Reputational Economies of Scale

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Abstract

For many years, most scholars have assumed that the strength of reputational incentives is positively correlated with the frequency of repeat play. Firms that sell more products or services were thought more likely to be trustworthy than those that sell less because they have more to lose if consumers decide they have behaved badly. That assumption has been called into question by recent work that shows that, under the standard infinitely repeated game model of reputation, reputational economies of scale will occur only under special conditions, such as monopoly, because larger firms not only have more to lose from behaving badly, but also more to gain. This article argues that reputational economies of scale exist even when there is competition and without other special conditions, if the probability of detection is positively correlated with the frequency of repeat play. It also shows that reputational economies of scale exist in a finite horizon model of reputation. Reputational economies of scale help explain why law and accounting firms can act as gatekeepers, why mass market products are more likely to be safe, why firms are less likely to exploit one-sided contracts than consumers, and why manufacturers market new products under the umbrella of established trademark.

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1. Introduction

For many years, most scholars have assumed that the strength of reputational incentives is positively correlated with the frequency of repeat play. The conventional wisdom was that firms that sell more products or services are more likely to be trustworthy than those that sell less, because they have more to lose if consumers decide they have behaved badly. This assumption helps explain why law and accounting firms can act as gatekeepers, why mass market products are more likely to be safe, why firms are less likely to exploit one-sided contracts than consumers, and why manufacturers market new products under the umbrella of

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established trademarks (Kraakman 1984 p. 891, Coffee 2006 p. 136, Choi 1998, Barnett 2012, Polinsky & Shavell 2010 p. 1491, Bebchuk & Posner 2006 p. 832, but see Macey 2013 Chapters 6 and 7).

Nevertheless, recent articles by Eric Rasmusen (2016) and Edward Iacobucci (2012) call into question the assumption of reputational economies of scale. They assert that, under the infinitely repeated game model of reputational enforcement, there is no advantage to firms that sell products with greater frequency. While firms that sell more have more to lose if they misbehave, they also have more to gain from misbehaving, and these two effects offset each other precisely. Instead, Rasmusen and Iacobucci assert that there are reputational economies of scale only under special circumstances, such as monopoly. Rasmusen and Iacobucci's work is consistent with prior work that shows reputational economies, because those articles assume an infinite horizon model in which sellers are able to price monopolistically (Rob & Fishman 2005, Cai & Obara 2009, Choi 1998, Wernerfelt, 1988).[1]

This article argues that reputational economies of scale exist under the infinitely repeated game model of reputation, even when there is competition and without other special conditions. The infinitely repeated game reputation model requires only minor adjustment in order to generate reputational economics of scale. The only modification necessary is to assume that low quality is detected with probability less than one and that the probability of detection is positively correlated with the frequency of repeat play. This assumption is valid for nearly all situations to which reputational enforcement is usually applied. For example, if a manufacturer skimps on the safety of its products, the probability that any one product will cause an accident is likely to be less than one. Nevertheless, if the manufacturer sells many products, it is quite likely that there will be accidents. As it sells more low quality products, there will be more accidents and more bad publicity, and consumers are more likely to choose to buy other products. Similarly, if an accounting firm is not rigorous in an audit of a single company in a single year, the probability that its lack of rigor will become known is less than one. Nevertheless, if an accounting firm is consistently sloppy in its audits of many companies, the low quality of its audits will eventually damage its reputation.

Rasumusen and Iacobbucci's argument against reputational economies of scale applies with equal force to finite horizon models of reputation, such as that developed by Kreps and Wilson (1982) and Milgrom and Roberts (1982). In fact, even with monopoly, there are no reputational economies of scale in simple finite horizon models. Nevertheless, the same modification – that the probability of detection is positively correlated with the frequency of

[1] Rogerson (1983) shows that higher quality firms tend to have more customers in a model with competition, although he assumes that firms "make a once-and-for-all quality choice upon entering the market" and therefore do not face that moral hazard problem (temptation to shirk on quality) that characterizes most of the literature. Helpful surveys on reputation include Bar-Isaac & Tadelis 2008, MacLeod 2007, Mailath & Samuelson 2006.

repeat play – is also sufficient to generate reputational economies of scale in a finite horizon model of reputation.

Section 2 briefly sets out the point made by Rasmusen and Iaccobucci that, under the basic infinitely repeated game model of reputation, there are no reputational economies of scale. Section 3 modifies the basic model by assuming that the probability of detection is less than one. It shows that the minimum quality-assuring price decreases with the volume of sales. Section 4 extends the analysis to the situation where firms are of different sizes. Because the quality-assuring price is lower for larger firms, firms which sell more will force smaller competitors out of the market. Section 5 analyzes the umbrella branding context and shows that a trademark owner can leverage its trademark from one competitive market to another and that doing so may make the market for high quality viable. Section 6 analyzes reputational economies of scale under a finite horizon model with two types. Section 7 discusses caveats and extensions, and Section 8 concludes.

2. Lack of Economies of Scale in the Basic Model

This section generally follows Eric Rasmusen's 1989 and 2007 formalization of Klein and Leffler's 1981 article. There are an infinite number of potential firms and consumers. In each period, there are n firms that participate in the market by selling at least one unit of the relevant good. Each participating firm incurs a fixed cost, $F > 0$, which is paid at the beginning of the first period in which it participates. A firm that chooses not to participate gets payoff of zero. Each period, each firm can choose to make products of high or low quality. Quality cannot be observed by consumers at the time of purchase. It costs a firm $c > 0$, paid at the end of the relevant period, to produce a unit of the high quality good; it costs zero to produce the low quality good. Each period, each firm chooses its price p. Each period, consumers decide how much of the product to buy from each firm. The amount consumers buy in each period from firm i is denoted q_i, and firms receive the payment at the end of the period. After purchasing the good, consumers learn the quality of the goods they purchased and can use that knowledge to determine which firm to purchase from in the next period. Knowledge is shared among all consumers. The payoff to a consumer of buying a low quality product is zero. The payoff to buying a high quality product varies among consumers, but for every price, there are a sufficient number of consumers willing to pay it to support several competing firms, although, of course, total demand, $q(p) = \sum_{i=1}^{n} q_i > 0$, falls with price, $\frac{dq}{dp} < 0$, where total demand is a differentiable and thus continuous function of price. The discount rate is $r > 0$. The game repeats infinitely. Since this is an infinitely repeated game, there are many equilibria. The equilibria of interest, however, are those that sustain the production of high quality goods. The exposition below is confined to ascertaining the conditions for such equilibria.

