) - M \), where the actor chooses a total amount of advocacy and allocates it optimally among both types of advocacy. This is straightforward to generalize to a larger number of types of advocacy.

B Proof of Proposition 2

The private sector chooses \( e_m \) to maximize \( \pi_m(e_m) \), and the public sector chooses \( e_g \) to maximize:

\[ \pi_g(e_g) \equiv \alpha_g(1 + p(e, y)\varepsilon)\Delta W - e_g. \]  
(B1)

The anti-incarceration forces choose \( y \) to minimize:

\[ \pi_g(y) \equiv (1 + p(e, y)\varepsilon)B - y. \]  
(B2)

The first-order conditions are:

\[ \begin{cases} 
\alpha_mp_1(e^*, y^*)\varepsilon\Pi \leq 1 \\
\alpha_qp_1(e^*, y^*)\varepsilon\Delta W \leq 1 \\
-p_2(e^*, y^*)B \geq 1 
\end{cases} \]  
(B3)

For the reasons explained in Proposition 1, it is likely that one of \( e_m \) or \( e_g \) is zero (and that variable’s first-order condition holds with inequality). Because of the assumptions, the other two first-order conditions hold with equality and imply single unique maxima. (By a reasoning analogous to that in Proposition 1, \( e^* > e_t(y^*) \) and \( y^* > y_t(e^*) \), and since \( p_{11} < 0 \) and \( p_{22} > 0 \) for those values, the second-order conditions are satisfied.)

For instance, consider the case that \( \alpha_g\Delta W > \alpha_m\Pi \). Differentiating the first-order conditions (and omitting the arguments of \( p \) for simplicity), we obtain:

\[ \begin{pmatrix} 
\frac{de_m}{de_g} \\
\frac{de_g}{dy} \\
\frac{de_m}{dy} 
\end{pmatrix} = \begin{pmatrix} 
\frac{p_1p_2}{(1-\alpha_m)(-p_{11}p_{22}+p_{12}p_{21})} < 0 \\
\frac{p_{11}p_2}{(1-\alpha_m)(-p_{11}p_{22}+p_{12}p_{21})} \\
\frac{p_{12}p_1}{(1-\alpha_m)(-p_{11}p_{22}+p_{12}p_{21})} 
\end{pmatrix}. \]  
(B4)
The sign of $\frac{dy}{dm}$ depends on that of $p_{21}$, that is, on the interaction between the effectiveness of pro- and anti-incarceration advocacy. Thus, total pro-incarceration advocacy declines with increased privatization (since $e_g$ declines and $e_m$ is 0), while the effect of increased privatization on anti-incarceration advocacy is ambiguous.

It is straightforward to show that if $g_W < 0$, total pro-incarceration advocacy increases with increased privatization, while the effect of increased privatization on anti-incarceration advocacy is ambiguous.

### C Proof of Proposition 3

**Lemma 1** Suppose $f$ is differentiable, $\arg \max_x \{f(x) - x\} \equiv x^* > 0$, $f'(\infty) = 0$, and $\alpha \in (0, 1)$. Then $\arg \max_x \{\alpha f(x) - x\} \equiv x' < x^*$.

**Proof.** Because:

$$\arg \max_x \{f(x) - x\} \equiv x^* > 0 \quad \text{(C1)}$$

and $f$ is differentiable, we know that $f'(x^*) = 1$ and $f''(x^*) < 0$. Moreover (assuming for simplicity that $x^*$ is a unique maximum):

$$\forall x \neq x^*, f(x^*) - x^* > f(x) - x. \quad \text{(C2)}$$

Now suppose that:

$$\arg \max_x \{\alpha f(x) - x\} \equiv x'' > x^*, \quad \text{(C3)}$$

and suppose this is a unique maximum. Then we have $f'(x'') = \frac{1}{\alpha}$ and $f''(x'') < 0$. But, because $f'(\infty) = 0$, $\exists x^{**} > x''$ such that $f'(x^{**}) = 1$. (This is the “next” $x$ such that $f' = 1$ “after” $x''$.) Because $x^*$ is a maximum for $f(x) - x$, we know that:

$$f(x^{**}) - x^{**} < f(x^*) - x^* \quad \text{(C4)}$$

Now define $x'$ as the “previous” $x$ such that $f' = \frac{1}{\alpha}$ “before” $x^*$:

$$x' \equiv \max \{x | f(x) = \frac{1}{\alpha}, x < x^*\}, \quad \text{(C5)}$$

or (if there is no such $x$) define $x' \equiv 0$. Because $x''$ is a maximum for $\alpha f(x) - x$, we know that:

$$\alpha f(x'') - x'' > \alpha f(x') - x', \quad \text{(C6)}$$

which implies that:

$$f(x'') - \frac{x''}{\alpha} > f(x') - \frac{x'}{\alpha}. \quad \text{(C7)}$$

Equation (C4) implies that:

$$\int_{x^*}^{x^{**}} (f'(x) - 1) dx < 0. \quad \text{(C8)}$$
And equation (C7) implies that:

\[ \int_{x'}^{x''} (f'(x) - \frac{1}{\alpha}) dx > 0. \]  
(C9)

