Contractual Remedies to the Hold-Up Problem: A Dynamic Perspective

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Abstract

An important theme of modern contract theory is the role contracts play to protect parties from the risk of hold up and thereby encouraging their relationship specific investments. While this perspective has generated valuable insights about various contracts, the underlying models abstract from realistic investment dynamics. We re-examine the role of contracts in a dynamic model that endogenizes the timing of investments and trade. The resulting interaction between bargaining and investment significantly alters the insights learned from static models. We show that contracts that would exacerbate the parties’ vulnerability to hold up – rather than those protecting them from the risk of hold up – can be desirable. Specifically, joint ownership of complementary assets can be optimal, and an exclusivity agreement can protect the investments of its recipient, much in contrast to the existing results – based on static models.

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1 Introduction

The hold-up problem arises when a business partner makes relationship specific investments that are susceptible to *ex post* expropriation by his associates. Examples of such investments include acquisition of firm specific skills by workers, subcontractors’ efforts to customize their parts to the special needs of manufacturers, and a firm’s relocation of its plant adjacent to its trading partner’s. These investments create more surplus within a relationship than without. Hence, absent any special safeguard, the fear of hold up may lead the parties to under-invest relative to the efficient level. As a result, an important theme of the modern contract literature is to look at how contracts can provide protection for the investors.¹

While this literature has generated valuable insights about various contracts, the underlying models abstract from the realistic investment dynamics present in many business relationships. Specifically, the extant models of incomplete contracts assume that the partners make a single investment decision, after which the renegotiation of the contract commences. In practice, however, the timing of investment and bargaining is – at least to some extent – chosen endogenously by the parties, and the investment and bargaining stages are often intertwined. Crucially, trade can potentially be postponed in the expectation of further investment. For instance, the Department of Defense may negotiate to order a weapons system from a contractor based on his current technical knowledge, or it may decide to wait until the latter invests more in R & D and develops a better technology. A similar dynamic interaction of investment and bargaining arises in the development of new building construction, advertising pilots or software projects.²

In this paper, we study the effects of *ex ante* (incomplete) contracts in an environment where each party can make relationship specific investments in every period until either the parties agree to trade or the negotiation breaks down – say, because trade ceases to be efficient. In our model, two risk-neutral parties initially sign a contract, that may specify

¹A range of organizational and contract forms have been rationalized as safeguards against hold up: Examples include vertical integration (Klein, Crawford and Alchian, 1978; Williamson, 1979), a property rights allocation (Grossman and Hart, 1986; Hart and Moore, 1990), contracting on renegotiation rights (Chung, 1991; Aghion, Dewatripont and Rey, 1994), option contracts (Nöldeke and Schmidt, 1995), and trade contracts (Edlin and Reichelstein, 1996; Che and Hausch, 1999).

²Such dynamic interaction is also present in the publishing of academic articles. Consider the editorial procedure at the Berkeley Electronic Journals. Here submission is simultaneous to four vertically differentiated journals. Unless the initial submission is rejected, in principle, the author is offered a choice between immediate acceptance at a lower level or acceptance at a higher level conditional on a substantial revision (incremental investment after the negotiation has commenced).
various aspects of their relationship, such as asset ownership, future trade decisions or exclusivity of their trading relationship. This contract may be renegotiated over time, but it affects the outcome of renegotiation by determining its threat point. Once the contract is signed, the parties begin by (potentially) making a sunk investment of predetermined size, and then a randomly chosen proposer offers to his partner a share of the (gross) surplus that would obtain through trading given the current level of investment. If that offer is accepted, trade occurs according to the agreement, and the game ends. If the offer is rejected, then negotiation breaks down irrevocably with some given probability. In that case, the contract in place takes effect, assigning the payoffs to the parties. With the remaining probability, the game moves to the next period without trade, and the same process is repeated; i.e., either party who has not yet done so can invest, what is followed by a new bargaining round with a random proposer, and the game goes on until either trade occurs or bargaining breaks down.

If investment is allowed to occur only in the first period – or equivalently, if breakdown is certain following disagreement – then, given risk neutrality, our model coincides with the majority of existing contract models, which use the Nash Bargaining Solution (NBS) with the contract payoffs serving as the threat point. Our full-fledged model provides a natural dynamic extension of these that endogenizes the timing of investment and trade decisions. For this reason, the equilibrium results of our model are easily comparable to those in most of the incomplete contract literature.

Absent the possibility to invest, the unique subgame perfect equilibrium of this bargaining model replicates the NBS, just as in Binmore, Rubinstein and Wolinsky (1986). Thus, our model can also be seen as a generalization of their model by making both the stake and the disagreement payoffs of bargaining endogenous through investment.

We show that allowing for investment dynamics yields a new insight on the effects of

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3 We have decided to present our results within the framework of a simple, binary model of investment. This model captures most of the relevant issues, while it allows us to concentrate on the intuition and the applications, without the obfuscation caused by the technical issues arising from considering continuous investment (see Che and Sákovics 2004a, 2004b).

4 Examples include Grossman and Hart (1986), Hart and Moore (1990), Edlin and Reichelstein (1996), Che and Hausch (1999), Hart and Moore (1999), Segal (1999), and Segal and Whinston (2000, 2002). The alternative approach treats contracts as affecting the outside option payoffs of bargaining (MacLeod and Malcolmson, 1993; Chiu, 1998; De Meza and Lockwood, 1998).

5 Binmore, Rubinstein and Wolinsky (1986) present the alternating-offer version of this model. They need to take the limit as the breakdown probability tends to zero in order to eliminate the first-mover advantage and to show the full equivalence with the NBS. Instead, we assume that in each bargaining round the proposer is chosen randomly, which – given risk neutrality – yields the same effect (see Binmore, 1987).
contracts. The recent contract literature has focused on the inappropriability of returns at the margin as a source of inefficiency and a rationale for contract intervention. More precisely, the literature has focused on how the parties’ contract payoffs vary with investment. If an investor’s contract payoff increases with her specific investment, it effectively reduces the specificity of her investment at the margin, thus reducing the exposure of her marginal investment return to expropriation. Such a contract thus improves her incentives to make the specific investment. Alternative contracts can be ranked along this logic: If a contract reduces the investor’s marginal specificity exposure more than another contract, the former contract will protect her investment return better than the latter, thus inducing a higher level of investment. By the same token, if two contracts entail precisely the same marginal specificity, their effect on investment incentives will be precisely the same, even when they differ in terms of the investors’ exposure to absolute specificity that they induce. Hence, the absolute degree of specificity investors are exposed to does not matter in the existing models. In our dynamic model, absolute specificity does matter (for a sufficiently low probability of breakdown).

