Rights of First Refusal

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ABSTRACT

This paper analyses rights of first refusal. The holder of a right of first refusal has the option to purchase a subject asset on the same terms as those accepted by a third-party buyer. A variant of a right of first refusal is a right of first offer. A right of first offer requires a seller who wishes to sell a subject asset to offer the right-holder to buy that asset before it is offered to other potential buyers. If the right-holder declined the seller’s offer, the seller may sell the asset to a third party but only on terms no better (for the third party) than those offered to the right-holder.

The main proposition of the paper is that in a multiple-buyer, sequential bargaining setting, rights of first refusal increase the joint profits of the seller and right-holder by increasing the probability that the right-holder is offered to purchase the seller’s asset and/or by providing the right-holder an opportunity to purchase the asset at a lower price than that he would be offered otherwise. The value of rights of first refusal derives from the facts that an unencumbered seller (i) is indifferent to buyers’ sampling order if the right-holder is identical to other buyers’; and (ii) exploits his monopoly position to make the right-holder an offer which is higher than that which maximizes the joint profits of the seller and right-holder.

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1. Introduction

This paper presents an analysis of rights of first refusal. A right of first refusal is triggered when a seller of an asset subject to such right has agreed to sell the asset to a third-party buyer. The holder of a right of first refusal then has the option to purchase the seller’s asset on the same terms as those accepted by the third-party buyer.

Rights of first refusal are commonly employed in a variety of contractual settings. They are found, for example, in real estate sales and lease contracts, in agreements among shareholders of closely-held corporations, in joint venture and franchise agreements, and in professional sports collective bargaining agreements. See, e.g., Mueller (1989); Bartok (1991); Daskal (1995); Johnson and Stanford (1997); Smith (1997); Platt (1999). In addition, state law creates miscellaneous rights of first refusal (for example, for franchisees, with respect to the establishment of new franchises). See, e.g., Keenan (1987); Lawless (1988); Hess (1995).

Contracts may also provide for a right of first offer, a variant of a right of first refusal. A right of first offer requires a seller wishing to sell the subject asset to offer the right-holder to buy that asset before it is offered to other potential buyers. If the right-holder declined the seller’s offer, the seller can sell the asset to a third party but only on terms no better (for the third-party) than those offered to the right-holder. To distinguish between rights of first refusal and rights of first offer, we will refer to them collectively as first-purchase rights.\(^1\)

Despite their prevalent use, there has been little formal analysis of first-purchase rights. The previous literature has focused primarily on rights of first refusal. Walker (1999) analyzes right of first refusal in a standard English auction with two bidders. Choi (2006) generalizes Walker’s analysis by showing that rights of first refusal increase the seller and right-holder’s joint profit as compared to standard auctions when the unprivileged bidder wins the contest (while not affecting the seller and right-holder’s joint profit otherwise). The reason is that the outside bidder must bid higher than the right-holder’s valuation to preempt the latter’s right, whereas in the standard auctions that bidder would win the auction at a price equal to or lower than the right-holder’s valuation. Bikhchandani, Lippman and Ryan (2005) consider the effect of a right of first refusal in a sealed-bid second-price auction in which bidders privately observe signals about their valuations. They show that when bidders’ valuations are correlated, a right of first refusal exacerbates the winner’s curse of ordinary bidders, thereby causing bidders to bid less aggressively. They conclude that a right of first refusal may result in inefficiency and caution sellers to exercise care in granting prospective bidders such right. Finally, Grosskopf and Roth (2006) analyze a peculiar combination of a right of first offer and a right of first refusal, whereby the right of first refusal is activated if the right of first offer is violated (the right of first offer thus precedes the right of first refusal chronologically). They show that in a two-buyer, sequential bargaining framework, this

\(^1\) Note that the term ‘rights of first refusal’ refers to rights of first refusal as defined above as well as rights of first offer as defined below.
hybrid right strengthens the seller’s bargaining position vis-à-vis the right-holder, thereby disadvantaging the right-holder.

The analysis here differs from previous works in a few respects: First, this paper is the first to formally model and analyze rights of first offer and to compare them to rights of first refusal. It thereby provides a unified explanation for the use of rights of first refusal and rights of first offer, consistent with the circumstances under which these rights are commonly observed.

Second, the paper employs a sequential bargaining, rather than an auction, framework, in which the seller makes take-it-or-leave-it offers to potential buyers. We motivate this framework on several grounds. As a practical matter, the seller may not be able to assemble all potential buyers at the same time (or it may be costly to do so) in order to conduct an auction. Relatedly, when the number of potential bidders is high and investigation costs are not trivial, an auction may not be incentive compatible. A seller, therefore, may instead approach a few buyers sequentially, and then conduct an auction among the remaining buyers. Indeed, most assets are not sold in auctions but rather through private negotiations. Moreover, an implicit assumption underlying the design of rights of first offers is that the seller approaches buyers sequentially.

Third, the analysis here highlights the importance of potential buyers’ investigation constraint – i.e., the maximum offer which ensures investigation by potential buyers – in determining the value of first-purchase rights. The value of potential buyers’ investigation constraint depends on their distribution of valuation, their investigation costs, and the presence of a first purchase right. In particular, potential buyers’ investigation constraint is more restrictive when a right of first refusal is present as compared to the no-rights case and the right-of-first-offer case. In addition to potential buyers’ investigation constraint, other factors considered in the analysis here are the number of potential buyers, the relation between investigation costs incurred by the potential right-holder and those incurred by other potential buyers, and the seller’s cost of having offers rejected.

The paper’s analysis proceeds by comparing the seller’s optimal sequence of offers to potential buyers in the no-rights case and in the case in which a first-purchase right is present. Consider first the no-rights case. As there are fewer potential buyers remaining, the seller would lower his offers to potential buyers, thereby trading off a higher profit margin for a higher probability of sale. The seller’s optimal sequence of unconstrained offers is thus decreasing. In addition, in order to ensure that potential buyers incur investigation costs, the seller may not make offers higher than potential buyer’s investigation constraint.

Next consider the right-of-first-refusal case. As in the no-rights case, the seller’s optimal sequence of unconstrained offers is decreasing. However, the right-holder’s option to purchase the asset after a potential buyer has incurred investigation costs reduces potential buyers’ expected profit from investigation as compared to the no-rights case. Consequently, the set of offers that induce investigation by potential buyers may be
smaller than in the no-rights case. When the investigation constraint is binding under a right of first refusal, therefore, the seller’s offers to potential buyers may be either higher or lower than in the no-rights case.

Finally, consider the right-of-first-offer case. A seller encumbered with a right of first offer is subject to two constraints. First, a right of first offer requires that offers made subsequent to the first offer may not be lower than the first offer. Second, all offers may not be higher than the value of potential buyers’ investigation constraint. As opposed to the no-rights case and the right-of-first-refusal case, the seller’s optimal sequence of unconstrained offers under a right of first offer has an inverted U-shape which has its maximum at the second offer and where the first offer is equal to the last one.