Therefore, subtracting equation (C9) from equation (C8):

\[ \int_{x'}^{x''} (f'(x) - 1) dx - \int_{x'}^{x''} \left( f'(x) - \frac{1}{\alpha} \right) dx < 0. \]  
(C10)

But evaluating the left-hand side of equation (C10) yields:

\[ \int_{x'}^{x''} (f'(x) - 1) dx - \int_{x'}^{x''} \left( f'(x) - \frac{1}{\alpha} \right) dx = \int_{x'}^{x''} \left( f'(x) - \frac{1}{\alpha} \right) dx - \int_{x'}^{x''} \left( f'(x) - \frac{1}{\alpha} \right) dx \]

\[ = - \int_{x'}^{x''} \left( f'(x) - \frac{1}{\alpha} \right) dx + \int_{x'}^{x''} \left( \frac{1}{\alpha} - 1 \right) dx + \int_{x'}^{x''} (f'(x) - 1) dx. \]  
(C11)

By construction of \( x' \) and \( x^* \), \( f'(x) \in [1, \frac{1}{\alpha}] \) when \( x \in [x', x^*] \), so:

\[ - \int_{x'}^{x''} \left( f'(x) - \frac{1}{\alpha} \right) dx > 0. \]  
(C12)

Also, because \( \frac{1}{\alpha} > 1 \):

\[ \int_{x'}^{x''} \left( \frac{1}{\alpha} - 1 \right) dx > 0. \]  
(C13)

Finally, by construction of \( x'' \) and \( x^{**} \), \( f'(x) \in [1, \frac{1}{\alpha}] \) when \( x \in [x'', x^{**}] \), so:

\[ \int_{x''}^{x^{**}} (f'(x) - 1) dx > 0. \]  
(C14)

Equations (C12), (C13), and (C14) together yield:

\[ \int_{x'}^{x''} (f'(x) - 1) dx - \int_{x'}^{x''} \left( f'(x) - \frac{1}{\alpha} \right) dx > 0, \]  
(C15)

which contradicts equation (C10). By contradiction, equation (C3) must be wrong, and so we must have:

\[ \arg \max_x \{ \alpha f(x) - x \} \equiv x'' < x^*. \]  
(C16)
Proposition 3  If the substantive content of the reform can be influenced by expenditures (i.e., the reform is to increase the industry by $\varepsilon(e)$), and the probability that this reform prevails is $p(\varepsilon(e), e)$, where $p_\varepsilon < 0$, $p_e > 0$, and $\varepsilon' > 0$, the comparative static results of Proposition 1 remain unchanged.

Proof. The public employees choose $e_g$ to maximize:

$$\pi_g(e_g) \equiv \alpha_g(1 + p(\varepsilon(e), e)e)\Delta W - e_g.$$  \hspace{1cm} (C17)

The first-order condition of this problem (omitting the arguments of $p$ and $\varepsilon$ for clarity) is:

$$L(e) \equiv (p_e \varepsilon' + p_e)e + p_e' \leq \frac{1}{\alpha_g\Delta W} \text{ (with equality if } e_g^* > 0).$$  \hspace{1cm} (C18)

Similarly, the first-order condition of the private sector is the same expression, with $e_g$ replaced by $e_m$ and $\alpha_g\Delta W$ replaced by $\alpha_i\Pi$. Suppose, without loss of generality, that $\alpha_g\Delta W > \alpha_i\Pi$. (This discussion assumes we are in the no-collusion case, but the proof is analogous for the other cases.) For reasons explained in the proof of Proposition 1, only the public sector gives anything:

$$\left\{ \begin{array}{l} e_g^* > 0 \\ e_m^* = 0 \end{array} \right.$$

If $L(e)$ is decreasing, then it is clear that decreasing $\alpha_g$ will decrease $e$ — that is, privatization will decrease public-sector advocacy and thus total advocacy. However, it is unclear that $L(e)$ is decreasing:

$$\frac{dL}{de} = (p_e \varepsilon' + p_e)e + p_e' \varepsilon' + p_e e' + p_e''.$$  \hspace{1cm} (C20)