Consider a firm that is deciding between two alternatives: (1) producing high quality products in every period and receiving a high price, and (2) producing low quality in every period, getting the high price in the first period and then zero profits in every other period. p^* is the price at which a firm would be indifferent between these two alternatives:

$$\frac{q_i(p^*-c)}{r} = \frac{q_i p^*}{1+r} \tag{1}$$

A little algebra shows that:

$$p^* = (1+r)c \tag{2}$$

This p^* is the quality-assuring price. It is the minimum price that gives a firm an incentive to produce high quality goods. Note that the quality-assuring price does not vary with the quantity produced by each firm, q_i.

It is a perfect equilibrium (a) for consumers to purchase randomly in the first period from firms selling at p^*, but, after that, to purchase randomly only from firms that sell at p^* and that have never sold a low quality product, and (b) for firms to produce high quality and sell at p^*. To make this price consistent with free entry, the profit for each firm must be zero. That is,

$$F = \frac{q_i(p^*-c)}{r} \tag{3}$$

Substituting (2), $p^* = (1+r)c$, we can derive the equilibrium quantity produced by each firm, q_i^*

$$q_i^* = \frac{F}{c} \tag{4}$$

The equilibrium number of firms, n^*, entering the market in each period is derived by setting supply equal to demand:

$$n^* q_i^* = q(p^*) \tag{5}$$

Combining (3), (4) and (5):

$$n^* = \frac{cq(p^*)}{F} = \frac{cq((1+r)c)}{F} \tag{6}$$

where $q((1+r)c)$ denotes q, total quantity, as a function of $(1+r)c$, not q times $(1+r)c$. The most important point is that the equilibrium price in equation (2) does not vary with quantity. Suppose, for example, that a firm were somehow able to double its quantity from q_i to $2q_i$. The quality-assuring price would still be given by the price that makes the firm indifferent between high quality and low quality. Modifying equality (1) to take into account the doubling of the quantity:

$$\frac{2q_i(p^*-c)}{r} = \frac{2q_i p^*}{1+r} \tag{7}$$

Doubling the quantity both doubles the benefit of faithfully producing high quality -- the left-hand side of equation (7) – and doubles the benefit of producing low quality -- the right-hand side of equation (7) . As a result, increasing quantity has no effect -- the 2s cancel each other out. So equation (7) easily reduces to equation (1), and the quality-assuring price in (2) remains the same regardless of the quantity produced by each firm. This is the essence of the argument made by Iacobucci (2012 p. 310). In slightly more complicated form, it is the argument made by Rasmusen (2016 pp. 267-78).

3. Reputational Economies of Scale When Low Quality Is Detected With Probability Less Than One

A key assumption in the prior section was that consumers detect and punish low quality with probability one at the end of each period. That is obviously unrealistic. It is more reasonable to assume that consumers detect and punish low quality with probability less than one, but that the probability increases as the firm sells more low quality goods. That is, let $s(k_i)$ be the probability that consumers detect and punish low quality if k_i low quality units are produced, where $0 \leq k_i \leq q_i$, $s(0) = 0$, $\lim_{k_i \to \infty} s(k_i) = 1$, $s' > 0$, and $s'' < 0$. The assumption that the second derivative is negative follows largely (but not entirely) from the idea that $s(k_i)$ is increasing but must take values between zero and one, whereas k_i can be any positive real number. That means that the second derivative would need to be negative over most of its range (although it could be zero or positive over some intervals). It simplifies the math to assume that the second derivative is always negative. The probability of detection and punishment might increase because media are more likely to publicize defects in products that are widely distributed.[2] Another possible mechanism would be to assume that the probability with which the low quality of any particular unit purchased is detected is ρ , $0 < \rho < 0$ and independent. One could then interpret s to be the probability that low quality is detected in at least one unit produced by a firm in a given period, under the assumption that consumers will punish the firm if it produces any low quality. Under this interpretation, if k_i low quality units are sold, $s = 1 - (1-\rho)^{k_i}$. Another interpretation might be that $s(k_i)$ is the fraction of consumers who refuse to buy from the firm that has sold low quality. That probability might go up with the number of low quality units produced, because knowledge of low quality is more likely to diffuse, either through word of mouth or through media, when the number of defective products is larger.

Assume provisionally that the firm produces either all high quality or all low quality, that is $k_i \in \{0, q_i\}$. Appendix B shows that this assumption is justified, because it would not be rational

[2] Large firms may, however, may be better able to manipulate the media or mitigate the effects of negative publicity through their own advertising and public relations. The author thanks Joshua Teitelbaum for this insight.

for a firm to produce some high and some low quality.[3] Because the probability of detection, s, increases with quantity of low quality goods, the equation for the quality-assuring price – see (1) above -- needs to be modified to take into account that cheating is discovered with probability less than one. Let p'^{*} denote the quality-assuring price under the assumptions in this section (e.g. that the probability of detection is less than one). The payoff of consistently producing high quality remains the same, but the payoff to producing low quality is more complicated, because a firm producing low quality goods may now get the high payoff for several periods until the low quality of its products is detected. As before, the quality-assuring price, p'^{*}, is determined by setting the payoff for producing high quality in every period equal to the payoff for producing low quality in every period, getting the high price in the first period or periods and then zero profits in every other period:

$$\frac{q_i(p'^{*}-c)}{r} = \frac{q_i p'^{*}}{1+r} + (1-S)\frac{q_i p'^{*}}{(1+r)^2} + (1-s)^2\frac{q_i p'^{*}}{(1+r)^3} + \cdots \tag{7}$$

$$= \sum_{j=1}^{\infty}(1-s)^{j-1}\frac{q_i p'^{*}}{(1+r)^j}$$

$$= \frac{q_i p'^{*}}{(1+r)}\sum_{j=0}^{\infty}\left(\frac{1-s}{1+r}\right)^j$$

$$= \frac{q_i p'^{*}}{(1+r)}\left(\frac{1}{1-\left(\frac{1-s}{1+r}\right)}\right)$$

$$= \frac{q_i p'^{*}}{r+s}$$

Solving for the quality-assuring price, p'^{*}:

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[3] Although the literature often assumes that firms are constrained to offering either all high-quality or all low-quality, it is useful to check that the quality assuring price does not give firms an incentive to produce some high quality and some low quality product. After all, this and most other models in the literature assume fixed marginal cost, so there is no loss in economies of scale if a firm chose to produce some high quality and some low quality. Appendix B proves that, under the quality assuring price in Equation (8), a firm rationally produces all high quality rather than a mixture of qualities. This result could also by justified if one assumed that producing the fixed cost, F, was incurred for each quality, e.g. if each quality required a different factory and/or separate management structure.

$$p'^* = \frac{(r+s)c}{s} \quad (8)$$

As one would expect, as the probability of detection, s, goes to one, p'^* in (8) converges to p^* in (2). Note, however, that since s is a function of quantity, the quality-assuring price now varies with the quantity produced by each firm, q_i. In particular, as q_i approaches infinity, s approaches one, and price approaches $(1+r)c$. On the other hand, as q_i approaches zero, s approaches 0, and p'^* approaches infinity. It is relatively easy to see that p'^* is a strictly monotonically decreasing function of q_i. This makes intuitive sense. When quantity is low, the probability of detection is low, so the firm needs a high price to make it worthwhile not to cheat. This is closely related to the standard result in the economic analysis of crime, where optimal sanctions increase as the probability of detection goes down. In the reputation model, the "sanction" for cheating is loss of the rents produced by receiving the high price in every period. On the other hand, as the probability of detection increases, the firm finds it profitable to produce high quality even if the price is lower.

This is the key result of the paper. Since the quality-assuring price for a firm goes down as the firm produces higher quantities, the larger firm has an advantage. It can sell at a lower price and still have an incentive to produce high quality. This is the meaning of reputational economies of scale.

As with the basic model, there are many equilibria, but the equilibrium of interest is the one which sustains the production of high quality goods. It is a perfect equilibrium (a) for consumers to purchase randomly in the first period from firms that sell at p'^*, but, after that, to purchase randomly only from firms that sell at p'^* and that have never been detecting as selling a low quality product, and (b) for firms to produce high quality and sell at p'^*. To make this price consistent with free entry, the profit for each firm must be zero. That is, the present discounted value of rents must equal the fixed cost of entry.

$$F = \frac{q_i(p'^* - c)}{r} \quad (9)$$

Substituting $p'^* = \frac{(r+s)c}{s}$, we can derive the equilibrium quantity, $q_i'^*$

$$\frac{q_i'^*}{s} = \frac{F}{c} \quad (10)$$

Again, it is instructive to compare equation (10) with equation (4), the equivalent expression when the probability of detection equals one. As one would expect, as the probability that that low quality is detected, s, approaches one, equation (10) converges to equation (4). When $q_i'^* = 1$, the left-hand side of equation (10) is $\frac{1}{s(1)}$. As $q_i'^*$ increases from one, the left-hand side approaches infinity. This means that, as long as $\frac{F}{c} \geq \frac{1}{s(1)}$, there exists a quantity, $q_i'^*$, such that equation (10) is satisfied, because $s(q)$ is differentiable and thus continuous.[4] The condition, $\frac{F}{c} \geq \frac{1}{s(1)}$, means that there may not be an equilibrium quantity with zero profits in situations where fixed costs are very low in relation to the variable costs of producing high quality and/or the probability of detection is very low. In those situations, the quality assuring price needs to be so high that, even if a firm produced only a single unit, it would earn positive profits.

Given that $q_i'^*$ exists, it is trivial to find the equilibrium number of firms, n'^*, by finding the value that satisfies:

$$n'^* q_i'^* = q(p) \tag{11}$$

$$n'^* = \frac{q(p'^*)}{q_i'^*} \tag{12}$$

It should be noted that the equilibrium described in this section is informationally demanding. Both consumers and producers need to know the quality-assuring price, and this requires that both consumers and producers know the cost of producing high quality goods, producers' fixed costs, and the probability that low quality will be detected. The need for the first piece of information is a characteristic of the basic infinitely repeated game model of reputation. The need for information on fixed costs and the probability that low quality is detected is, however, an additional requirement of the modified model presented in this section. Rasmusen and Perri (2001) suggest, using a more complicated model, that it might be possible to relax some of these informational requirements. Further research could explore the extent to which the results in this section are robust to parties estimating the parameters with error. In the real world, consumers do seem to have some sense that certain low prices are "too good to be true." This suggests that the model's prediction that consumers would refuse to buy from producers who charged too low a price has some plausibility.

4. Different Sized Firms

In the model in the prior section, firm size (the quantity sold by each firm) was endogenous, and, in equilibrium, all firms were of the same size. Suppose, however, some firms are able to sell more than others. For example, perhaps, in addition to individual consumers there are a few bulk corporate or governmental buyers who buy on long-term contracts.[5] Even if the

[4] Depending on the exact form of the function $s(k)$, here may be more than one quantity, $q_i'^*$, that satisfies equation (10). If so, let $q_i'^*$ be the highest such value. It is also possible that equation (10) will be satisfied for some values $\frac{F}{c} < \frac{1}{s(1)}$.

[5] To preserve the incentive to produce high quality goods, these contracts would need to give the bulk buyers the right to cancel the contract without penalty if any buyer detected low quality.

bulk buyers chose from whom to purchase randomly, those firms would acquire a price advantage, because, as noted in the prior section, the quality-assuring price falls with quantity. The larger firms with the bulk contracts would underprice the other firms in the individual market and force the smaller firms out of the market. Similar size differences could emerge if individual consumers were influenced to prefer the products of some firms over others, perhaps through advertising, favorable press, or other factors.