The main proposition of the paper is that in a multiple-buyer, sequential bargaining setting, first-purchase rights increase the joint profits of the seller and right-holder by increasing the probability that the right-holder is offered to purchase the seller’s asset and/or by providing the right-holder an opportunity to purchase the asset at a lower price than that he would be offered in the no-rights case. The value of first-purchase rights to the contracting parties derives from the facts that an unencumbered seller is indifferent to buyers’ sampling order if the right-holder’s investigation costs are identical to other buyers’ and makes the right-holder an offer which is higher than that which maximizes the joint profits of the seller and right-holder.

More specifically, consider a right of first refusal. When the seller’s optimal offers are unconstrained, the sequence of offers under a right of first refusal is identical to that in the no-rights case. As a result, granting a right of first refusal is costless to the seller. For a sufficiently low discount rate, the right-holder is bound to benefit from a right of first refusal since (i) the right increases the probability that the right-holder will be offered to buy the asset; and (ii) the right-holder may exercise the option embedded in the right at a lower price than that he would be offered in the no-rights case (recall that the seller’s optimal sequence of offers is decreasing). Thus, when the seller’s optimal offers are unconstrained and the discount rate is sufficiently low, a right of first refusal increases the joint profits of the seller and right-holder.

When the seller’s optimal offers under a right of first refusal are constrained, by contrast, the seller may be forced to lower his early offers to potential buyers in order to ensure investigation. The lower offers made by the seller, in turn, provide the right-holder an opportunity to purchase the asset at a lower price that that he would be offered in the no-rights case. Whether a right of first refusal increases the parties’ joint profits depends on the relation between the right-holder’s benefit from the option to buy the asset at a lower price and/or the higher probability to buy the asset versus the seller’s cost of making suboptimal offers.

Consider next a right of first offer. To comply with a right of first offer, the seller has to make the first offer to the right-holder. That first offer would typically be lower than the seller’s optimal first offer in the no-rights case. We show that the seller’s lower first offer always increases the parties’ joint profits as compared to the case in which the right-
holder is approached first in the no-rights case. But the right of first offer also forces the seller to make suboptimal subsequent offers to other potential buyers. Whether a right of first offer increases the parties’ joint profits depends on the relation between the seller and right-holder’s joint surplus from the reduced first offer versus the seller’s cost of making lower subsequent offers.

Our results comports well with the common perception that first-purchase rights are more likely to be employed when the right-holder has lower investigation cost or has a higher valuation of the asset for-sale than other potential buyers. This is frequently the case when the potential right-holder had a previous relationship with the seller with respect to the asset-for-sale. In such circumstances, the seller’s ability to exploit his monopoly position vis-à-vis the potential right-holder is greater. Consequently, inducing the seller to offer the right-holder to buy the asset at a lower price may be in the mutual interest of the seller and right-holder.

The paper proceeds as follows. Part 2 presents the model. Part 2.1 examines the seller’s and a designated buyer’s expected profits in the benchmark case in which the designated buyer has no first-purchase right. Part 2.2 examines the respective expected profits where the right-holder has a right of first refusal. Part 2.3 examines the case of rights of first offer. Part 3 analyzes the circumstances in which the seller and the right-holder can increase their joint profits by contracting for a right of first refusal or a right of first offer. Part 4 illustrates the paper’s results in the specific case in which buyers’ valuation are drawn from a uniform distribution. Part 5 concludes.2

2. The Model

Consider a single risk-neutral seller, $S$, with one indivisible asset for sale. $S$’s valuation of the asset is normalized to zero. There are $2 \leq n \leq \infty$ potential buyers for the asset. For simplicity, we refer to any potential buyer as “buyer.” All buyers are risk-neutral. A buyer’s valuation of the asset is a random draw from a common differentiable distribution $F$ on support $[0, \tilde{v}]$ and a strictly positive density $f$. We designate one buyer, a potential “right-holder,” by $R$. All buyers other than $R$ can learn their valuation of the asset at a cost $c \in [0, E(v))$, where $E(v) = \int_0^{\tilde{v}} vdv$. We consider two alternative assumptions on $R$’s investigation costs, $c_R$: (1) $c_R = c$; and (2) $c_R < c$. The motivation for the assumption that $R$’s investigation costs are lower than other buyers’ is that $R$ often has a previous relationship with $S$ with respect to the asset potentially subject to a first-purchase right, and therefore is likely to know his valuation of the asset at lower costs than other buyers’ (e.g., $R$ may have leased the asset from $S$ or may be a co-owner of $S$). Our qualitative result would not change if we assumed instead that $R$’s valuation of the asset were drawn from the probability distribution $F$ on support $[0 + a, \tilde{v} + a]$, for some $a > 0$ (provided that $a$ is sufficiently small).

2 The Appendix to the paper is available from the authors.
Bargaining takes place sequentially without recall. In each period, $S$ makes an offer to a buyer. The buyer then chooses whether to consider the offer. If the buyer does not consider the offer, the game proceeds to the next period. If the buyer chooses to consider the offer, he pays investigation costs $c$ and observes his valuation of the asset. If the buyer’s valuation is higher than or equal to $S$’s offer, the buyer accepts the offer and the game ends. If the buyer’s valuation is lower than $S$’s offer, the buyer rejects the offer and the game proceeds to the next period. If the asset has not been sold after all buyers have been approached, the game terminates and $S$ receives a payoff of zero. We consider three cases. In the first, baseline case, the game proceeds as discussed above. In addition, we assume that $R$ is the buyer who is approached first. In the second case, $R$ has a right of first refusal. In the third, $R$ has a right of first offer.

Before proceeding to the analysis, a comment on the structure of the game is in order. The assumption that the seller makes prospective buyers take-it-or-leave-it offers is made to enhance the analytical tractability of the model. The qualitative nature of our results, however, is driven by the properties that the seller has some bargaining power and that this power declines as the number of remaining buyers decreases. Using a more complex bargaining game would complicate the analysis without altering the nature of our conclusions.

2.1. The Benchmark Model – The No-Rights Case

We start our analysis assuming that $R$’s investigation costs are identical to other buyers’. We comment later on the case in which $R$’s investigation costs are lower than other buyers’. We begin by describing $S$’s set of feasible offers, i.e., offers that would induce a buyer to consider $S$’s offer. A buyer whose valuation of the asset is $v$ will accept an offer $k$ if $v \geq k$. If the buyer accepts the offer, his profit is $v - k$. If $v < k$, the buyer will reject the offer. The buyer’s expected profit from investigation is thus $\int_k^v (v - k) f(v) dv$. A buyer will incur investigation costs if and only if his expected profit from investigation is positive. $S$’s offer must therefore satisfy the following condition:

$$E[(v - k)^+] \geq c,$$

where $\theta^+ = \max \{\theta, 0\}$.

We denote by $\overline{k}$ the investigation constraint (“IC”) – i.e., the value of $k$ that satisfies (1) as an equality. Note that $\overline{k}$ is a decreasing function of $c$. The set of feasible offers is

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3 We make this assumption in order to segregate the direct effect of a first-purchase right on the right-holder from the indirect effect of such right on the order in which the right-holder is approached. We will make some observations on how our results would change if the order in which the right-holder is approached in the no-rights case were determined randomly.
thus a set of the form $K^n = [0, \bar{k}]$ (the subscript stands for ‘no-rights’). The assumption $c \in [0, E(v))$ ensures that there exists $\bar{k} > 0$ for which buyers’ expected profit from investigation is positive.