Consider the expression $(p_e \varepsilon' + p_e)$ contained in the second term; $p_e e'$ is negative while $p_e$ is positive. So just from that term alone, we can see that $L(e)$ is not necessarily decreasing. However, by Lemma 1, decreasing $\alpha_g$ decreases $e$. $\blacksquare$

D  Proof of Proposition 4

The public sector chooses $e_g$ to maximize:

$$\pi_g(e_g) \equiv \alpha_g(1 + (p(e_g) + q(e_m))\varepsilon)\Delta W - e_g,$$  \hspace{1cm} (D1)

so it sets:

$$p'(e_g^*) \leq \frac{1}{\alpha_g\varepsilon\Delta W} \text{ (with equality if } e_g^* > 0).$$  \hspace{1cm} (D2)

This first-order condition and the next one have unique solutions for any $\alpha_i$ for analogous reasons to those stated above. Similarly, the private sector chooses $e_m$ to maximize:

$$\pi_m(e_m) \equiv \alpha_m(1 + (p(e_g) + q(e_m))\varepsilon)\Pi - e_m,$$  \hspace{1cm} (D3)
so it sets:

$$q'(e_m^*) \leq \frac{1}{\alpha_m \varepsilon \Pi} \quad \text{(with equality if } e_m^* > 0) \quad \text{(D4)}$$

The assumptions here guarantee an interior solution. However, if the relevant assumption is weakened and we have

$$\alpha_g \varepsilon \Delta W < \frac{1}{p(e_g)} \text{ or } \alpha_m \varepsilon \Pi < \frac{1}{p(e_m)}$$

for some parameter values, one or both of the first-order conditions cannot be solved with equality, in which case it would not be profitable for the relevant sector or sectors to advocate at all. (This could also explain why the private sector might not advocate — $\alpha_m$ is not high, and neither is $\Pi$.)

The total effect of advocacy, at the optimum, is:

$$E \equiv p(e_g^*) + q(e_m^*), \quad \text{(D5)}$$

which we can express in terms of $\alpha_m$:

$$E(\alpha_m) \equiv p(e_g^*(\alpha_m)) + q(e_m^*(\alpha_m)). \quad \text{(D6)}$$

To gauge the effect of increased privatization (and dropping the $\alpha_m$ arguments for convenience), we examine:

$$\frac{dE}{d\alpha_m} = p'(e_g^*) \frac{de_g^*}{d\alpha_m} + q'(e_m^*) \frac{de_m^*}{d\alpha_m}$$

$$= \frac{p'(e_g^*)}{p''(e_g^*)\alpha_g^2 \varepsilon \Delta W} - \frac{q'(e_m^*)}{q''(e_m^*)\alpha_m^2 \varepsilon \Pi}$$

$$= \frac{1}{p''(e_g^*)\alpha_g^2 \varepsilon^2 \Delta W^2} - \frac{1}{q''(e_m^*)\alpha_m^2 \varepsilon^2 \Pi^2}. \quad \text{(D7)}$$

This expression is of indeterminate sign. The first term, which is negative, represents the decrease in the effectiveness of public-sector advocacy when privatization increases, since the public sector gets less of the benefit of its advocacy. The second term, which is positive, represents the increase in the effectiveness of private-sector advocacy when privatization increases.

### E Proof of Proposition 5

The public sector chooses $e_g$ to maximize:

$$\pi_g(e_g) \equiv \alpha_g (1 + (\beta q(e_g) + q(e_m)) \varepsilon ) \Delta W - e_g, \quad \text{(E1)}$$

so it sets:

$$q'(e_g^*) \leq \frac{1}{\beta \alpha_g \varepsilon \Delta W} \quad \text{(with equality if } e_g^* > 0). \quad \text{(E2)}$$

The private sector chooses $e_m$ to maximize:

$$\pi_m(e_m) \equiv \alpha_m (1 + (\beta q(e_g) + q(e_m)) \varepsilon ) \Pi - e_m, \quad \text{(E3)}$$

so it sets:

$$q'(e_m^*) = \frac{1}{\alpha_m \varepsilon \Pi} \quad \text{(E4)}$$
(the assumptions of Proposition 1 guarantee an interior solution).