5. Umbrella Branding

Now consider what would happen if some firms sell multiple goods while other sell only one good. For simplicity, suppose there are two goods, A and B, and three types of firms, firms that produce only A, firms that produce only B, and firms that produce both A and B. This section will show that firms that produce both A and B will be able to sell both A and B at lower prices than firms that produce only A or only B. As a result, firms which produce only A or B will not survive in equilibrium, and only those that produce both A and B will survive. This establishes the core idea of reputational economies of scale – larger firms which sell more products have a competitive advantage over smaller ones. The intuition for this result is the same as for the result in the previous section. The firm that sells both goods sells more total goods, so, because the quality assuring price falls with quantity, the firm selling both goods will be able to sell at a lower cost and drive single-good firms out of the market. Nevertheless, the math demonstrating this result gets rather complicated, because one must take into account the price, quantity, and number of firms in two different markets.

The analysis in this section depends crucially on how consumers react to the detection of low quality in one product produced by a firm that produces multiple products. If a consumer purchases product A and it turns out to be of low quality, the consumer might cease purchasing product A from that firm, but still purchase product B from it. Or, such a consumer might shun all products from that firm, that is, avoid purchasing both A and B from it. If consumers behave in the former way – treating low quality in one product as irrelevant to the quality of other products produced by the same firm – then there is no advantage to the two-product firm, and the equilibrium with different sized firms is the same as if firms were all the same size. On the other hand, if, as seem plausible, consumers interpret low quality in one product to mean that the firm is cutting corners on both products, then it makes sense for them to refuse to purchase A or B from that firm. If so, a new and interesting equilibrium arises. That is the equilibrium that will be analyzed in the rest of this section. This equilibrium is plausible in situations where a producer has chosen to market two or more products under the same trademark, -- for example several different car models under the trademark "Toyota" or several types of toothpaste under the trademark "Colgate." In these situations, it is plausible that consumers assume that quality standards are similar for all products marketed under the same "umbrella trademark."

Notation needs to be modified to reflect that there are now two different goods. Denote the quantity of good A produced by firm i as q_{A_i}, the quantity of good B produced by such a firm is q_{B_i}, and denote the cost of producing high quality of each good as $c_A > 0$, and $c_B > 0$. Let $s_A(k_{A_i})$ be the probability that low quality is detected and punished if the firm sells k_{A_i} low quality units of product A, but either no product B or only high quality of product B. Let $s_B(k_{B_i})$ be the probability that low quality is detected and punished, if the firm sells k_{B_i} low quality units of product B, but either no product A or only high quality of product A. Let $s_{AB}(k_{A_i}, k_{B_i})$ be the probability that low quality is detected and punished, if the firm sells k_{A_i} low quality units of A and k_{B_i} low quality units of B. $0 \leq k_{A_i} \leq q_{A_i}$ and $0 \leq k_{B_i} \leq q_{B_i}$. As with the one good case, assume $s_u(0) = 0$, $\lim_{k_{u_i} \to \infty} s_u(k_{u_i}) = 1$, $s_u' > 0$, and $s_u{}'' < 0$, where $u \in \{A, B\}$. Similarly, $s_{AB}(0,0) = 0$, $s_{AB}(0, k_{B_i}) = s_B(k_{B_i})$, $s_{AB}(k_{A_i}, 0) = s_A(k_{A_i})$, $\lim_{k_{A_i} + k_{B_i} \to \infty} s_{AB}(k_{A_i}, k_{B_i}) = 1$, $\frac{ds_{AB}}{dk_{u_i}} > 0$ and $\frac{d^2 s_{AB}}{d^2 k_{u_i}} > 0$. Note that these assumptions imply that if the firm produces positive quantities of both low quality A and B, $s_{AB} > s_A$ and $s_{AB} > s_B$. This makes sense, because if two firms produce the same number of low quality goods of kind A, and one of them also produces low quality goods of kind B, it is more likely that the firm that produces two types of low quality goods will be caught. After all, the probability that the two-good firm is caught making low quality goods of kind A is the same as the probability that the one good firm is caught making low quality goods of kind A, but the two-good firm also has some probability of being caught making low quality goods of kind B. Under the interpretation that the probability with which the low quality of any particular unit purchased of good A or B is detected as $\rho_A > 0$ and $\rho_B > 0$, where these two probabilities are independent, then $s_A = 1 - (1 - \rho_A)^{k_{A_i}}$, $s_B = 1 - (1 - \rho_B)^{k_{B_i}}$, and $s_{AB} = s_A + s_B - s_A s_B = 1 - (1 - \rho_A)^{k_{A_i}}(1 - \rho_B)^{k_{B_i}}$. Note that, under this interpretation, the assumptions made above, that $s_{AB} > s_A$ and $s_{AB} > s_B$, are always true when the firm produces both low goods of both goods (e.g. $k_{A_i} > 0$ and $k_{B_i} > 0$) Let $F_A > 0$, and $F_B > 0$ be the fixed costs of producing A and B respectively. As in the one-good case, these costs are paid at the beginning of the first period in which the firm produces the relevant good. Note that F_A does not depend on whether the firm produces one or two goods, nor does F_B. That is, there are no economies of scope attributable to the fixed costs. This assumption, like the assumption of constant marginal costs, allows the analysis to focus economies scale created by reputation rather than by technological factors. As in the one good case, the payoff to buying high quality of product A or B varies among consumers, but for every price and each good, there are a sufficient number of consumers willing to pay it to support several competing firms, although, of course, total demand, $q_A(p) = \sum_{i=1}^{n} q_{A_i} > 0$ and $q_B(p) = \sum_{i=1}^{n} q_{Bi} > 0$, falls with price, $\frac{dq_A}{dp} < 0$ and $\frac{dq_B}{dp} < 0$, where total demand for A and B is a differentiable and thus continuous function of the price of each good.

As in the previous section, assume provisionally that the firm does not mix high and low quality for a given product. That is, if the firm produces A, all units of A are high quality or all or low quality. Similarly, if the firm produces B, all units of B are high quality or all are low quality. Appendix B shows that this assumption is justified, because it would not be rational for a firm to produce some high and some low quality of each product.