Consider first the case in which the number of buyers, $n$, is finite. To solve for $S$’s optimal sequence of offers, we proceed by backward induction. Let $k_n$ denote $S$’s offer at the last period. The probability that an offer $k_n$ will be accepted is $1 - F(k_n)$. $S$’s profit if the offer is accepted is $k_n$. $S$’s maximization problem at the last period is thus:

$$\max_{k_n \in K^n} k_n[1 - F(k_n)].$$

(2)

Let $k_n^*$ denote $S$’s optimal offer at the last period and $V_n = V_n(k_n^*)$ the maximized objective function.

Now consider $S$’s maximization problem at period $n-1$, the second-to-last period. $S$’s problem is:

$$\max_{k_{n-1} \in K^n} k_{n-1}[1 - F(k_{n-1})] + F\delta(k_{n-1})V_n,$$

where $0 < \delta \leq 1$ is a discount factor by which next-period values are multiplied to obtain current period value. Let $k_{n-1}^*$ denote $S$’s optimal second-to-last offer and $V_{n-1}(k_{n-1}^*)$ the maximized objective function.

The procedure is completed by induction. $k_i^*$ – the optimal offer made at period $i$ when there are $n-i$ buyers remaining – is the solution to the following maximization problem:

$$\max_{k_i \in K^n} k_i[1 - F(k_i)] + F(k_i)\delta V_{i+1},$$

(4)

where $V_{i+1}$ is $S$’s expected profit at period $i+1$ from the remaining $(n-i-1)$ buyers. $S$’s expected profit at period $i$, $V_i$, is the maximum of the objective function in (4). We assume that $F$ has a strictly increasing failure rate (i.e., $\frac{f(k)}{1-F(k)}$ is strictly increasing in $k$) and therefore that any local maximum is unique and globally optimal (see the Appendix).

Recall that by assumption $R$ is approached first by $S$. $R$’s expected profit from investigation is thus:

$$B = E[(v - k_i^*)^+] - c.$$

(5)
A general expression for $S$’s expected profit when there are a finite number of buyers is:

$$
\sum_{i=1}^{n} \prod_{j=0}^{i-1} F(k_j^*) \delta^{i-1} [1 - F(k_i^*)] k_i^*,
$$

where $F(k_i^*) = 1$ by definition.

$[1 - F(k_i^*)] k_i$ is $S$’s expected profit from stage $i$, $\delta^{i-1}$ is the discount factor, and $\prod_{j=0}^{i-1} F(k_j^*)$ is the probability that stage $i$ will be reached. The summation operator adds up $S$’s discounted expected profits from periods 1 to $n$.

Correspondingly, the time-1 expected profit of a buyer who is approached at period $i$ is:

$$
B_i = \prod_{j=0}^{i-1} F(k_j^*) \delta^{i-1} E[(v - k_i^*)^+ - c].
$$

$\delta^{i-1} E[(v - k_i^*)^+ - c]$ is the buyer’s discounted expected profit at period $i$, whereas $\prod_{j=1}^{i-1} F(k_j^*)$ is the probability that period $i$ will be reached.

Note that there is a tradeoff for buyers between being approached earlier or later. Later buyers have a lower probability of being approached and any profits they earn may be discounted, but they will, in general, receive a lower offer price.

$S$’s optimal offer and value take on a simpler form when $n$ is infinite. In this case, it is optimal for $S$ to make the same offer to all buyers. To solve for $S$’s optimal offer, note that for any $k \in K^n$, $S$’s expected profit, $V_\infty(k)$, satisfies:

$$
V_\infty(k) = [1 - F(k)] k + \delta F(k) V_\infty(k).
$$

Thus, we have:

$$
V_\infty(k) = \frac{[1 - F(k)] k}{1 - \delta F(k)}.
$$

The optimal offer made to all buyers is the value of $k$ that maximizes $V_\infty(k)$ for $k \in K^n$. Denote that optimal offer as $k^*_\infty$ and $S$’s expected profit given that offer as $V_\infty(k^*_\infty)$.

The following Lemma summarizes the effects of buyers’ investigation cost, the number of buyers and the discount factor on $S$’s and $R$’s expected profit in the no-rights case:
Lemma 1.

1.1 S’s optimal sequence of offers in the no-rights case:

- if \( k_{i+1}^* < \bar{k} \) then \( k_{i+1}^* < k_i^* \) for \( i = 1, \ldots, n-1 \).
- if \( k_i^* = \bar{k} \) then \( k_j^* = \bar{k} \) for \( i = 1, \ldots, n \) and \( j = 1, \ldots, i-1 \).
- \( dk_i^*(\delta) \over d\delta > 0 \) for \( i = 1, \ldots, n-1 \) and \( k_i^* < \bar{k} \).

1.2 \( k_1^*(c_R) \geq k_1^*(c) \) for \( c_R < c \).

1.3 Let \( \hat{k}_1 \) be the offer which maximizes S and R’s joint profits. Then \( k_1^* > \hat{k}_1 \).

**Proof:** See Appendix.

Lemma 1.1 describes S’s optimal sequence of offers:

- The sequence of offers is decreasing when the IC is not binding on S’s optimal offers and constant otherwise (first part). The intuition is that, as the number of remaining buyers decreases, S becomes more eager to sell the asset to avoid a payoff of zero if the asset is not sold.

- When the IC is binding on any offer, it is also binding on all prior offers (second part). This follows from the fact that the optimal sequence of offers is monotonically decreasing.

- When the IC is not binding, S’s optimal offer is decreasing in the discount rate (fourth part). The intuition is that S makes lower offers when the cost of rejection is higher. Note that S’s last offer is independent of the discount rate.

Lemma 1.2 states that the first-period offer when R’s investigation costs are lower than other buyers’ (\( c_R < c \)) is higher than or equal to the first-period offer when R’s investigation costs are identical to other buyers’ (\( c_R = c \)). In particular, S’s optimal first-period offer when \( c_R < c \) is higher than when \( c_R = c \) if and only if the IC is binding on the optimal first-period offer when \( c_R = c \).

Lemma 1.3 describes the relations between S’s optimal first-period offer and the offer that maximizes the joint profits of S and R. Increasing the first-period offer has two opposite effects on S and R’s joint profits. On the one hand, a higher first-period offer decreases the parties’ joint profits from the first period only. (That value is maximized when S makes an offer of zero, S’s valuation of the asset.) On the other hand, a higher first-period offer increases S’s expected profit from subsequent periods by increasing the probability that S will approach other buyers. The first-period offer that maximizes the parties’ joint profits is such that the marginal joint cost of increasing the offer is equal to the (S’s) marginal benefit from subsequent periods of increasing the offer. But S’s optimal first-period offer is such that the marginal cost to S of increasing the first-period offer is...
offer is equal to $S$’s marginal benefit. Since increasing the first-period offer necessarily reduces $R$’s expected profit, $S$’s optimal first-period offer is higher than that which maximizes $S$ and $R$’s joint profits.