By analogy to equation (D6), the total effect of advocacy, at the optimum, is:

\[ E(\alpha_m, \beta) \equiv \beta q(e^*_g(\alpha_m, \beta)) + q(e^*_m(\alpha_m)). \]  

(E5)

To gauge the effect of increased privatization, we examine (dropping the \( \alpha_m \) and \( \beta \) arguments for convenience):

\[ \frac{\partial E}{\partial \alpha_m} = \beta q'(e^*_g) \frac{\partial e^*_g}{\partial \alpha_m} + q'(e^*_m) \frac{de^*_m}{d\alpha_m}. \]  

(E6)

Case 1. Suppose \( \lim_{e \to 0} q'(e) = \infty \). Then equation (E2) can be satisfied with equality \( \forall \beta > 0 \). To evaluate the derivatives in equation (E6), we differentiate equations (E2) and (E4) with respect to \( \alpha_m \):

\[ \frac{\partial e^*_g}{\partial \alpha_m} = \frac{1}{\beta q''(e^*_g) \alpha^2 \Delta W} \]  

\[ \frac{\partial e^*_m}{\partial \alpha_m} = \frac{1}{\beta q''(e^*_m) \alpha^2 \Delta W}. \]  

(E7)

Plugging the results of system (E7) into equation (E6), and using equations (E2) and (E4):

\[ \frac{\partial E}{\partial \alpha_m} = \beta q'(e^*_g) \frac{\partial e^*_g}{\partial \alpha_m} + q'(e^*_m) \frac{de^*_m}{d\alpha_m} \]

\[ = \frac{q''(e^*_g)}{\beta q''(e^*_g) \alpha^2 \Delta W} - \frac{q''(e^*_m)}{\beta q''(e^*_m) \alpha^2 \Delta W} - \frac{q''(e^*_g) \alpha^2 \Delta W}{\beta q''(e^*_g) \alpha^2 \Delta W} - \frac{q''(e^*_m) \alpha^2 \Delta W}{\beta q''(e^*_m) \alpha^2 \Delta W}. \]  

(E8)

As an initial matter, we show that \( \lim_{e \to 0} \beta q''(e^*_g(\alpha_m, \beta)) = -\infty \). Because \( \beta = \left( \frac{1}{q''(e^*_g) \alpha^2 \Delta W} \right) \) and \( \alpha^2 \Delta W > 0 \), this is equivalent to showing that \( \lim_{e \to 0} \frac{q''(e^*_g)}{q''(e^*_g)} = -\infty \) \( \frac{q''(e^*_m)}{q''(e^*_m)} \). Moreover, equation (E2) (with equality) implies that:

\[ e^*_g = (q')^{-1} \left( \frac{1}{\beta q''(e^*_g) \alpha^2 \Delta W} \right), \]  

(E9)

which is a strictly increasing function of \( \beta \), so this is equivalent to showing that \( \lim_{e \to 0} \frac{q''(e^*_g)}{q''(e^*_g)} = -\infty \).

The proof is by contradiction. Suppose \( \frac{q''(e^*_g)}{q''(e^*_g)} > M \forall e \), where \( M \) is a finite negative number. Then \( q''(e) > M q'(e) \forall e \). Thus:

\[ \int_0^e q''(e) \, de > \int_0^e M q'(e) \, de = M \left[ q(e) \right]^e_0 = M \left[ q(e) - q(0) \right], \]  

(E10)
which is a finite negative number. But this contradicts:

\[ \int_0^e q''(e)de = [q'(e)]_0^e = q'(e) - q'(0) = -\infty. \quad (E11) \]

By contradiction, \( \lim_{e \to 0} q''(e) = -\infty \), and thus \( \lim_{e \to 0} \beta q''(e^*(\alpha_m, \beta)) = -\infty \).

Thus, as \( \beta \to 0 \):

\[ \beta q''(e^*(\alpha_m, \beta)) \to -\infty, \quad (E12) \]

so:

\[ \frac{1}{\beta q''(e^*(\alpha_m, \beta))} \to 0, \quad (E13) \]

and thus:

\[ \frac{\partial E}{\partial \alpha_m} \to -\frac{1}{q''(e^*_m)\alpha^3_m e^2 \Delta W^2} > 0. \quad (E14) \]

There thus exists a \( \tilde{\beta} \in (0, 1) \) such that, for all \( \beta < \tilde{\beta} \), \( \frac{\partial E}{\partial \alpha_m} > 0 \).

Case 2. Suppose \( \lim_{e \to 0} q'(e) < \infty \). Then by equation (E2), \( \exists \tilde{\beta} \in (0, 1) \) such that \( \forall \beta < \tilde{\beta} \) and \( \forall \alpha_g \in [0, 1], e^*_g(\alpha_m, \beta) = 0 \). For this range of \( \beta \),

\[ \frac{\partial e^*_g}{\partial \alpha_m} = 0. \]

By system (E7), \( \frac{\partial e^*_g}{\partial \alpha_m} > 0 \). Therefore, by equation (E6), \( \frac{\partial E}{\partial \alpha_m} > 0 \).

The proof for \( q(e) \equiv \gamma p(e) \) is analogous.
References