Several papers have developed somewhat similar insights, though in differing modelling contexts. Halonen (2002) shows in a repeated-game model that joint ownership of an asset strictly dominates single ownership for intermediate values of players’ discount factor, $\delta$, since the former can make the repeated game punishment more severe. Baker et al. (2001, 2002) also demonstrate that the absolute payoff levels can affect the efficiency ranking of different ownership structures in a repeated trade setting. The repeated trade opportunities assumed in these papers make the folk theorem of repeated games applicable, which implies that an efficient outcome is sustainable as $\delta \to 1$, irrespective of the underlying organizational arrangements. In this sense, the organizational issues become irrelevant for a sufficiently large $\delta$ in these papers. By contrast, the parties have a single trading opportunity in our model (just as in the standard hold-up problem), which makes the folk theorem inapplicable. Indeed, the organizational issues remain relevant when $\delta \approx 1$ in our model. Matouschek (2004) studies the effects of ex ante contracts on the ex post trading (in)efficiencies when the parties have two-sided asymmetric information (à la Myerson-Satterthwaite), but have no opportunity to invest. Contracts inducing low disagreement payoffs can increase the efficiency of agreements, but prove more costly when agreement fails to obtain. Finally, in Che and Sákovics (2004a) we analyze a dynamic hold-up model much like6 the current one, but without the possibility of breakdown – and therefore, effectively in the absence of contracts. The role of contracts in providing incentives to invest

6 There we model impatience by exponential discounting instead of probabilistic breakdown.
is the focus of the current study.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 looks at the necessary and sufficient conditions that lead to the efficient investment in both the standard (“static”) and our dynamic contract model and establishes the effects of contracts. Section 4 then applies the results to some well-known contracts. Section 5 concludes.

2 The model

Two risk-neutral parties, 1 and 2, can create a surplus from trade. The joint surplus from trade at a given date increases with the aggregate investment the parties have accumulated. Investment is observable but not verifiable. The cost of investing is \( c_i > 0 \) for party \( i \). Let \( I_i \) denote the indicator function of whether party \( i \) has invested. If the parties agree on a trade decision, \( q \in Q \), when the cumulative investment is \( I = (I_1, I_2) \), then trade creates the joint surplus of \( \Phi(q, I) - \text{gross of the investment cost}. \) The set of feasible trades, \( Q \), is a compact subset of \( \mathbb{R}^n \) for some \( n \in \mathbb{N} \), and contains a null trade \( q_0 \) that yields a zero surplus, i.e., \( \Phi(q_0, \cdot) \equiv 0 \). As it will become clear, it is useful to focus on the efficient trade decision conditional on the level of investment,

\[
\phi(I) := \max_{q \in Q} \Phi(q, I),
\]

and let \( q^*(I) \) be the associated maximizer, which we assume is well defined. Further, we assume that the efficient surplus function, \( \phi(\cdot) \), is strictly increasing: \( \phi(I) < \phi(I') \) if \( I < I' \), (where the strict vector inequality means weak inequalities for both components with at least one of them strict).

Given any \( I \), the parties may realize \( \phi(I) \) by agreeing to trade efficiently. In case they fail to agree, they can collect payoffs of \( (\psi_1(I; \mu), \psi_2(I; \mu)) \), henceforth referred to as disagreement or contract payoffs, which depend on the contract \( \mu \) in place. Without specifying the precise nature of the contracts we consider, we make a few assumptions about the set of available contracts, \( \mathcal{M} \). First, we assume that, for each \( \mu \in \mathcal{M} \), \( \psi_1(I; \mu) \leq \psi_1(I'; \mu) \) for any \( I < I' \). Next, we assume that the investments are relationship-specific in the following sense:

**Assumption 1. (Specificity)** For each \( \mu \in \mathcal{M} \)

(a) \( \psi_1(0, \cdot; \mu) > \psi_1(1, \cdot; \mu) - c_1 \) and \( \psi_2(\cdot, 0; \mu) > \psi_2(\cdot, 1; \mu) - c_2 \);

(b) \( \phi(I; \mu) - \psi_1(I; \mu) - \psi_2(I; \mu) < \phi(I'; \mu) - \psi_1(I'; \mu) - \psi_2(I'; \mu) \) for any \( I < I' \).
Assumption 1-a implies that a party will not invest unless there is internal trade between the partners. It simplifies our equilibrium characterization. Assumption 1-b means that the parties’ investments are specific, in the sense that they generate higher total surplus when the parties trade efficiently than when they disagree, both in the *absolute* and *marginal* senses. These assumptions are sensible in the context of many applications. For instance, the disagreement outcome under any ownership may be inefficient since non-owners may not exert sufficient human capital input (as is assumed in GHM); a trading contract may not specify the efficient trade level (as with Edlin and Reichelstein (1996) and Che and Hausch (1999)); and external trading does not yield as much surplus as the efficient internal trading both under exclusive and nonexclusive regimes (see Segal and Whinston (2000)).

Assumption 1-b implies that the disagreement outcome can never be efficient, so the social optimum can be characterized independently of the contract in place.

We assume, for clarity’s sake, that it is always efficient to invest:

**Assumption 2. (Investment is efficient)** Letting $\mathbf{I}^*:=(1,1)$,

$$\phi(\mathbf{I}^*) - c \cdot \mathbf{I}^* > \phi(\mathbf{I}) - c \cdot \mathbf{I} \quad \forall \mathbf{I} \in \{0,1\}^2, \mathbf{I} \neq \mathbf{I}^*, $$

where $c = (c_1,c_2)$.

Our model is general enough to accommodate a broad set of circumstances in terms of the underlying environment and the allowed contracts/organizations. In particular, several well-known contracts are included.\(^7\)

**Example 1. (Asset Ownership)** Grossman-Hart-Moore (GHM) model of asset ownership concerns how different ways of allocating the assets to the parties affects their incentives for relationship specific investments. They postulate that asset ownership directly affects the status quo payoffs of the parties when they negotiate, a la Nash Bargaining, to determine the terms of the trade between them. This model is clearly subsumed in our current setup when $\psi_i$ is allowed to depend on the allocation of asset ownership.

\(^7\)In principle, a more sophisticated contract, for example, one requiring exchanges of messages, can be incorporated into our model, with $\psi_i$ interpreted as the equilibrium payoff of party $i$ in that contract (sub)game. Of course, there is the issue of how these latter payoffs are determined and what contract payoffs are feasible. These are difficult questions to address even in the static model (as is well known from the debates on the incomplete contract paradigm). The more complex extensive form makes our dynamic model even less suitable for analyzing, let alone likely to offer any new insight on, these problems. For these reasons, we do not address the question of optimal contracts, limiting attention instead to some well known contracts.
Example 2. (Exclusive Dealing) An agreement prohibiting a trade partner from dealing with a third party is often justified by the protection that it may provide for the relationship specific investments. Segal and Whinston (2000) investigate this hypothesis using an incomplete contract model wherein trade partners negotiate the terms of internal trade, and the status quo payoffs of the parties in negotiation depends on the presence of the exclusivity agreement. Our model accommodates such a model, with the contract payoff $\psi(\cdot; \mu)$ varying with the extent, $\mu$, to which one is allowed to trade with an external third party.

Example 3. (Contracting with cooperative investment) The parties may contract ex ante on the terms of trade, which may be later renegotiated. Many authors have analyzed the effects of such ex ante contracts on the incentives for specific investments (Edlin and Reichelstein, 1996; Che and Hausch, 1999, and Segal and Whinston, ???, among others). The value of such ex ante trade contract is crucial for the foundation of the incomplete contract paradigm. Of particular interest in this regard is the case in which investments are cooperative in the sense that investors do not directly benefit from their investments, as such investments have been found particularly difficult to motivate via ex ante trade contracts (Che and Hausch, 1999). These models and the related questions can be reexamined in our dynamic context, when the status quo payoffs are allowed to depend on the terms of trade they initially agreed upon. Of special interest is the case in which the...