2.2. Right of First Refusal

This section examines the effect of a Right of First Refusal (“RFR”) on $S$ and $R$. RFR provides $R$ with the right to buy the asset at a price offered by $S$ to another buyer conditional on the offer being accepted by that buyer. The RFR game proceeds as follows. Each period $S$ makes an offer to a buyer other than $R$. If the buyer fails to consider the offer or rejects the offer outright, the game proceeds to the next period. If the buyer accepts the offer, $R$ then incurs investigation costs and observes his valuation of the asset. If $R$’s realized valuation of the asset is higher than or equal to $S$’s offer, $R$ exercises his right; if, by contrast, $R$’s valuation is lower than $S$’s offer, the asset is sold to the other buyer. This procedure tracks the legal terms of an RFR.\textsuperscript{4} We further assume that if $S$ has made offers to all buyers other than $R$ and none of these offers were accepted, $S$ makes an offer directly to $R$ in the last period. This assumption reflects the notion that the presence of an RFR should not (and by its terms does not) prevent $S$ from approaching $R$ directly when $R$ is the only available buyer remaining.

We denote the case in which $R$ is granted RFR by attaching the superscript ‘$r$’ to the relevant expressions. We begin by characterizing the set of feasible offers under RFR. $R$ will incur investigation costs (either when the RFR is triggered or when $R$ receives a direct offer) for any $k < \bar{k}$. A buyer other than $R$ will incur investigation costs only if the buyer’s expected profit from investigation is positive. Any offer to a buyer other than $R$ must therefore satisfy the following condition:

$$F(k)E[(v - k)^+] \geq c,$$

(10)

$F(k)$ is the probability that $R$’s value is lower than $k$ so that $R$ does not exercise his right. The expectation expression is the buyer’s expected profit conditional on the occurrence of a sale. As opposed to the no-rights case, a buyer’s expected profit under RFR does not necessarily increase as $S$’s offer decreases. This is because as the offer decreases, the probability that $R$ will exercise his right increases as well. Let $c^m = \max_{k < \bar{k}} F(k)E[(v - k)^+]$ denote potential buyers’ maximum profit from investigation under RFR.

\textsuperscript{4} Note that the last-period offer in the RFR case is identical to that in the no-right case when $c_R = c$ but may be higher than that offer when $c_R < c$. 

http://law.bepress.com/alea/17th/art57
The presence of RFR introduces an additional constraint on S’s set of feasible offer to buyers other than R. It follows that \( K^r \subseteq K^n \), where \( K^r \) and \( K^n \) are the sets of feasible offers in the RFR case and the no-rights case, respectively. The additional constraints introduced by RFR on the set of feasible offers cannot make S better off, but may make R better off, as compared to the no-rights case.

S’s last-period offer to R is subject to the same investigation constraint as in the no-right case. S’s last-period offer to R will thus be identical to S’s last-period offer in the no-rights case except if (i) \( c_R < c \); and (ii) the IC is binding on the last offer in the no-rights case. In this case, S’s last-period offer in the RFR case will be higher than in the no-rights case. Note that, since the last-period offer is subject to the no-rights IC but the second-to-last-period offer is subject to the RFR IC, S’s optimal last-period offer in the RFR case may exceed the second-to-last-period offer even when \( c_R = c \) (a “spike”).

The solution for S’s optimal sequence of offers under RFR is analogous to the no-rights case (see Eqs. (2), (3), and (4)), except that S’s set of feasible offers at all periods other than the last period is now \( K^r \). In particular, the RFR investigation constraint may be binding on earlier offers as compared to the no-rights case. Let \( k_i^{r*,r} \) denote S’s optimal sequence of offers and \( V_i^r \) denote S’s maximized value under RFR. The expressions for S’s maximized value are analogous to the no-right case, except that the optimal offer is \( k_i^{r*,r} \) instead of \( k_i^{r*} \).

R’s value under RFR is given by:

\[
\sum_{i=1}^{n-1} \prod_{j=0}^{i-1} F(k_j^{r*})\delta^{i-j-1}[(1 - F(k_j^{r*}))E[(v - k_j^{r*})^+] - c] \\
+ \prod_{j=1}^{n-1} F(k_j^{r*})\delta^{n-j-1}E[(v - k_n^{r*})^+] - c], \\
\]

(11)

where \( F(k_0^{r*}) = 1 \) by definition.

The first expression is R’s discounted expected profit from period 1 to \( n - 1 \). R’s value at each period other than the last is equal to the joint probability that R’s valuation and buyer i’s valuation are higher than S’s offer, multiplied by R’s expected profit conditional on his valuation exceeding the offer to buyer i. The second expression is R’s discounted expected profit from the last period.

The following Lemma considers S’s optimal offers in the RFR case as compared to the no-rights case:
Lemma 2.

2.1 S’s optimal sequence of offers in the RFR case:

- \( k_i^* \geq k_{i+1}^* \) for \( i = 1, \ldots, n - 2 \).
- We call the case in which \( k_n^* > k_{n-1}^* \) a ‘spike’. A spike may exist if \( k_{n-1}^* < k_{n-1}^* \) or \( c_R < c \).

2.2 S’s optimal offers in the RFR case as compared to the no-rights case:

- If \( k_i^* = \bar{k} \) then \( k_i^* < k_i^* \) for \( i = 1, \ldots, n - 1 \).
- If \( k_n^* = \bar{k} \) and \( c_R < c \) then \( k_n^* < k_n^* \).

Proof. See Appendix.

- Lemma 2.1 states that S’s optimal sequence of offers in the RFR case is monotonically decreasing from the first-period offer to the second-to-last offer (first part). The intuition here is identical to the one provided in the no-rights case. The last offer, however, may be higher than the second-to-last-offer. We define such state of affairs as a ‘spike’ (second part). The reason is that the last offer in the RFR case is made to \( R \) and therefore is not subject to the RFR IC as are previous offers nor is it subject to the no-rights IC of other buyers if \( R \)’s investigation costs are lower than other buyers’.

- Lemma 2.2 compares S’s optimal offers in the no-rights case and the RFR case. When the IC is binding on S’s optimal offer in the no-rights case, S’s optimal offer in the RFR case is lower than the equivalent offer in the no-rights case (first part). The intuition is that buyers’ expected profit under RFR is strictly lower than in the no-rights case. When the IC is binding in the no-rights case, buyers’ expected profit from investigation in the no-rights case is zero. Thus, buyers’ expected profit in the RFR case would be negative if the offer under RFR were equal to (or higher than) the IC in the no-rights case.

When the IC is binding on the optimal last-period offer in the no-rights case and \( R \)’s investigation costs are lower than other buyers’, S’s optimal last-period offer in the no-rights case is lower than the equivalent offer in the RFR case (second part). The reason is that the last offer in the RFR case is made to \( R \) and thus is subject to a less restrictive IC when \( R \)’s investigation costs are lower than other buyers’.

2.3. Right of First Offer

This section explores the value of a Right of First Offer (“RFO”) for \( S \) and \( R \). The game proceeds as in the benchmark case with the following modifications. If \( R \) is granted a RFO, any sequence of offers must satisfy two conditions:
1. The first offer must be made to \( R \);\(^5\)
2. If \( R \) rejects the offer, \( S \) cannot offer the object to another buyer for a price lower than that offered to \( R \).