The goal of this section is to find quality assuring prices for the two good firm which are lower than those for the single good firm and consistent with competition. For this to be true, the following six conditions (C1-C6) must be satisfied.

C1. High quality on both A and B at least as profitable as low quality on both A and B:

$$\frac{q_{A_i}(p_A'^{*}-c_A)+q_{B_i}(p_B'^{*}-c_B)}{r} \geq \frac{q_{A_i}p_A'^{*}+q_{B_i}p_B'^{*}}{(r+s_{AB})}$$

C2. High quality on both A and B at least as profitable as high quality on B and low quality on A:

$$\frac{q_{A_i}(p_A'^{*}-c_A)+q_{B_i}(p_B'^{*}-c_B)}{r} \geq \frac{q_{A_i}p_A'^{*}+q_{B_i}(p_B'^{*}-c_B)}{(r+s_A)}$$

C3. High quality on both A and B at least as profitable as high quality on A and low quality on B:

$$\frac{q_{A_i}(p_A'^{*}-c_A)+q_{B_i}(p_B'^{*}-c_B)}{r} \geq \frac{q_{A_i}(p_A'^{*}-c_A)+q_{B_i}p_B'^{*}}{(r+s_B)}$$

C4. Prices are lower for a firm producing both A and B than for a firm selling only A or only B (assuming equal quantities):

$$p_A'^{*} < \frac{(r+s_A)c_A}{s_A} \quad \text{and} \quad p_B'^{*} < \frac{(r+s_B)c_B}{s_B}$$

C5. Competition drives down prices to the lowest values consistent with C1-C3:

Either C1 or C2 or C3 holds with equality

C6. New firms enter the market until profits drop to zero:

$$F_A + F_B = \frac{q_{A_i}(p_A'^{*}-c_A)+q_{B_i}(p_B'^{*}-c_B)}{r}$$

The existence of an ordered pair, $(p_A'^{*}, p_B'^{*})$, that satisfies these constraints is proved in Appendix A.

Note that if the quantity of A produced by the A-only firm and by the firm producing both A and B are equal, then the quality-assuring price will be lower for the firm producing both A and B than for the firm producing only A. This price advantage directly follows from the discussion in Section III of expression (8) where it was shown that the quality-assuring price for a firm goes down as the firm produces higher quantities. Here, the firm produces both A and B, and thus produces at higher overall quantities than the firm that produces only A. Similarly, the quality-assuring price for the two product firm will be lower than for a firm producing only good B. From the two-product firm's price advantage, it is easy to see that the two-product firm will dominate the market for both A and B. Consider the following set of strategies that form a perfect equilibrium.

> Let $p_A'^*$ and $p_B'^*$ be the pair of prices identified in Appendix A as satisfying constraints C1-C6.
>
> (1) Consumers purchase good A randomly in the first period from firms that sell both A and B and that sell good A at $p_A'^*$. After the first period, consumers purchase good A randomly from firms that sell both A and B and that sell good A at $p_A'^*$ and that have never been detected as having sold a low quality product.
>
> (2) Consumers purchase good B randomly in the first period from firms that sell both A and B and that sell good B at $p_B'^*$. After the first period, consumers purchase good B randomly from firms that sell both A and B and that sell good B at $p_B'^*$ and which have never been detected as having sold a low quality product.
>
> (3) Firms that produce both A and B produce both at high quality and sell them at prices $p_A'^*$ and at $p_B'^*$ respectively.
>
> (4) Firms that would produce only A or only B never enter the market.

Note that it is rational for consumers to purchase only from firms which sell both A and B, because a firm that produced only A or only B would not rationally produce high quality goods at $p_A'^*$ or $p_B'^*$. For example, if a firm that produced only good A sold its goods at $p_A'^*$, it would have no incentive to produce high quality, because, as pointed out above, $p_A'^*$ is lower than the quality-assuring price for the A-only firm. So it would produce low quality. So no rational consumer would purchase from it at that price. Since consumers won't purchase from a firm that produced only good A or good B, such firms rationally won't pay the fixed cost to enter the market.

Note, of course, that there are many equilibria, including equilibria in which both two-product and single-product produce goods of similar quality. The most obvious (and uninteresting) is the equilibrium in which consumers purchase only products sold at the low quality price (here normalized to zero) and in which all producers produce only low quality. A more interesting equilibrium is one in which consumers favor single-product firms by buying from them with higher probability, even if the single product firm offers the good at the same price as the two-product firm. By doing so, consumers could negate the scale advantage of the two-product firm by purchasing, in aggregate, more of the single good from the single-product firm. For example, suppose goods A and B are similar in that the cost of producing high quality is equal, $c_A = c_B$, the probability that low quality is detected depends solely on the total number of low quality goods produced by that firm, k, $s_A(k) = s_B(k) = s_{AB}\left({k}/{2}, {k}/{2}\right)_B$ and demand for the two products is the same. Under those assumptions, there would be an equilibrium in which each consumer purchased randomly from all firms, but in which the probability with which the consumer purchased from each single-good firm was twice the probability with which the consumer purchased from each two-good firm. If consumers behaved in this fashion, each two-product firm would sell half as much of each good as each single-product firm. So the total sales (A and B combined) of the two-product firms would equal the sales of the single-product firm. So the quality-assuring price for both products would be the same for two-product and one-product firms. While interesting, this equilibrium assumes implausible consumer behavior.

So far this paper has assumed that consumer demand is structured so that, no matter the price, there is always sufficient demand to support multiple competing firms. As Rasmusen (2016) points out, however, it is possible that consumers will prefer low quality to high quality at the quality-assuring price that could be offered by a single-product firm. The existence of multi-product firms can, in some circumstances, solve this problem by lowering the quality-assuring price. In this way, umbrella branding (the use of a single trademark for several products produced by the same firm) makes high quality viable in situations where, if the same product were produced by a single-product firm, high quality would require a quality-assuring price that consumers were unwilling to pay. Note, contrary to Rasmusen, umbrella branding makes high quality viable even in the presence of perfect competition.