Example 4. (Contracting in a complex environment) The foundation of incomplete contracts has been also found with a model in which parties to an ex ante contract may trade in a complex environment which makes it difficult for the parties to forecast the type of trade that will harness their specific investments. Segal (1999) and Hart and Moore (1999) consider a model in which a seller (party 1) and a buyer (party 2) can trade one of $n$ different types of “widgets.” One of the different types becomes ex post optimal to trade, and the parties’ investments raise the value of trading the special type of widget but no other types. They find ex ante trade contract to be of little value when there are so many types that it is ex ante difficult to predict the special type of widget. This model again lends itself to our dynamic model, with the the status quo payoffs allowed to depend on the type of widget that the parties may agree to trade initially.

As Example 2 illustrates, the absence of a contract is a special case of our model.\(^8\) The previous authors studying these problems have employed the framework in which the

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\(^8\)In Che and Sákovics (2004a), the status quo payoffs are assumed to be zero. But this is just a normalization, and the reader should not interpret the current paper as assuming that the status quo payoffs will indeed rise as the parties sign some ex ante contract. As the exclusivity example illustrates, a contract may increase or decrease the parties’ status quo payoffs.
parties invest first and then bargain over the terms of trade according to the Generalized Nash Bargaining Solution with bargaining shares \((\alpha_1, \alpha_2)\) (henceforth GNBS), with the contract payoffs \((\psi_1(\cdot; \mu), \psi_2(\cdot; \mu))\) serving as status quo point. For ease of comparison, it is indeed useful to establish the resulting outcome as a benchmark. Suppose the parties choose investment pair \(I\), and subsequently bargain over the terms of trade according to the GNBS. Then, they will choose \(q^*(I)\), and party \(i = 1, 2\) will collect the payoff (gross of investment costs) of

\[
U_0^i(I; \mu) := \alpha_i[\phi(I) - \psi_{-i}(I; \mu)] + \alpha_{-i}\psi_i(I; \mu).
\]

We consider a natural dynamic extension of this GNBS framework. The precise extensive form of game is described as follows. We first fix a contract \(\mu\), say chosen at time \(t = 0\) from some set of contracts, \(M\), satisfying Assumption 1. The parties subsequently play an investment and bargaining game as follows. In period \(t = 1\), the parties make sunk investment \(I = (I_1, I_2) \in \{0, 1\}^2\). A party \(i\) is then chosen with probability \(\alpha_i \in [0, 1]\), \(\alpha_1 + \alpha_2 = 1\), to make a proposal on the terms of trade, consisting of \(q \in Q\) and a transfer \(t_1, t_1 + t_2 = 0\). If the proposed terms are accepted by the recipient, the game ends. If not, then with probability \(1 - \delta \in [0, 1]\), the bargaining breaks down, and the parties collect their contract payoffs, \((\psi_1(I; \mu), \psi_2(I; \mu))\). With probability \(\delta\), the game moves on to the next period, and the same process is repeated as in the first period, i.e., the parties may invest (if they haven’t done so before) followed by (random-proposer) bargaining, and so on and so forth, until there is an agreement and trade or the bargaining breaks down. Our solution concept is that of Subgame Perfect Equilibrium (SPE).

If \(\delta = 0\), so the game must end by the end of the first period, then this game replicates the standard two-stage investment-trade model. The SPE of our extensive form in that case replicates the GNBS, yielding payoffs \(U_0^i(I; \mu)\) to the players.

3 Characterization of Equilibrium and Contracts

3.1 Characterization of equilibrium under arbitrary contract

The parties’ strategies map from any history into their behavior in the investment and bargaining phases of each period. Naturally, we are interested in a SPE in which both parties invest in the first period. An important step of the analysis is to evaluate the continuation payoffs following the investment decision. To this end, suppose that both parties invest. Since then there is no further investment opportunity for either party, the
game becomes a pure bargaining game. Hence, the standard argument shows that the continuation payoffs are uniquely determined in that case.

**Lemma 1.** Fix any $\delta \in [0, 1)$. Given $\mu$, in any SPE the continuation payoffs following $\Gamma^* = (1, 1)$ are given by the GNBS payoffs, $U^0_i(\Gamma^*; \mu)$, $i = 1, 2$.

**Proof.** Let $\bar{v}_i$ and $\underline{v}_i$ be the supremum and infimum payoffs of party $i = 1, 2$ attainable in any SPE, immediately following $\Gamma = (1, 1)$. Then, for each $i = 1, 2$,

$$
\bar{v}_i = \alpha_i[\phi(\Gamma^*) - \delta \varphi_i - (1 - \delta)\psi_i(\Gamma^*; \mu)] + \alpha_i \delta \varphi_i(\Gamma^*; \mu),
$$

$$
\underline{v}_i = \alpha_i[\phi(\Gamma^*) - \delta \varphi_i - (1 - \delta)\psi_i(\Gamma^*; \mu)] + \alpha_i \delta \varphi_i(\Gamma^*; \mu).
$$

Solving this system of four equations for $\{\bar{v}_i, \underline{v}_i\}_{i=1,2}$ such that $\bar{v}_i \geq \underline{v}_i$ yields $\bar{v}_i = \underline{v}_i = U^0_i(\Gamma^*; \mu)$. 

It is striking that the parties' equilibrium payoffs following $\Gamma^* = (1, 1)$ coincide with the GNBS payoffs. This clearly shows that the exposure to hold up is not diminished by the introduction of investment dynamics. As will be seen, however, this fact does not lead to the same condition or conclusion about the incentives for investment without the investment dynamics. The next theorem characterizes the precise condition for the attainability of the efficient investment $\Gamma^* = (1, 1)$. To this end, it is useful to define, for any given $\delta \in [0, 1)$ and contract $\mu$, a payoff for party $i$:

$$
U^\delta_i(\Gamma; \mu) := \alpha_i[\phi(\Gamma) - (1 - \delta)\psi_i(\Gamma; \mu)] + \alpha_i(1 - \delta)\psi_i(\Gamma; \mu).
$$

Notice that this payoff coincides with those associated with GNBS when $\delta = 0$, but differ from them when $\delta \neq 0$. Let $\Gamma^*_{-i}$ be the investment pair arising when party $i = 1, 2$ unilaterally deviates from $\Gamma^*$. That is, $\Gamma^*_{-1} = (0, 1)$ and $\Gamma^*_{-2} = (1, 0)$. We now present the characterization.

**Theorem 1.** Given Assumptions 1 and 2, the investment $\Gamma^* = (1, 1)$ in the first period is implementable in a SPE by contract $\mu$, if and only if

1. (IR) $U^0_i(\Gamma^*; \mu) - c_i \geq \psi_i(\Gamma^*_{-i}; \mu)$, for $i = 1, 2$
2. (IC) $U^\delta_i(\Gamma^*; \mu) - c_i \geq U^\delta_i(\Gamma^*_{-i}; \mu) - \alpha_i \delta c_i$, for $i = 1, 2$. 

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Proof: See the Appendix.

As \( \delta \to 0 \), (IC) collapses to

\[
U_i^0(\Gamma; \mu) - c_i \geq U_i^0(\Gamma_i^-; \mu).
\]

Further, the RHS of (1) is no less than \( \psi_i(\Gamma_i^-; \mu) \), by Assumption 1-(b). This means that (IR) and (IC) together reduce to (1) when \( \delta \to 0 \). Condition (1) coincides with that from the standard models, suggesting that the incentives for investments — and thus the role of contracts — conform to that identified by the extant literature in that case. In fact, by the same argument, for \( \delta \) small enough (IC) always implies (IR), and therefore the latter one is the binding constraint.