We denote the case in which \( R \) is granted RFO by attaching the superscript ‘\( \circ \)’ to the relevant expressions.

We begin by characterizing \( S \)'s set of feasible offers under RFO as a function of the first offer. \( S \)'s set of feasible offers under RFO is a set of the form \( K^\circ = [k_i^\circ, \overline{k}] \), where \( k_i^\circ \) is the first-period offer in the RFO case and \( \overline{k} \) is buyers’ investigation constraint. Note that the investigation constraint in the RFO case is identical to that in the no-rights case. It follows that \( K^\circ \subseteq K^n \), where \( K^n \) is the set of feasible offers in the no-rights case.

The additional constraint introduced by RFO on the set of feasible offers cannot make \( S \) better off. Recall that Lemma 1.1 established that the optimal sequence of offers in the no-rights case is non-increasing. Introducing a lower bound on the set of feasible offers thus affects the feasibility of some of these offers. As a consequence, \( S \)'s expected profit under RFO is lower than or equal to his expected profit in the no-rights case.

To solve for \( S \)'s optimal sequence of offers, we use the fact that the first-period offer under RFO (i.e., the offer that is made to \( R \)) is equal to the last-period offer. This is because (i) the first-period offer may not be higher than the last-period offer due to the RFO constraint; and (ii) the optimal first-period offer is not lower than the optimal last-period offer for any choice of last-period offer. This follows from the assumption that \( F(\cdot) \) has a strictly increasing failure rate, which implies, in turn, that \( S \)'s expected profit is increasing on \([0,k^*_i]\). Note that this reasoning applies only to the relation between the first- and last-period offers. Offers other than the last offer may be (in optimum) higher than the first-period offer because these offers depend on the optimal offers made to the remaining buyers.

\( S \)'s optimal offer at each period other than the first- and last periods can thus be expressed as a function of the first offer:

\[
k^0_{i} (k^0_{i}, \overline{k}) = \arg \max_{k_{i} \in [k^0_{i}, \overline{k}]} k_{i} (1 - F(k_{i})) + F(k_{i}) \delta V_{i+1},
\]

where \( V_{i+1} \) is \( S \)'s expected profit from the remaining periods (\( i+1 \) to \( n \)). That is, each offer, other than the first-period and last-period offers, is equal to the maximum of the optimal unconstrained offer and the first offer, subject to buyers’ investigation constraint. Let \( k^\circ_i \) denote the optimal offer made to buyer \( i \) under RFO as a function of the first offer, \( k^0_{i} \), and the investigation constraint, \( \overline{k} \).

\(^5\) This was an assumption in the benchmark case. In the RFO case, it follows for the legal terms of an RFO.
Setting \( F(k_0^*) = 1 \), we can now present \( S \)'s maximization problem in the following simplified form:

\[
\max \sum_{k_{n(0,j)} \mid j=1}^{n-i} \prod_{j=1}^{i-1} F(k_j^o) \delta^{i-1} k_i^o [1 - F(k_i^o)]
\]

s.t. \( k_1^o = k_n^o \). \hspace{1cm} (13)

The objective function is \( S \)'s expected profit from periods 1 to \( n \). The constraint ensures that the last offer is equal to the first offer. Since offers made to buyers positioned between the first- and last buyers are dependent on \( k_n^o \) and \( k_n^o = k_1^o \), \( S \)'s maximization problem is reduced to finding an optimal first offer that maximizes \( S \)'s expected profit. This formulation of \( S \)'s maximization problem helps to characterize \( S \)'s sequence of optimal offers and is useful in solving for \( S \)'s optimal sequence of offers within a specific parameterization setting. We denote by \( k_1^o^* \) the optimal first offer and \( k_i^o^* \) the optimal subsequent offers.

\( R \)'s expected profit under RFO is:

\[
E[(v - k_1^o^*)^+] - c_R.
\]

When \( c_R = c \), \( R \)'s expected profit when IC is binding on \( S \)'s optimal first offer is zero (by definition of IC). It follows that when \( c_R = c \), RFO is valuable for \( R \) only if \( S \)'s optimal first offer, \( k_1^o^* \), is unconstrained.

When \( c_R = c \) but IC is not binding on \( S \)'s optimal first offer under RFO (\( k_1^o^* \leq \bar{k} \)), RFO can become valuable for \( R \). Specifically, RFO forces \( S \) to lower his first-period offer below the optimal first-period offer in the no-rights case. RFO may thus provide \( R \) with an opportunity to buy the asset for a price lower than that he would be offered if he were sampled first in the no-rights case. When \( c_R < c \), this effect may be enhanced. \( S \) may make a higher offer to \( R \) in the no-rights case when \( c_R < c \) (relative to \( c_R = c \)), but will make the same offer in the RFO case. However, for \( c_R << c \), it may be optimal for \( S \) to make a single, high offer that precludes \( S \) from making subsequent offers to other buyers (because the IC precludes other buyers from investigating an offer equal to that made to \( R \) and the RFO constraint prevents \( S \) from making other buyers a lower offer than that made to \( R \) (‘skipping buyers.”)

We summarize the main results in the following lemma:

**Lemma 3.**

3.1 Skipping buyers
when $V_n > V_1^0$, $S$ makes a single offer to $R$ which is higher than $\bar{k}$ ("skipping buyers").

necessary (but not sufficient) conditions for $V_n > V_1^0$ are:
(i) $c_R < c$; and (ii) $k_n^* > k_n^{o_0} - \bar{k}$.

3.2 $S$’s optimal sequence of offers in the RFO case:

- $k_1^{o_0} = k_n^{o_0}$.
- if $k_1^{o_0} < \bar{k}$ then (i) $k_i^{o_0} \geq k_{i+1}^{o_0}$ for $i = 2, \ldots, n-1$; and (ii) $k_2^{o_0} \geq k_1^{o_0}$.

3.3 $S$’s optimal offers in the RFO case as compared to the no-rights case:

- if $k_1^{o_0} < \bar{k}$ then $k_1^{o_0} < k_1^*$.
- if $k_1^{o_0} = \bar{k}$ then (i) $k_i^{o_0} = k_i^* = \bar{k}$; and (ii) $k_i^{o_0} = \bar{k}$ for $i = 2, \ldots, n$.
- if $k_2^{o_0} < \bar{k}$ then $k_2^{o_0} < k_2^*$; if $k_2^{o_0} = \bar{k}$ then $k_2^{o_0} = k_2^* = \bar{k}$.
- $k_n^{o_0} \geq k_n^*$.
- if $k_n^* = \bar{k}$ then $k_i^{o_0} = k_i^* = \bar{k}$ for $i = 1, \ldots, n$.

Lemma 3.1 states that in the RFO case $S$ may make a single offer to $R$ which is higher than the investigation constraint of other buyers ("skipping buyers"). For this it is necessary that $R$’s investigation costs are lower than other buyers’ and that the IC is binding on $S$’s optimal last offer under the RFO case if all buyers are approached.