While this section focused on umbrella branding, the law firm size issue explored by Iacobucci (2012) is analytically identical. Instead of a manufacturer making two goods, he considers a law firm producing two kinds of legal services, where the two services are distinguished by the fact that each is produced by a different lawyer. Nevertheless, the model would be the same. For each lawyer, there is a probability that low quality will be detected. If consumers punish both lawyers in a firm if they detect low quality by one lawyer, then low quality is likely to be punished more swiftly in a multi-member firm. This enables the multi-member firm to credibly offer high quality at a lower quality-assuring price than either lawyer could offer if she practiced alone.

6. Reputational Economics of Scale in a Finite Horizon Model

This section shows that reputational economies of scale exist also in a finite horizon model similar to that pioneered by Kreps and Wilson (1982) and Milgrom and Roberts (1982). In this model, there are two types of firms, good and bad. Firms know their types, but consumers do not know firm types. Good firms always produce high quality goods, even if doing so is not profitable, while bad firms are opportunistic and produce high quality only if it maximizes the present discounted value of their profits. The literature sometimes calls good firms "commitment type" firms, while bad firms are "strategic type" firms. With probability $\emptyset$, a firm is good, and with probability $1-\emptyset$ a firm is bad. $0<\emptyset<1$. Assume that high quality goods cost c to produce, $0<c<1$, while low quality goods cost zero to produce. Consumer are willing to pay 1 for high quality goods and zero for low quality goods. If consumers are uncertain about the quality of goods, the price they are willing to pay is proportional to the probability that they think quality will be high. So, if consumers think only good firms will produce high quality, and they cannot distinguish between good and bad firms, they are willing to pay $\emptyset$. As in the main model, let $s(k)$ be the probability that consumers detect and punish low quality, where k is the number of low quality units sold, $s(k)=0$, $\lim_{q\to\infty} s(q)=1$, $s'>0$, and $s''<0$. Note that these assumptions imply that if a firm produces any low quality goods, then $0<s<1$. There is only one firm. For simplicity, it is assumed that the firm produces either all high quality or all low quality, so $k=q$.[6] The firm sets the price, and consumers decide whether or not to buy. The discount factor is δ, $0<\delta<1$. Consumers know all parameters -- $\emptyset$, c, s, and δ --, but consumers do not know what type a firm is. For convenience, if the payoffs to high and low quality are the same, it is assumed that the bad firm produces high quality.

Consider first a one-period game. Obviously, the bad firm will produce low quality and the good firm will produce high quality. As a result, the price will be $\emptyset$.

Now consider the strategy of a bad firm in a two-period game. It will produce high quality if doing so maximizes the present discounted value of its profits. Assuming that consumers will believe it produces high quality in the first period (an assumption justified below), the bad firm will produce high quality if:

$$1-c+\delta\emptyset \geq 1+\delta(1-s)\emptyset \quad (13)$$

The left side represents the bad firm's profits if it produces high quality in the first period and it is believed to produce high quality. It gets 1 in the first period (the price consumers are willing to pay for high quality), and it pays c to produce it. Because it produced high quality in the first period, consumers are willing to purchase from it in the second period. Nevertheless, because consumers know that the bad firm will produce low quality in the second (last) period, and since they do not know whether the firm is low quality, the price in the last period is $\emptyset$, as in the one-

[6] No proof of the irrationality of mixing high and low quality is provided for this model, but there is no reason to think the proof in Appendix B would not, with appropriate modifications, apply to this model as well. As with the infinitely repeated model, the key is that the second derivative of $s(k)$ is negative. This means that the marginal cost of low quality (detection) is decreasing in quantity. Since the marginal benefit of low quality (cost savings) is constant with respect to quantity, if it is profitable to produce one unit of low quality, it is profitable to produce all low quality.

period game. Since it costs the bad firm nothing to produce low quality, its discounted profits in the second period are $\delta\emptyset$. If the bad firm produces low quality in the first period, it gets profits of 1 (the price of high quality minus the costs of production, which are zero). Low quality is detected with probability s, and consumers rationally do not purchase from bad firms in the last period, so the present-discounted second-period profits are $\delta(1-s)\emptyset$. Rearranging the terms of expression (13), the bad firm produces high quality in the first period if $\emptyset \geq \frac{c}{\delta s}$. As in Kreps and Wilson's model, under some parameters, it is equilibrium behavior for bad firms to mimic good firms in all periods other than the last period, and it is rational for consumers to believe that bad firms, under some parameters, produce high quality, except in the last period.

If this inequality is satisfied, then it is a perfect Bayesian equilibrium for a bad firm to produce high quality in the first period (and to sell goods for 1 in the first period) and for a bad firm to sell low quality in the last period (and for the bad firm (and good firm) to sell its goods for $\emptyset$ in the second period). In this equilibrium, consumers purchase goods for a price of 1 in the first period, and purchase goods for $\emptyset$ in the second period, unless a firm has been detected as having sold low quality in the first period, in which case consumers refuse to buy from the firm in the second period (or buy only at price zero). If consumers in the first period do not buy at the prices stated above, the firm does not sell in the second period. If firms sell at prices other than those stated above, consumers do not buy anything. Appropriate beliefs can be constructed to make these off-equilibrium path behaviors rational.

Note the effect of s. As s goes up, the inequality in (13) is more likely to be satisfied, because s appears only on the right side, and the right side decreases as s increases. That is, the inequality is satisfied for lower values of $\emptyset$ and δ and for higher values of c. Since s is an increasing function of quantity, q, this means that consumers are more likely to trust large firms. Thus, as under the infinitely-repeated game model analyzed in prior sections, there are reputational economies of scale. Since, in this model, the sellers are assumed to have pricing power, the reputational advantage is not in offering lower prices, but rather that a bad firm is more likely to be trusted to produce high quality (and more likely to do so) over a wider range of parameters. The model could be easily modified to show a pricing advantage of the larger firm, because c can be thought of not as absolute cost, but rather as the ratio of cost to price. Thus, as s goes up, it is rational for a bad firm to produce high quality even at a lower price. So a larger firm will be able to underprice a smaller firm. That is, suppose consumers were willing to pay price, p ¸$c < p < 1$, for high quality goods, then there are prices such that it would be an equilibrium for a larger bad firms to produce high quality for price p, but not for a smaller bad firm.