The parties’ incentives for investments are different for a higher \( \delta \), however, altered by the presence of the investment dynamics. In particular, higher \( \delta \) allows more incentives to be generated from the same contract. To see this, write out the payoff loss party \( i = 1, 2 \) would suffer from deviating from \( \Gamma^* \) to \( \Gamma_i^- \):

\[
\Delta V_i^\delta(\mu) = \alpha_i[\Delta_i \phi - (1 - \delta)\Delta_i \psi_i(\mu)] + \alpha_{-i}(1 - \delta)\Delta_i \psi_i(\mu) - (1 - \alpha_{-i})c_i \geq 0,
\]

where \( \Delta V_i^\delta(\mu) := U_i^\delta(\Gamma; \mu) - U_i^\delta(\Gamma_i^-; \mu) - (1 - \alpha_{-i})c_i \), \( \Delta_i \phi := \phi(\Gamma) - \phi(\Gamma_i^-) \) and \( \Delta_i \psi_i(\mu) := \psi_i(\Gamma; \mu) - \psi_i(\Gamma_i^-; \mu) \). It then follows that, for any \( \delta' > \delta \),

\[
\Delta V_i^{\delta'}(\mu) = \Delta V_i^\delta(\mu) + (\delta' - \delta) [\alpha_i \Delta_i \psi_i(\mu) + \alpha_{-i}(c_i - \Delta_i \psi_i(\mu))].
\]

Notice that the terms between the square brackets are nonnegative, by Assumption 1-(a). Thus, if (IC) holds for \( \delta \), it holds for \( \delta' > \delta \). Now suppose \( U_i^0(\Gamma^*; \mu) - c_i \geq U_i^0(\Gamma_i^-; \mu) \), \( i = 1, 2 \), so it is an equilibrium in the static model for both parties to invest. Then, (IC) holds for any \( \delta > 0 \). As noted, (IR) also holds in this case. Therefore, it is also an equilibrium for both parties to invest when \( \delta > 0 \).

The reason for the strengthened incentives can be explained as follows. As noted in Lemma 1, an investor shares the return from his investment precisely the same way as in \( \delta = 0 \) on the equilibrium path. Yet, he may not share the payoff in the same way when he deviates. In particular, if, following his deviation, the party is expected to invest in the next period (if no agreement is reached in the current period), then the pie will grow next period if no agreement is reached this period, so the investor’s partner will demand a bigger share of the current pie to forego that option. Essentially, the forecasted investment dynamics in this case toughens the partner’s bargaining position following deviation, which leads to the investor receiving smaller than the usual share when under-investing relative to the target. This creates a stronger incentive for an investor, which in turn confirms the belief that investment will be made in the future, making it self-fulfilling.
3.2 The effects of contracts

Our central focus is how investment dynamics may affect the relative performance of alternative contracts. For expositional simplicity, it is useful to develop some terminology.

Definition 1. A contract implements $I^*$, if given the contract $I^*$ is reached in some SPE of the game.

Definition 2. (i) Contract $\mu'$ weakly dominates contract $\mu$ if $\mu'$ implements $I^*$ whenever $\mu$ implements it. A contract $\mu \in \mathcal{M}$ is optimal in $\mathcal{M}$, if it weakly dominates all other contracts in $\mathcal{M}$.

(ii) Contract $\mu'$ dominates contract $\mu$ if $\mu'$ weakly dominates $\mu$ and there exists $(c_1, c_2)$ such that the former implements $I^*$ but not the latter.

(iii) Contracts $m$ and $m'$ are equivalent if $m'$ implements $I^*$ if and only $m$ implements it.

The central insight from the static models concerns the extent to which a contract in question enables the investor to appropriate his return, or equivalently reduce his exposure to hold up, “at the margin.” This effect is fully captured by the terms $\Delta_i \psi_j(\mu)$, for investor $j = i$ and non-investor $j = -i$. Specifically, $\Delta_i \psi_i(\mu)$ represents the marginal return of investment that the investor $i$ appropriates, and $\Delta_i \psi_{-i}(\mu)$ reflects the marginal return of $i$’s investment “leaked” to the non-investor, $-i$. The former protects the investor, while the latter undermines him, from the hold-up problem at the margin. Indeed, as can be seen from (IC'), the condition (IC) can be written solely in these marginal terms and conform to the standard insight.

The condition (IR) does not necessarily conform to this insight, however. To see this, rewrite (IR) as:

\[
\alpha_i \phi(I^*) - c_i + \Delta_i \psi_i(\mu) \geq \alpha_i (\psi_i^*(\mu) + \psi_{-i}^*(\mu)),
\]

where $\psi_i^*(\mu) := \psi_i(I^*; \mu)$. Notice that, in addition to the marginal term $\Delta_i \psi_i(\mu)$, the condition also depends on $\psi_i^*(\mu) + \psi_{-i}^*(\mu)$ — i.e., the degree to which the parties are jointly exposed to hold up in the absolute sense. Hence, this condition can yield a different insight on the role of contracts than has been previously proposed.

The important question is which of the two conditions is binding. As we argued above, (IC) implies (IR) when $\delta$ is close to zero, in which case the role of contracts will conform to the standard insight. As will be demonstrated next, (IR) can only matter if $\delta$ is sufficiently large. In this latter case, a new insight emerges.
Proposition 1. Consider any pair of contracts $\mu', \mu$.

a) If $(-\Delta_i(\psi)(\mu'), \Delta_i(\psi)(\mu')) > (-\Delta_i(\psi)(\mu), \Delta_i(\psi)(\mu))$ for $i = 1, 2$, then there exists $\delta > 0$ such that contract $\mu'$ dominates contract $\mu$ for any $\delta < \tilde{\delta}$.

b) If $\psi^*_1(\mu') + \psi^*_2(\mu') < \psi^*_1(\mu) + \psi^*_2(\mu)$, then there exist $\hat{\delta} < 1$ and $M < 0$ such that contract $\mu'$ dominates contract $\mu$ if $\delta > \hat{\delta}$ and $\Delta_i(\psi)(\mu') - \Delta_i(\psi)(\mu) > M \forall i = 1, 2$.

Proof. See the Appendix.

A few points are worth noting. First, our results show the standard insight to be robust to introducing a small $\delta$, i.e., a small probability of the bargaining continuing after a disagreement. In other words, the extent to which contracts influence the investor’s exposure to hold up “at the margin” explain the relative performance of contracts well. The same cannot be said, however, when $\delta$ is large. In this case, bargaining is likely to continue after initial disagreement, permitting more investments to be made in the future. Such a possible investment dynamics changes the incentives of the parties and more importantly the way contracts influence them. In particular, (IC) is no longer important for it is satisfied regardless of the underlying contract; and only (IR), or equivalently (IR'), matters. First of all, as can be seen from (IR') the “leakage” $\Delta_i(\psi)(\mu)$ of investment returns to the non-investor does not have an adverse effect on the incentive. Most important, the extent to which the contracts affect an investor’s exposure to holdup “in absolute terms” matter.