Lemma 3.2 describes the sequence of optimal offers in the RFO case if $S$ does not skip buyers.

- The optimal first- and last-period offers under RFO are identical (first part).
- The optimal sequence of offers in the RFO case is either constant or forms a reverse U-shape peaking at the second offer (second part). The intuition is that, as in the no-rights case, the optimal sequence of offers from the second- to the last-period is monotonously decreasing. However, due to the RFO constraint, the first-period offer may not be higher than the last-period offer. Thus, if the IC is binding on the optimal first-period offer in the RFO case, the optimal sequence of offers must be constant since the IC prevents $S$ from making offers higher than the first-period offer, whereas the RFO constraint prevents $S$ from making offers lower than the first-period offer.

Lemma 3.3 compares the RFO case to the no-rights case.

- If the IC is not binding on $S$’s optimal first offer under RFO, the optimal first-period offer in the RFO case is lower than the first-period offer in the no-rights case (first part).
It follows that, in the presence of RFO, \( S \) may make \( R \) a lower first-period offer as compared to the no-rights case. As will be shown below, this may increase the parties’ joint profits from contracting for RFO.

- If the IC is binding on \( S \)’s optimal first-period offer in the RFO case, then the optimal first-period offer in the no-rights case is identical to that in the RFO case and all offers subsequent to the first-period offer in the RFO case are equal to the investigation constraint (second part). The intuition here is that \( S \)’s continuation value at the second period is higher in the no-rights case than in the RFO case and therefore \( S \)’s optimal unconstrained offers in the no-rights case is higher than that in the RFO case.

- If the IC is not binding on \( S \)’s optimal second-period offer in the RFO case, that offer is lower than \( S \)’s optimal second offer in the no-rights case (third part). The reason is that the RFO imposes a constraint on subsequent offers that reduces \( S \)’s continuation value relative to the no-rights case. Because \( S \)’s continuation value at the second period (i.e., \( S \)’s expected profit from period three on) is lower in the RFO case than in the no-rights case, \( S \) will make a lower second offer in the RFO case, which is more likely to be accepted. It follows that if the IC is binding on the optimal second offer in the RFO case, it is also binding on the optimal second offer in the no-rights case; both offers thus equal the IC.

- The optimal last-period offer in the RFO case is higher than that in the no-rights case (fourth part). The intuition here is that it is less profitable to \( S \) to make a lower last-period offer in the RFO case—where \( S \) would then also be forced to make a lower first-period offer—than in the no-rights case, where the last-period offer is constrained only by buyers’ investigation costs.

- If the IC is binding on the optimal last offer in the no-rights case than it is binding on all offers in both the no-rights case and the RFO case. The sequences of offers in the no-rights case and the RFO case are thus identical (fifth part). The intuition is that the set of feasible offers in the no-rights case is bigger than that in the RFO case. If \( S \)’s optimal sequence of offers in the no-rights case is constant (and therefore is compatible with RFO), then this sequence of offers is also optimal in the RFO case.

### 3. Joint Profits of First-Purchase Rights

A first-purchase right will be contracted for ex ante if \( R \)’s and \( S \)’s joint profits if they contracted for such right are greater than those in the no-rights case. Thus, RFR will be contracted for if and only if:

\[
B^r + V^r > B + V \tag{15}
\]

Likewise, RFO may be contracted for if and only if:

\[
B^o + V^o > B + V \tag{16}
\]
We denote the difference between the joint profits in the right-of-first-purchase case and the no-rights case as ‘joint surplus,’ whether or not such difference is positive. Formally, let $\psi^j = B^j + V^j - (B + V) \text{ for } j = r, o$.

Proposition 1 considers the effect of $R$’s investigation costs on the surplus generated by RFR and RFO.

**Proposition 1.**

**R’s investigation costs:**

Let $C_1$ be the set of $R$’s investigation costs such that $S$’s optimal first-period offer to $R$ in the no-rights case is strictly increasing as $R$’s investigation costs decrease; i.e., $C_1 = \{c_R : k(c_R) \geq k^*_1(c_R) \geq k^*(c) \mid k^*_1(c) = k(c)\}$. Then, if $k^*_n < k^*$ the surplus generated by RFR and RFO is increasing in $R$’s investigation costs for $c_R \in C_2$ and is invariant to $R$’s investigation costs otherwise.

- The surplus generated by RFR and RFO is monotonically increasing as $R$’s investigation costs decrease when (i) the IC is binding on $S$’s optimal first-period offer, but not on the optimal last-period offer, in the no-rights case; and (ii) other buyers’ investigation costs are held constant (Proposition 1.2). Note that the assumption that the IC is binding on $S$’s optimal first-period offer implies that $S$ would prefer to approach $R$ first in the no-rights case.

The intuition here derives from the fact that the optimal first-period offer that maximizes $S$ and $R$’s joint profits in the no-rights case is lower than $S$’s optimal first offer (Lemma 1.3). As $R$’s investigation costs decrease, $S$ will increase its first-period offer in the no-rights case, thereby reducing the parties’ joint profits. By contrast, $S$’s optimal offers in the RFR case are invariant to $R$’s investigation costs (provided that the IC is not binding on the last offer in the no-rights case).

---

6 This result fits the common notion that parties often contract for first-purchase rights when $R$ has some prior relationship with $S$—e.g., when $R$ leases the asset-for-sale from $S$ or when $R$ and $S$ are partners or co-owners—that reduces $R$’s investigation costs. $S$ might take advantage of $R$’s lower investigation cost by making him a higher offer than that he would make an ordinary buyer. Since the joint value is higher when $S$’s offer is lower than the offer that is optimal to $S$ in the no-rights case, first-purchase rights may be used to share the surplus brought about by avoiding such offers.
Proposition 2 considers the surplus generated by RFR.

**Proposition 2.**

2.1 Let $C_2$ be the set of buyers’ investigation costs such that $S$’s optimal sequence of offers in the no-rights case is identical to that in the RFR case; i.e., $C_2 = \{c : k_i^r = k_i^* \forall i = 1, ..., n \mid n, \delta \}$ (the set $C_2$ is not empty since it contains $c = 0$).

Then:
- If $c \in C_2$ and $\delta = 1$ then RFR generates a positive surplus ($\psi^r > 0$).
- When $c \in C_2$, the surplus generated by RFR is invariant to buyers’ investigation costs (i.e., $\frac{d\psi^r}{dc} = 0$).

2.2 If there is no spike ($k_{n-1}^* < k_{n-1}^r$) and $S$’s optimal first-period offer in the no-rights case is constrained ($k_i^* = \bar{k}$), then the surplus generated by RFR is decreasing in the number of buyers ($\psi^r(n) < \psi^r(n')$ for $n > n'$).

**Proof.** See the Appendix.

According to Proposition 2:

**Low investigation costs and low discount rate:**

- When $S$’s optimal sequence of offers under RFR is unconstrained and the discount rate is zero, RFR generates positive surplus (Proposition 2.1). The intuition is as follows. When the RFR ICs are not binding, the optimal offers in the no-rights case are identical to those in the RFR case. $S$ therefore bears no cost from granting RFR. The surplus generated by RFR thus depends solely on $R$’s expected profit in the no-right case as compared to the RFR case. The probability that $R$ will be offered to buy the asset is equal to one in the no-rights case as well as the RFR case. The price paid by $R$ in the RFR case, by contrast, is lower than, or equal to, the price paid by $R$ in the no-rights case, since under RFR the asset may be sold to $R$ at periods later than the first period and hence at a lower price. As long as the discount rate is sufficiently small (and always when the discount rate is zero), $R$’s expected profit under RFR is higher than his expected profit in the no-rights case.