Note also that quantity manifests itself in (13) only through s, the probability that bad quality in at least one product will be detected. If, as in the standard model, bad quality were detected with certainty, $s = 1$, and quantity would be irrelevant. There would be no reputational economies of scale. A firm producing one good per period would behave in the same way as a firm producing a million goods per period, and a small firm would be just as trustworthy as a large one. As with the infinite horizon model of reputation, the key to reputational economies of scale is the idea that poor quality is more likely to be detected and punished when quantity is high.

Now consider a three-period game. There are two cases to consider, where $\emptyset \geq \frac{c}{\delta s}$, so the bad firm can be assumed to produce high quality in the second period if low quality is not detected in the first period, and where $\emptyset < \frac{c}{\delta s}$, so the bad firm can be assumed to produce low quality in the second period. Consider first the situation where $\emptyset \geq \frac{c}{\delta s}$. An equilibrium in which the bad firm produces high quality in the first period is plausible if:

$$1 - c + \delta(1 - c) + \delta^2\emptyset \geq 1 + \delta(1 - s) + \delta^2(1 - s)^2\emptyset \quad (14)$$

The left side is the payoff if the bad firm produces high quality in the first and second periods and low quality in the third. The right hand side is the payoff if the bad firm produces low quality in all periods. It is assumed (and justified below) that consumers expect the firm to produce high quality in the first and second period and low quality in the third. Note that the expression above is more likely to be satisfied when s is high, because s appears only in the right-hand side, and higher values of s always makes the right-hand side smaller, thus increasing the range of parameters for which the inequality holds. Thus, as in the two-period game, there are reputational economies of scale, and consumers are more likely to trust large firms. It is, of course, necessary to check that it is rational for the firm to produce high quality in the first period. Solving (14) for $\emptyset$, dividing top and bottom ∂, and rearranging the numerator, expression (14) can be rewritten as:

$$\emptyset \geq \frac{c + \frac{c}{\delta} - s}{\delta(1 - (1 - s)^2)} \quad (15)$$

It is relatively easy to show that this expression is satisfied whenever $\emptyset \geq \frac{c}{\delta s}$, which is the assumption for this case, because the numerator of expression (15) is always smaller than c and the denominator of expression (15) is always larger than δs, so any $\emptyset \geq \frac{c}{\delta s}$ satisfies (15). Compare the numerator of the right-hand side (15) to the numerator of $\frac{c}{\delta s}$. That is, compare $c + \frac{c}{\delta} - s$ to c. The numerator of the former is always smaller, because $\frac{c}{\delta} - s < 0$ whenever $1 > \emptyset \geq \frac{c}{\partial s}$, which is our assumption for this case together with the fact that $\emptyset$ is a proportion and so must be less than 1. Similarly, compare the denominator of the right-hand side of (15) to the denominator of $\frac{c}{\delta s}$. That is, compare $\delta(1 - (1 - s)^2)$ to δs. The former is always larger, because $0 < s < 1$, whenever any low quality is produced. Thus, a bad firm will provide high quality in the first (and second) period of the three period game if it would be rational for it to produce high quality in the first period of the two period game.

Under this equilibrium, it is rational for consumers to believe that the bad firm will produce high quality in the first and second periods (and thus for consumers to pay 1) and that the bad firm will produce low quality in the third period (and thus for the consumers to pay $\emptyset$), as long as the consumer refuses to pay these prices from any firm that has been detected as producing low quality in the past. Appropriate actions and beliefs off the equilibrium path can be easily constructed.

Now consider the three-period game where $\emptyset < \frac{c}{\delta s}$, that is, where the bad firm will produce low quality in the second and third periods. As in Kreps and Wilson's (1982) model, it is possible that the bad firm will produce high quality in the first period, even though it will produce low quality in later periods. Such an equilibrium would be plausible if:

$$1 - c + \delta\emptyset + \delta^2(1-s)\emptyset \geq 1 + \delta(1-s)\emptyset + \delta^2(1-s)^2\emptyset \quad (16)$$

As before, the left side is the payoff if the bad firm produces high quality in the first period, but not in any other period, whereas the right side is the payoff if the bad firm produces low quality in all periods. It is assumed (and justified below) that consumers expect the firm to produce high quality in the first period and low quality in the second and third periods. Solving for $\emptyset$, inequality (16) is satisfied if and only if:

$$\emptyset \geq \frac{c}{\delta s[1 + \delta(1-s)]} \quad (17)$$

Note that the right hand side of this inequality, is always less than $\frac{c}{\delta s}$, because $0 < \delta < 1$ and $0 < s < 1$. This means that it will be rational for some firms to produce high quality in the first period of the three-period game, even though it would not be rational for them to produce high quality in the first period of the two period game.

Reputational economies of scale can be observed be differentiating the right side of (17) by s. Because the numerator is a constant, it is sufficient to show that the derivative of the denominator is positive. The derivative of the denominator is $\delta[1 - \delta(2s - 1)]$, which is always positive because $0 < \delta < 1$ and $0 < s < 1$. This means that there are reputational economies of scale, because as s increases, the threshold $\emptyset$ which makes it rational for firms to product high quality goes down, so larger firms will be trustworthy and produce high quality goods under a wider array of parameters.

As in the case where $\emptyset \geq \frac{c}{\delta s}$, consumers are rational to believe that, when inequality (16) is satisfied, a bad firm will produce high quality in the first period and low quality in later periods. Of course, if inequality (16) is violated, the bad firm will produce low quality in all periods and equilibrium prices will fall accordingly.

Appendix C generalizes these results to games of any number of periods.