This latter point can be seen most clearly, when the parties’ investments are totally relationship specific so that a failure to consummate trade between the two parties leads to the parties unable to appropriate any investment returns. In this case, for all $\mu$, $\Delta_i(\psi)(\mu) = 0, \forall i, j = 1, 2$. In this case, the condition (IC) will be the same for all contracts, i.e., they will be indistinguishable with regard to this condition. Yet, they are not same with respect to (IR'). In particular, the extent to which contracts expose the parties to holdup in absolute level — $\psi^*_1(\mu) + \psi^*_2(\mu)$ — matters. Strikingly, (IR') tells us that a contract maximizing parties’ exposure to holdup is the one that relaxes the constraint the most! The same insight applies if the investments need not be totally relationship specific, but rather if alternative contracts provide the same marginal protection to the investors from the hold-up problem.

Corollary 1. (Status quo minimization) Suppose $\Delta_i(\psi)(\mu) = \Delta_i(\psi)(\mu')$, $i, j = 1, 2$, and

$$\psi^*_1(\mu') + \psi^*_2(\mu') < \psi^*_1(\mu) + \psi^*_2(\mu).$$

Then, there exists $\hat{\delta} \in (0, 1)$ such that i) contract $\mu'$ and contract $\mu$ are equivalent if $\delta < \hat{\delta}$, and ii) $\mu'$ dominates $\mu$, if $\delta > \hat{\delta}$.
Proof. The statement follows from Proposition 1, together with the observation that 
\( \tilde{\delta} = \hat{\delta} \) since whenever \( \mu \) satisfies (IR') at any \( \delta \) it does so for any other \( \delta \), whereas the (IC') becomes easier to satisfy for higher value of \( \delta \). □

Any two contracts that have the same "marginal" features are equivalent for low values of \( \delta \), but not for high values of \( \delta \). In the latter case, a contract exacerbating the investors’ exposure to hold up performs well. Next, we explore how this new insight alters our understanding of contracts in various circumstances.

4 Applications

In this section, we apply our results to a range of problems to illustrate how the investment dynamics influences the specific prescriptions on organizational design.

4.1 GHM Model of Asset Ownership

Suppose there are two assets, \( A = \{a_1, a_2\} \). An ownership structure, \( \mu \), is then represented by a pair of exclusive subsets of \( A \), \( (A^\mu_1, A^\mu_2) \), where \( A^\mu_i \subset A \), \( i = 1, 2 \), stand for the asset(s) party \( i \) owns under ownership structure \( \mu \). There are four alternative structures: 
1. separate ownership or non-integration: \( \mu_N := (\{a_1\}, \{a_2\}) \); 
2. common ownership (or integration) by party 1: \( \mu_1 := (\{a_1, a_2\}, \emptyset) \); 
3. common ownership (or integration) by party 2: \( \mu_2 := (\emptyset, \{a_1, a_2\}) \); and 
4. joint ownership \( \mu_J := (\emptyset, \emptyset) \).

According to the GHM theory, a party’s contract payoff, \( \psi_i(I; \mu) \), represents the revenue that he/she can generate by exercising his/her residual rights in the event of disagreement, so the payoff depends on the assets owned by that party. As with Hart (1995), it is thus reasonable to assume that the payoff depends on the assets owned by that party: i.e.,
\[
\psi_i(\cdot; \mu) = \psi_i(\cdot; \mu') \text{ if } A^\mu_i = A'^\mu_i.
\]
In the same vein, it is sensible to assume that more assets give higher contract payoffs:
\[
\psi_i(I, \mu) \leq \psi_i(I, \mu') \text{ if } A^\mu_i \subset A'^\mu_i, \text{ for all } I.
\]
Further, since the investments are interpreted as acquisition of human capital not embodied in the assets, it is reasonable to assume that a party’s contract payoff does not depend on his partner’s investment: \( \Delta_i \psi_{-i}(\mu) \equiv 0 \). This setup easily lends itself to analysis in our dynamic model in which following the choice of asset ownership the parties play our investment-trading game.

The crucial feature of the GHM theory is that the contract payoffs can be characterized in terms of the extent to which each ownership structure determines one’s exposure to hold up at the margin, with the assumption that additional assets owned reduces this exposure:
$\Delta_i \psi_i(\mu) \leq \Delta_i \psi_i(\mu)$ if $A_i^\mu \subset A_i^\nu$.

To highlight our new insight in comparison with the existing one, it is useful to characterize the nature of the assets in the following way. First, two salient cases of interests are defined in terms of how asset ownership affects the parties’ overall exposure to the hold-up problem.

**Definition 3.** The assets are **substitutive** if

$$\psi_1(I; \mu_i) + \psi_2(I; \mu_i) < \psi_1(I; \mu_N) + \psi_2(I; \mu_N), \ i = 1, 2.$$  

The assets are **complementary** if

$$\psi_1(I; \mu_i) + \psi_2(I; \mu_i) > \psi_1(I; \mu_N) + \psi_2(I; \mu_N), \ i = 1, 2.$$  

Suppose the assets are substitutive. Then, starting from separate ownership, if a party gains an asset, his status quo value does not rise as much as the other person’s status quo payoff declines. In this sense, the assets are more valuable in status quo when owned separately than when owned under a common ownership. In the same sense, complementary assets are more valuable in status quo when owned by the same party than when they are owned separately. The assets can be characterized also in “marginal” terms; i.e., by the way in which the raising of investments affects the status quo payoffs.¹⁹

**Definition 4.** The assets are **marginally substitutive** if

$$\Delta_i \psi_i(\mu_i) = \Delta_i \psi_i(\mu_N) > \Delta_i \psi_i(\mu_{-i}) = \Delta_i \psi_i(\mu_J), \ i = 1, 2.$$  

The assets are **marginally complementary** if

$$\Delta_i \psi_i(\mu_i) > \Delta_i \psi_i(\mu_N) = \Delta_i \psi_i(\mu_{-i}) = \Delta_i \psi_i(\mu_J), \ i = 1, 2.$$  

Invoking Proposition 1, a series of observations follow.

**Proposition 2.** a) If assets are marginally substitutive, then separate ownership of the assets dominates their common ownership by either party for any $\delta < \tilde{\delta}_1$ for some $\tilde{\delta}_1 > 0$.

b) If assets are marginally complementary, then common ownership of assets by either party dominates separate ownership for any $\delta < \tilde{\delta}_2$ for some $\tilde{\delta}_2 > 0$.

¹⁹Hart (1995) defines these notions and labels them differently there. We changed the terms to be more cohesive with the alternative notions, defined above. Note also that the above definitions are a little more general than the “absolute payoffs” counterparts of the next definitions.
c) If assets are substitutive, then there exist $M > 0$ and $\hat{\delta} < 1$ such that separate ownership of the assets dominates their common ownership by either party, if $\delta > \hat{\delta}$ and $\Delta_i\psi_i(\mu) < M, \forall i, \mu$.

d) If assets are complementary, then there exist $M > 0$ and $\hat{\delta} < 1$ such that common ownership of assets by either party dominates their separate ownership, if $\delta > \hat{\delta}$ and $\Delta_i\psi_i(\mu) < M, \forall i, \mu$.

e) There exist $M > 0$ and $\hat{\delta} < 1$ such that joint ownership is optimal if $\delta > \hat{\delta}$ and $\Delta_i\psi_i(\mu) < M, \forall i, \mu$.