Note that this result does *not* depend on the assumption that $R$ is sampled first in the no-rights case. Thus, so long as the discount rate is sufficiently low, $R$’s expected profit is higher under RFR than in the no-rights case because the probability that $R$ is offered to buy the asset is higher in the RFR case as compared to the no-rights case, whereas the price paid by $R$ in the RFR case is lower than, or equal to, the price paid by $R$ in the no-rights case.

**Number of buyers:**
The surplus generated by RFR is decreasing as the number of buyers increases, if there is no spike (see Lemma 2) and the IC is binding on the first-period offer in the no-rights case (Proposition 2.2). When there is no spike, R’s expected profit in the RFR case decreases when the last period is reached later because offers made in earlier periods are higher than the last-period offer. In addition, when the IC is binding on the optimal first-period offer in the no-rights case, R’s expected profit in the no-rights case is invariant to the number of buyers. As to S, when the IC is binding in the no-rights case, it is binding at a lower level of investigation costs in the RFR case. An increase in the number of buyers makes granting an RFR more costly to S because there are additional offers that are more tightly constrained.

The next proposition considers the surplus generated by RFO.

Proposition 3.

3.1 If the IC is binding on the optimal first-period offer in the RFO case \((k_1^* = \overline{k})\) and R’s investigation costs are equal to other buyers’ \((c_R = c)\), then RFO generates negative surplus \((\psi^0 \leq 0)\); in particular when the number of buyers is infinite and \(c_R = c\) then RFO generates zero surplus \((\psi^0 = 0)\).

3.2 If the IC is binding on the last-period offer in the no-rights case \((k_n^* = \overline{k})\) and R’s investigation costs are lower than other buyers’ \((c_R < c)\), then RFO generates positive surplus \((\psi^0 > 0)\); in particular, when the number of buyers is infinite and R’s investigation costs are lower than other buyers’, then RFO generates positive surplus.

According to Proposition 3:

- When the investigation constraint is binding on the optimal first-period offer in the RFO case and R’s investigation costs are identical to other buyers’, RFO generates zero or negative surplus (Proposition 3.1). The intuition is that when the IC is binding on the optimal first offer under RFO, it is also binding on the optimal first-period offer in the no-rights case (Lemma 3.3). R’s expected profit in both the RFO case and the no-rights case is thus zero. But S’s expected profit in the RFO case is never higher than S’s expected profit in the no-rights case. RFO therefore cannot generate surplus.

Note that this result might change if R’s were not sampled first. If R were not sampled first or were sampled randomly, then his expected profit in the no-rights case might be positive (in contrast to the case where R is sampled first). In this case, the surplus generated by RFO would be positive.

- When the IC is binding on the optimal last-period offer in the no-rights case and R’s investigation costs are lower than other buyers’, RFO generates positive surplus (Proposition 3.2). When the IC is binding on the last-period offer in the no-rights case, it
is binding on all offers (other than that made \( R \)) in the no-rights case (see Lemma 3.3). The optimal sequence of offers to such buyers in the no-rights case is thus constant. Therefore, the optimal offers other than the first-period offer in the RFO case are identical to those in no-rights case. It follows that \( S \)'s cost of granting RFO stems only from the lower first-period offer in the RFO case as compared to the no-rights case. As shown in Lemma 1.3, the parties’ joint profits from the first-period offer increases as the first-period offer decreases.

3.2. A Comment on Discount Rate

A decrease in the discount rate has an indeterminate effect on the surplus generated by RFR. We briefly describe why that is for RFR and RFO.

**RFR:**

Consider the effect of a decrease in the discount rate on \( S \)'s and \( R \)'s expected profits in the no-rights case and the RFR case. A decrease in the discount rate does not affect \( R \)'s value in the no-rights case (recall that \( R \) is sampled first), but may increase or decrease \( R \)'s expected profit in the RFR case. (A lower discount rate increases \( S \)'s optimal offers under RFR; this in turn increases the exercise price of \( R \)'s RFR option, thereby decreasing \( R \)'s expected profit; but a decrease in the discount rate also increases the likelihood that \( R \) will be approached last, thereby increasing \( R \)'s expected profit if the last offer is lower than the second-to-last one). In addition, a lower discount rate increases the discounted value of \( R \)'s expected profit from later periods. The overall effect on \( R \)'s expected profit is indeterminate. As to \( S \), a lower discount rate increases \( S \)'s expected profit in the no-rights case (both because a lower discount rate increases the present value of \( S \)'s expected profit from later periods and because it allows \( S \) to raise his offers) as well as \( S \)'s expected profit in the RFR case. The overall effect of a decrease in the discount rate on the joint surplus generated by RFR is thus indeterminate.

**RFO:**

A decrease in the discount rate decrease \( S \)'s optimal first-period offer in both the no-rights case and the RFO case. In addition, a decrease in the discount rate flattens \( S \)'s optimal sequence of offers in both the no-rights case and the RFO case (see Lemma 1.3), thereby affecting the difference between \( S \)'s optimal offers in periods subsequent to the first period in both cases. The difference between \( S \)'s optimal sequence of offers in the no-rights case and the RFO case in turn affects the cost to \( S \)'s from granting RFO. Whether the surplus generated by RFO is increasing or decreasing in the discount rate thus depends on the relation between the effect of the discount rate on \( S \)'s optimal offers in the no-rights case and the RFO case.

4. Specific Parameterization

To illustrate the paper’s main insights and to derive concrete results we consider the case in which buyers’ valuation is distributed uniformly on \([0,1]\). We make two alternative assumptions on \( R \)'s investigation cost: (i) \( R \)'s investigation costs are equal to
other buyers’ investigation cost; and (ii) \( R \)’s investigation costs, as contrasted to other buyers’, are equal zero. We provide results for the cases of \( n = 5, 10 \) and \( \delta = 1, 0.95, 0.9, 0.85 \).

### 4.1. The No-Rights Case

Given that his valuation is distributed uniformly on \([0,1]\), a buyer will consider an offer \( k \) if and only if \( \int_{k}^{1} (v - k) dv \geq c \); i.e., if and only if

\[
k \leq 1 - \sqrt{2c}.
\]

The investigation constraint is thus \( \bar{k} = 1 - \sqrt{2c} \), where \( 0 \leq c \leq 0.5 \). The set of feasible offers is a set of the form \( K^* \in [0,1-\sqrt{2c}] \mid 0 \leq c \leq 0.5 \).