Proposition 2-a) and -b) find the robustness of the GHM prescription that marginally substitutive assets should be owned separately and marginally complementary assets should be owned together, to introducing a small $\delta$ of continuance of bargaining and possible investment dynamics. Parts c), d) and e) show results in the “opposite flavor” to hold, however, if $\delta$ is sufficiently large. They show, for instance, that complementary assets should be owned separately and that joint ownership, where neither party has firm control over assets, and thus any appreciable residual right, is optimal, when $\delta$ is large. This surprising result again is traceable to the status quo minimization principle: the parties’ incentives to shirk can be controlled better when they can credibly commit themselves to a high risk of hold up.

4.2 Exclusive Dealing

An agreement to deal with a partner at the exclusion of others has been the subject of much debate. Antitrust authorities have either banned or held in suspicion any exclusive practices that may foreclose on competition. [Court practices? Per se illegal?] Others suggested that the voluntary nature of such agreements may reflect some efficiency benefits they may bring. One such hypothesis is that the security of trading relationship such agreement may bring can motivate the partners to engage in relationship specific investments: in other words, exclusivity may protect the partners from future hold up.

Whether this hypothesis holds true can be studied within the framework of the current model. Suppose two parties, 1 and 2, can realize the trading benefit of $\Phi(I)$, given their investment $I$. If they fail to reach an agreement to trade, they can collect the payoffs of $\psi_i(I; \mu)$, depending on the contractual arrangement $\mu$. There are four possibilities in this regard, as exclusivity may be granted to either 1 or 2 or to both, or to neither. Let $X_i$ denote an agreement for party $i = 1, 2$ not to engage in external trade, $X_b$ the
agreement for both parties not to trade externally, and \( NX \) means no such agreement. Thus \( \mathcal{M} = \{X_1, X_2, X_b, NX\} \). It is reasonable to assume that the opportunity to trade externally is valuable:

\[
\psi_i(\cdot; NX) = \psi_i(\cdot; X_{-i}) > \psi_i(\cdot; X_i) = \psi_i(\cdot; X_b), i = 1, 2.
\]

There are several cases of potential interest. The first is the case where the specific investments are not transferable to trading outside the current relationship. This implies that \( \Delta_i \psi_j(\mu) = 0 \) for all \( \mu \in \mathcal{M} \). This is reasonable if the nature of the external partners or the trading with them is such that the investment cannot be utilized. For instance, a particular logistics/inventory arrangement made with a particular retailer may be lost when a manufacturer must switch to a new retailer. Segal and Whinston (2000) found the exclusivity agreement to be of no value in promoting investments in this context.

Next is the case where investment is transferable but in a way that benefits the investor, in the sense that \( \Delta_i \psi_i(NX) = \Delta_i \psi_i(X_{-i}) > 0 = \Delta_i \psi_{-i}(X_i) = \Delta_i \psi_{-i}(NX) \). That is, there is no leakage. This will be a reasonable assumption.... The last case is one where the investment has “leakage” toward the trading partner: \( \Delta_i \psi_i(NX) = \Delta_i \psi_i(X_{-i}) = 0 < \Delta_i \psi_{-i}(X_i) = \Delta_i \psi_{-i}(NX) \). Leakage of investment is an issue for sporting teams and entertainment agencies, which often discover, train and groom their talents, only to see them switching to different teams or different agencies, taking with them the human capital and marketing assets cultivated by the original partner. The following series of results hold.

**Proposition 3.**

a) Suppose the investments are non-transferable in the sense that \( \Delta_i \psi_j(\mu) = 0 \) for all \( \mu \in \mathcal{M} \). Then, there exist \( \delta > 0 \) and \( \hat{\delta} < 1 \) such that, for \( \delta < \hat{\delta} \), all arrangements in \( \mathcal{M} \) are equivalent but that, for any \( \delta > \hat{\delta} \), \( X_b \) dominates \( X_i \), which in turn dominates \( NX \).

b) Suppose \( \Delta_i \psi_i(NX) = \Delta_i \psi_i(X_{-i}) > 0 = \Delta_i \psi_{-i}(X_i) = \Delta_i \psi_{-i}(NX) \). Then, there exist \( \delta > 0 \) and \( \hat{\delta} < 1 \) such that, for \( \delta < \hat{\delta} \), \( NX \) dominates \( X_i \), which in turn dominates \( X_b \). For any \( \delta > \hat{\delta} \), \( X_b \) dominates \( X_i \), which in turn dominates \( NX \) if \( \Delta_i \psi_{-i}(X_i) = \Delta_i \psi_{-i}(NX) \) is sufficiently small.

c) Suppose \( \Delta_i \psi_i(NX) = \Delta_i \psi_i(X_{-i}) = 0 < \Delta_i \psi_{-i}(X_i) = \Delta_i \psi_{-i}(NX) \). Then, \( X_b \) dominates \( X_i \), which in turn dominates \( NX \).

Part a) contrasts the differences between the cases with small \( \delta \) and large \( \delta \). In the former case, exclusive dealing has no effect on the investment incentives, as it was found by
Segal and Whinston (2000), since exclusivity affects the scope of the hold-up problem only in absolute terms. With a large $\delta$, this latter effect matters, however, so exclusivity does promote the investment, the more so with more exclusivity. Part b) deals with the case in which the investments transferable to the investors in their external trade. In this case, the possibility of external trade actually harnesses the incentives at the margin, so with small $\delta$, exclusivity is undesirable. With large $\delta$, however, the opposite result holds as long as the extent of the transferability is small. Part c) concerns the case of “leakage.” In this case, exclusivity promotes investments regardless of $\delta$. Leakage of investment returns undermines the incentives, which can be prevented by prohibiting the investor’s partner from engaging in external trade. For instance, suppose $\Delta_1 \psi_2(NX) > 0$, with $\Delta_i \psi_j(NX) = 0$ for all $(i, j) \neq (1, 2)$. Then, granting exclusivity to party 1, which prohibits party 2’s external trade, promotes the former’s investment for $\delta$ small. If $\delta$ is close to 1, however, the leakage by itself does not pose a problem, but the parties’ aggregate exposure to the holdup becomes important. Exclusivity increases the exposure, which increases their ability to punish, and thus improves the incentives even of those who grant the exclusivity clause.

De Meza and Selvaggi (2006) also find that exclusive dealing may promote specific investment, but in a markedly different model. In addition to a buyer and a seller, they explicitly model a third party (another buyer), who makes no investment but can either trade with the seller directly or can buy the good off the other buyer – when this is efficient. If the investing buyer has exclusivity protection, the second buyer can only participate in an eventual resale. Their result and ours complement each other towards establishing a positive role exclusivity may play in promoting specific investments.

4.3 The Foundations of Incomplete Contracts

Much of modern organization theory rests on the assumption that some crucial decisions such as specific investments are difficult to contract on. Such an incompleteness can often be overcome indirectly through contracting on the price and quantity of ex post trading. However, a number of scenarios have been identified in which the underlying incompleteness is such that a trade contract does not deliver full efficiency. We will discuss two of these. Both scenarios recognize the renegotiability of contracts as an important ingredient to obtain this result. In addition, they require some assumptions about the nature of specific investments: either cooperativeness or unpredictability of investment benefit, which will be described more fully below. Again, our extensive form is well suited to subsume these scenarios.
4.3.1 Cooperative Investments

Two parties, seller (party 1) and buyer (party 2), have an opportunity to trade \( q \in Q \subset \mathbb{R}^+ \) units of a good, which will cost party 1 \( c(q, I) \) but generate a surplus of \( v(q, I) \), if they invest \( I \). The efficient surplus is then \( \phi(I) = \max_q v(q, I) - c(q, I) \). The parties can initially sign a contract \((\hat{q}, \hat{t}) \in Q \times \mathbb{R} =: \mathcal{M}\) which obligates them to trade \( \hat{q} \) at the payment \( \hat{t} \). The set \( \mathcal{M} \) includes the possibility of the null contract, \((\hat{q}, \hat{t}) = (0, 0)\), with an associated outcome \( v(0, \cdot) = c(0, \cdot) = 0 \).

The parties can renegotiate the terms of any contract, including the null contract, and revise the investment along the way, according to our extensive form. If they fail to renegotiate the contract, then they collect the payoffs, \( \psi_i(I; \hat{q}, \hat{t}) = \hat{t} - c(\hat{q}, I) \) and \( \psi_2(I; \hat{q}, \hat{t}) = v(\hat{q}, I) - \hat{t} \), respectively. Of particular interest for our purpose is the situation in which the investments are cooperative in the sense that investors do not directly benefit from their investments — \( c(\cdot, I) = c(\cdot, I_2) \) and \( v(\cdot, I) = v(\cdot, I_1) \). Given this property of investment, it is easy to see that

\[
\Delta_i \psi_i(\mu) = 0 < \Delta_i \psi_{i-1}(\mu), \quad i = 1, 2
\]

for any \( \mu = (\hat{q}, \hat{t}) \) with \( \hat{q} > 0 \), whereas \( \Delta_i \psi_j(0, 0) = 0, \forall i, j = 1, 2 \). In other words, any nontrivial contract increases one’s exposure to hold up at the margin. Accordingly, Che and Hausch (1999) find that the null contract dominates any nontrivial trade contract.

Whether this conclusion holds true in the current dynamic model depends on whether there exists an (excessive) trade level that will generate a loss. Suppose

\[
\hat{q}^+ \in \arg\min_{q \in Q} v(q, \Gamma^*) - c(q, \Gamma^*).
\]

If \( v(\hat{q}^+, \Gamma^*) - c(\hat{q}^+, \Gamma^*) < 0 \), then a contract to trade \( \hat{q}^+ \) can be optimal for a sufficiently large \( \delta \).

**Proposition 4.**

a) The null contract is optimal, for all \( \delta < \tilde{\delta} \) for some \( \tilde{\delta} > 0 \).

b) If \( v(\hat{q}^+, \Gamma^*) - c(\hat{q}^+, \Gamma^*) \geq 0 \), then the null contract is optimal, regardless of \( \delta \).

c) If \( v(\hat{q}^+, \Gamma^*) - c(\hat{q}^+, \Gamma^*) < 0 \), then a contract to trade \( \hat{q}^+ \) is optimal, for any \( \delta > \hat{\delta} \) for some \( \hat{\delta} \in [\tilde{\delta}, 1) \).

4.3.2 Complexity

Suppose a seller (party 1) and a buyer (party 2) can trade one of \( n \) different types of “widgets.” Let the set \( Q^n \) be the set all feasible types of widgets, with \( |Q^n| = n \), and
$q \in Q^n$ represents a particular type of widget and $q = 0 \in Q^n$ represents the null contract. After the parties make investments $I = (I_1, I_2)$, they learn one of the widget types to be special in that it generates higher joint surplus. Each type in $Q$ has the equal chance of becoming the special widget. The special widget, regardless of its type, costs party 1 $c(I)$ and and yields the surplus of $v(I)$ to party 2. If a type $q$ is ordinary, then it costs $c_q$ to party 1 and yields the surplus of $v_q$ to party 2, with $v_0 = c_0 = 0$. We assume $\Phi(i) := v(I) - c(I) > v_q - c_q, \forall I \in \{0, 1\}^2$ and $\forall q \in Q^n$, so that it is efficient for the parties to trade the special widget.

Notice that the investments need not be cooperative here, but their value is realized only when the special good is traded. This latter property entails the same sort of difficulties with ex ante contracts in generating incentives. Specifically, the parties may sign a contract that requires them to trade a particular type $\hat{q} \in Q$ of widget for some transfer payment $\hat{t}$. The disagreement payoffs for parties 1 and 2 are random, since the type $\hat{q}$ becomes special with probability $\frac{1}{n}$ and ordinary with the remaining probability. Segal (1999) and Hart and Moore (1999) considered such a model. Of special interest is the limiting case in which the environment gets complex in the sense that $n \to \infty$. Let $Q = \lim_{n \to \infty} Q^n$. We consider this limiting case. Assume $Q$ is compact. (Alternatively, we could start with a set $Q$ that contains infinitely many types of widgets.)

Suppose the parties contract to trade any particular type $\hat{q}$. There is zero probability that that type will be special, so the contract payoffs are $\psi_1(I; \hat{q}, \hat{t}) = \hat{t} - c_\hat{q}$ and $\psi_2(I; \hat{q}, \hat{t}) = v_\hat{q} - \hat{t}$, respectively for parties 1 and 2. Notice that these payoffs do not depend on the investments at all. Hence, $\Delta_i\psi_j(\mu) = 0$, for all $\mu \in \mathcal{M} := Q \times \mathbb{R}$. Again, it is useful define the level of trade,

$$\hat{q}^+ \in \arg\min_{\hat{q} \in Q} v_\hat{q} - c_\hat{q},$$

that would lead to the worst joint payoff unless renegotiated. We obtain a result similar to that with the cooperative investment.

**Proposition 5.** *It is optimal for the parties to contract to trade $\hat{q}^+$.*

The intuition behind this result is clear. All different types of contracts are not distinguishable based on the marginal features, since $\Delta_i\psi_j(\mu) = 0$, for all $\mu \in \mathcal{M}$, so they are equivalent with respect to (IC). Hence, alternative contracts can be only be differentiated by (IR). The status quo minimization principle in Corollary 1 then suggests that the contract to trade the worst type of widget is optimal. Of course, such a contract may boil down to the null contract:

**Corollary 2.** *The null contract is optimal if and only if $v_{\hat{q}^+} - c_{\hat{q}^+} \geq 0$.*
Hence, our results from both cases suggest that the foundations of incomplete contracts can be justified in the dynamic setting but require some qualifications.

5 Concluding Remarks

We have shown that allowing for a simple and plausible investment dynamics in a hold-up model produces much different implications on the design of important contracts and organizations than have been suggested in the literature. The novel theme in our prediction is that the incentives for specific investments depend not just on how a contract affects the investor’s exposure to hold up at the margin – the focus on the recent contract/organization literature – but, more importantly, on how the contract affects the investor’s exposure to hold up in absolute terms. Absolute exposure to hold up per se was never a concern in the static models, since the individual rationality constraint is never binding there, but it is an important consideration in our dynamic model since the steeper incentives provided by investment dynamics may cause the latter constraint to be binding.