Consider the case in which the number of buyers is finite. \( S \)’s optimal offer at the last stage solves the following maximization problem:

\[
\max_{k \in K^*} k_n (1 - k_n).
\]

We thus obtain:

\[
k_n^* = \min[\frac{1}{2}, 1 - \sqrt{2c}] \\
V_n = \begin{cases} \frac{1}{4}, & \text{if } k_n^* = \frac{1}{2} \\ \sqrt{2c - 2c}, & \text{if } k_n^* = 1 - \sqrt{2c}. \end{cases}
\]

For \( i < n \), \( S \) solves the following maximization problem:

\[
\max_{k_i \in K^*} k_i (1 - k_i) + k_i \delta V_{i+1}.
\]

This gives the recursive solution for the optimal offer at stage \( i < n \):

\[
k_i^* = \min[\frac{1}{2} (1 + \delta V_{i+1}), 1 - \sqrt{2c}].
\]

Table 1 illustrates \( S \)’s optimal sequence of unconstrained offers when \( n = 3, 5, 10 \):

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<th>( n )</th>
<th>( k_{n,9} )</th>
<th>( k_{n,8} )</th>
<th>( k_{n,7} )</th>
<th>( k_{n,6} )</th>
<th>( k_{n,5} )</th>
<th>( k_{n,4} )</th>
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<td>0.62500</td>
<td>0.50000</td>
</tr>
</tbody>
</table>

20

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Table 1

$S$’s value at stage $i < n$ is given by the following recursive solution:

\[
v_i = \begin{cases} 
\frac{1}{4} (1 + \delta v_{i+1})^2, & \text{if } k_i^* = \frac{1}{2} (1 + \delta v) \\
\sqrt{2c - 2c + \delta (1 - \sqrt{2c})v_{i+1}}, & \text{if } k_i^* = 1 - \sqrt{2c}.
\end{cases}
\]

When the number of buyers is infinite, $S$’s maximization problem is:

\[
\max_{k \in K^*} \frac{k(1-k)}{1-\delta k}
\]

The solution to this problem is

\[
k_{*\infty} = \min \left\{ \frac{1}{\delta} (1 - \sqrt{1-\delta}), 1 - \sqrt{2c} \right\}.
\]

4.2. Right of First Refusal

This Section considers the value of a Right of First Refusal. A buyer other than $R$ will consider an offer $k$ if and only if $\int_k^1 (v-k)dv \geq c$; that is, if and only if:

\[
\frac{k(1-k)^2}{2} \geq c.
\]

The RFR investigation constraint does not admit a simple closed-form representation. Note, however, that since the left-hand side reaches a maximum on $[0,1]$ at $k = \frac{1}{2}$, the highest value of investigation costs at which a buyer will investigate is $c = 0.07407 \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right)^2 = 0.07407$.\(^7\) Note also that since $S$’s value is increasing on $[\frac{1}{3},1]$, $S$ will never make an offer of $k < \frac{1}{3}$. As a result, the lower RFR investigation constraint is never binding. The set of feasible offers is thus a set of the form $K^{**} \in \{k : \frac{1}{2} k(1-k)^2 \geq c \land k \in [\frac{1}{3},1] \mid 0 \leq c \leq 0.07407\}$.\(^8\)

Setting $k_0^{**} = 1$, $R$’s value under RFR is given by:

\[
\sum_{i=1}^{n-1} \prod_{j=0}^{i-1} k_i^{**} \delta^{i-1} (1-k_i^{**}) \cdot \left(\frac{1}{2} k_i^{**} (1-k_i^{**})^2 - c\right)
\]

\(^7\) When $\delta < 1/3$ and $c = 0.07407$, $S$’s value is maximized when the first offer is made to $R$. The marketing constraint is thus lower than 0.07407.

\(^8\) The expression for $S$’s values is analogous to the no-rights case.
\[ + \prod_{j=1}^{n-1} F(k_j^*) \delta^{n-1} \cdot \left( \frac{1}{2} (1 - k_j^*)^2 - c \right) \]

### 4.3 Right of First Offer

Consider \( S \)'s maximization problem under RFO when buyers’ valuation is distributed uniformly on \([0, 1]\):

\[
\max_{k_i \in [0, k]} \sum_{i=1}^{n-1} \prod_{j=0}^{i-1} k_j^o \delta^{i-1} k_i^o (1 - k_i^o) \\
\text{s.t. } k_1^o = k_n^o.
\]

Using Solver, we obtain that for \( n \leq 3 \), \( k_i^o = k_i^o \) for \( i = 1, \ldots, n-1 \) and every \( c \); the optimal sequence of offers is thus constant. For example, when \( n = 3 \) and \( c \leq 0.06846 \), \( S \) makes an offer of 0.62996 to all buyers. For \( c > 0.06846 \), the optimal offer is equal to buyers’ investigation constraint, \( 1 - \sqrt{2c} \), and is decreasing in \( c \). (the case of \( n = 2 \) is simple; the first and last offer must be equal.) \( S \)'s value is obtained by plugging in the optimal first offer into Eq. (28). When IC is binding on the optimal first offer, \( S \)'s value is

\[
\sqrt{2c} (1 - \sqrt{2c}) \left( 1 - (1 - \sqrt{2c})^n \right) \frac{1}{1 - \delta \sqrt{2c}}.
\]

Table 2 illustrates \( S \)'s optimal sequence of *unconstrained* offers under RFO when \( n = 3, 5, 10 \):

<table>
<thead>
<tr>
<th>( N )</th>
<th>( k_n^o )</th>
<th>( k_{n-1}^o )</th>
<th>( k_{n-2}^o )</th>
<th>( k_{n-3}^o )</th>
<th>( k_{n-4}^o )</th>
<th>( k_{n-5}^o )</th>
<th>( k_{n-6}^o )</th>
<th>( k_{n-7}^o )</th>
<th>( k_{n-8}^o )</th>
<th>( k_{n-9}^o )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td></td>
<td>0.62996</td>
<td>0.62996</td>
<td>0.62996</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>0.67223</td>
<td>0.69271</td>
<td>0.67223</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.73032</td>
<td>0.74971</td>
<td>0.74492</td>
<td>0.73775</td>
<td>0.73032</td>
<td>0.73032</td>
<td>0.73032</td>
<td>0.73032</td>
<td>0.73032</td>
<td></td>
</tr>
</tbody>
</table>

**Table 2**

### 4.4. Joint Profits of First-Purchase Rights

The following table presents the values of investigation cost for which a first-purchase-right generates positive surplus for different value of \( n \) and \( \delta \):
5. Conclusion

This paper analyzed the value of first-purchase rights within a sequential-bargaining framework. The analysis proceeded by comparing the seller’s optimal offers in the no-rights case and in the case in which a first-purchase right is present. We showed that the value of rights of first-purchase depend on the number of buyers, buyers’ investigation costs, the right-holder’s investigation costs and the discount rate. Whether first-purchase rights will be contracted for thus depend on the bargaining environment. The analysis identified two ways by which first-purchase rights may increase the seller and right-holder’s joint profits. First, first-purchase rights may increase the likelihood that the right-holder will be made an offer to buy the seller’s asset. Second, first-purchase rights may provide the right-holder an opportunity to buy the seller’s asset at a price lower than that he would be offered in the no-rights case. In particular, we showed that the value of first-purchase rights increases with the degree of the seller’s ability to exploit the right-holder.
References