A shift of emphasis from how organizations affect the extent to which investments alleviate the hold-up problem at the margin to how they affect their exposure to hold up directly takes us back to the original “transaction cost analysis” (TCA) authors (Klein et al., 1978; Williamson 1979, 1985), who were largely concerned about the absolute level of hold up parties are subject to as the source of inefficiencies and the rationale for organizational interventions. While we agree that absolute degree of hold up matters, our specific predictions differ from these authors as well. Our theory predicts that contracts that would exacerbate the parties’ vulnerability to hold up – rather than those protecting them from it (as proposed by the TCA authors) – can be desirable. As discussed in the paper, this view throws a more positive light on a variety of “hostage taking” or “hands-tying” arrangements such as exclusivity agreements, joint ownership of assets, and trade contracts compelling parties to trade excessive amounts. These contracts/organization forms can perform well in our dynamic model since they can create a strong equilibrium punishment for deviation.

The fact that our predictions are largely based on the absolute level of quasi-rents could also make them more empirically testable. As Whinston (2003) points out, the GHM theory is difficult to test, since the (marginal) effects of investment on the disagreement payoffs are difficult to estimate, especially since most feasible levels of investment are not made in equilibrium. By contrast, hypotheses pertaining to the effects of absolute degree of asset specificities can be tested without observing payoff consequences of all investment choices, especially when investments are totally specific.
6 Appendix

Proof of Theorem 1: (Necessity) By Lemma 1, in any efficient investment SPE, party $i = 1, 2$ must obtain a payoff of $U_i^0(\Gamma^*; \mu) - c_i$. Suppose now party $i$ deviates unilaterally by not investing. There are only two possible subgame perfect continuations following the deviation: either the deviator will make the investment in the following period or never, in case no agreement is reached in the current period. Consider the latter case, i.e., without investment in the next period. Let $w_i$ be the infimum of party $i$'s subgame perfect equilibrium payoff attainable in any subgame following $\Gamma_i^*$. Then,

$$w_i \geq \delta w_i + (1 - \delta) \psi_i(\Gamma_i^*; \mu),$$

since party $i$ has an option of avoiding trade. It follows that

$$w_i \geq \psi_i(\Gamma_i^*; \mu).$$

Since party $i$ earns at least $\psi_i(\Gamma_i^*; \mu)$ from deviating, the efficient investment equilibrium can be supported only if

$$U_i^0(\Gamma^*; \mu) - c_i \geq \psi_i(\Gamma_i^*; \mu),$$

as is required by (IR).

Consider next a deviation followed by party $i$ investing in the subsequent period in case no agreement is reached in the current period. If the next period is indeed reached, the associated continuation payoffs for $i$ and $-i$ are respectively $U_i^0(\Gamma^*; \mu) - c_i$ and $U_{-i}^0(\Gamma^*; \mu)$, by Lemma 1. Hence, party $i$'s payoff from such a deviation must be at least

$$\alpha_i[\phi(\Gamma_i^*; \mu) - \delta U_{-i}^0(\Gamma^*; \mu) - (1 - \delta) \psi_{-i}(\Gamma_{-i}^*; \mu)] + \alpha_{-i}[\delta(U_i^0(\Gamma^*; \mu) - c_i) + (1 - \delta) \psi_i(\Gamma_i^*; \mu)].$$

Since such a deviation should not be profitable,

$$U_i^0(\Gamma^*; \mu) - c_i \geq \alpha_i[\phi(\Gamma_i^*; \mu) - \delta U_{-i}^0(\Gamma^*; \mu) - (1 - \delta) \psi_{-i}(\Gamma_{-i}^*; \mu)] + \alpha_{-i}[\delta(U_i^0(\Gamma^*; \mu) - c_i) + (1 - \delta) \psi_i(\Gamma_i^*; \mu)]$$

$$\Leftrightarrow U_i^0(\Gamma^*; \mu) - c_i \geq U_i^0(\Gamma_i^*; \mu) - \alpha_{-i} \delta c_i,$$

as is required by (IC).

(Sufficiency) To show that these conditions are sufficient for existence of an efficient investment SPE, consider the following investment strategy profile: “Each party invests whenever he has not invested previously.” This simple investment strategy profile clearly
implements the efficient investment. Further, given (IR) and (IC), this strategy profile, along with the optimal bargaining behavior, forms a SPE. This can be seen by the fact that, given (IR), any unilateral single-period deviation by party 1, say, gives him precisely the payoff in (2), which is dominated by his equilibrium payoff, as is guaranteed by (IC).

**Proof of Proposition 1:** The argument following Theorem 1 shows that (IC) strictly implies (IR) when \( \delta = 0 \). By the continuity of payoffs in \( \delta \), for any any \( \mu, \mu' \), the same is true for \( \delta < \tilde{\delta} \) for some \( \tilde{\delta} > 0 \). Fix any such \( \delta \) and suppose \((-\Delta_i \psi_i(\mu'), \Delta_i \psi_i(\mu')) > (-\Delta_i \psi_i(\mu), \Delta_i \psi_i(\mu)) \) for \( i = 1, 2 \). Suppose \( \mu \) implements \( \Gamma \). Then, contract \( \mu \) must satisfy (IC), so \( \Delta V_{i}^{\delta}(\mu) \geq 0, i = 1, 2 \). Given the hypothesis, it follows that, for \( i = 1, 2 \),

\[
(3) \ \Delta V_{i}^{\delta}(\mu') - \Delta V_{i}^{\delta}(\mu) = (1-\delta)\{\alpha_i(\Delta_i \psi_i(\mu') - \Delta_i \psi_i(\mu)) - \alpha_i(\Delta_i \psi_i(\mu') - \Delta_i \psi_i(\mu))\} > 0.
\]

Hence, we must have \( \Delta V_{i}^{\delta}(\mu') > 0, i = 1, 2 \), so \( \mu' \) satisfies (IC). Since (IC) implies (IR), \( \mu' \) must also implement \( \Gamma \). Given the strict inequality in (3), there exists \( (c_1, c_2) \) such that \( \Delta V_{i}^{\delta}(\mu') > 0, i = 1, 2 \), but that \( \Delta V_{i}^{\delta}(\mu) < 0 \) for some \( i \). In this case, \( \mu' \) implements \( \Gamma^* \) but \( \mu \) cannot. We thus conclude that \( \mu' \) dominates \( \mu \).

Next, observe that, for any \( \mu \), as \( \delta \to 1 \),

\[
\Delta V_{i}^{\delta}(\mu) \to \alpha_i(\phi(\Gamma') - c_i) > 0,
\]

by Assumption 2. In other words, every contract satisfies (IC) for \( \delta \) close to 1. Consequently, for any contracts \( \mu, \mu' \) there exists \( \hat{\delta} < 1 \) such that, for any \( \delta > \hat{\delta} \), only condition (IR) matters. Fix any such \( \delta \), and suppose \( \psi_{i}^*(\mu') + \psi_{i}^*(\mu') < \psi_{i}^*(\mu) + \psi_{i}^*(\mu) \). Then, there exists \( M < 0 \) such that whenever \( \mu \) satisfies (IR'), so does \( \mu' \) if \( \Delta_i \psi_i(\mu') - \Delta_i \psi_i(\mu) > M \) for each \( i \). Since both contracts satisfy (IC), whenever \( \mu \) implements \( \Gamma' \), so does \( \mu' \). Given the strict inequality, there exists \( (c_1, c_2) \) such that (IR') holds only for \( \mu' \). Hence, \( \mu' \) dominates \( \mu \).}

**References**