7. Caveat and Extension

The models in this paper are, like all models, unrealistic in some respects. A key way in which these models are unrealistic is that they assume that if the firm chooses to produce with high quality, no low quality goods are produced. That is unrealistic, because even the best quality control cannot prevent production of an occasional defective product. Relaxing this assumption will reinforce the reputational advantage of larger firms, because consumers can more easily discern whether defects are endemic or idiosyncratic when the firm produces a large number of goods. For example, if a firm produces ten goods and one is of poor quality, consumers cannot infer with confidence that the firm has bad quality control, because it is possible that the one good of poor quality reflects simply bad luck from a firm that produces high quality goods with probability much higher than 90%. On the other hand, if a firm produces one million goods and one hundred thousand are defective, the consumer can very reliably infer that the firm has poor quality controls that result in a high (10%) rate of defects.[7]

An interesting extension of the analysis in this article is to certification marks. Certification marks are a type of trademark in which one entity uses its reputation to back the idea that products produced by others meet certain quality standards (Holtzman 1991). Examples include such as Underwriter's Laboratory (for electrical safety) and the Union of Orthodox Rabbis (for kosher food). One puzzle is why certification is necessary. Why isn't the reputation of the company producing the goods sufficient to ensure quality? One possibility is that certification is helpful when the probably that low quality will be detected is very low. In that situation, the reputation of a single firm, even a large one, may not be sufficient to bond good behavior. The theory of umbrella branding set out in Section V is helpful here. Just as a firm producing two goods may be in a better position to bond the reputation of its products, so a certifier who certifies hundreds of goods may be in a better position to bond the reputation of all those products. If consumers blame the certifier when one of the certified firms is detected as shirking, then the certifier has an incentive to closely monitor all the firms it certifies, and consumers will rationally trust certified products more than uncertified. In this way, the effectiveness of certification is an application of the idea of reputational economies of scale. When the probability of detection is very low, the quantity produced by a single firm may not be sufficient to ensure no shirking. Spreading the reputational umbrella of the certifier over the goods produced by multiple firms restores reputational incentives. This may explain why certification is especially prevalent for credence attributes, such as safety or kashrut, where even discerning consumers cannot tell whether the producer has shirked simply by consuming or experiencing the good (Dulleck, Kerschbamer and Sutter 2011).[8]

[7] The author thanks Steve Shavell for making the point in this paragraph.

[8] The author thanks Megan Stevenson for encouraging him to include this paragraph, which, in fact, motivated the author's interest in reputational economies of scale.

8. Conclusion

Relaxing the assumption that low quality is detected with certainty at the end of each period helps explain the widely assumed phenomenon of reputational economies of scale. Once this assumption is relaxed, reputational economies of scale emerge under the infinitely repeated game model of reputation, even in competitive markets. Similarly, if the assumption that low quality is detected with certainty, reputational economies of scale also occur in a finite horizon game model of reputation with two types. Reputational economies of scale help explain many market phenomena, including gatekeeper liability, one-sided consumer contracts, umbrella branding, and the large size of firms in industries where product quality is hard to enforce through inspection or contract.

Appendix A. Proof of the Existence of Quality-Assuring Umbrella Prices

Lemma 1. If firms producing both A and B choose prices and quantities that would be quality-assuring for firms producing just A or just B, then constraints C1, C2, and C3 would be satisfied with strict inequalities. That is, if $p_A'^* = \frac{(r+s_A)c_A}{s_A}$ and $p_B'^* = \frac{(r+s_B)c_B}{s_B}$, then C1, C2 and C3 would hold with strict inequalities.

First consider C1. To Prove:

$$\frac{q_{A_i}(p_A'^* - c_A) + q_{B_i}(p_B'^* - c_B)}{r} > \frac{q_{A_i}p_A'^* + q_{B_i}p_B'^*}{(r+s_{AB})}$$

Substitute $p_A'^* = \frac{(r+s_A)c_A}{s_A}$ and $p_B'^* = \frac{(r+s_B)c_B}{s_B}$ and simplify:

$$\frac{q_{A_i}\left(\frac{(r+s_A)c_A}{s_A} - c_A\right) + q_{B_i}\left(\frac{(r+s_B)c_B}{s_B} - c_B\right)}{r} > \frac{q_{A_i}\frac{(r+s_A)c_A}{s_A} + q_{B_i}\frac{(r+s_B)c_B}{s_B}}{(r+s_{AB})}$$

$$\frac{q_{A_i}c_A}{s_A} + \frac{q_{B_i}c_B}{s_B} > \frac{q_{A_i}c_A}{s_A}\left(\frac{r+s_A}{r+s_{AB}}\right) + \frac{q_{B_i}c_B}{s_B}\left(\frac{r+s_B}{r+s_{AB}}\right)$$

$s_{AB} > s_A > 0$ and $s_{AB} > s_B > 0$ (see p. 10), so the above inequality always holds.

Next consider C2: To Prove:

$$\frac{q_{A_i}(p_A'^* - c_A) + q_{B_i}(p_B'^* - c_B)}{r} > \frac{q_{A_i}p_A'^* + q_{B_i}(p_B'^* - c_B)}{(r+s_A)}$$

As with C1, substitute $p_A'^* = \frac{(r+s_A)c_A}{s_A}$ and $p_B'^* = \frac{(r+s_B)c_B}{s_B}$ and simplify:

$$\frac{q_{A_i}c_A}{s_A} + \frac{q_{B_i}c_B}{s_B} > \frac{q_{A_i}c_A}{s_A} + \frac{q_{B_i}c_B}{s_B}\left(\frac{r}{r+s_A}\right)$$

$r > 0$ and $s_A > 0$, so the above inequality always holds.

Similar reasoning shows that C3 also holds with strict inequality. Q.E.D.

Now consider the main proposition to be proved, that there exist prices, $(p_A'^*, p_B'^*)$, satisfying constraints C1 through C6. Consider ordered pairs, (p_A, p_B) and, in particular the set of ordered pairs constituting the line segment between $(c_{A,} c_B)$ and $\left(\frac{(r+s_A)c_A}{s_A}, \frac{(r+s_B)c_B}{s_B}\right)$. The first point is defined by the marginal cost of producing each good, and the second point is defined by the quality-assuring prices when a firm produces only good A or only good B. Note that at $(c_{A,} c_B)$, C1, C2, and C3 will be each violated, because the left-hand sides will be zero and the right-hand sides will be positive. Note also that, according to Lemma 1, at $\left(\frac{(r+s_A)c_A}{s_A}, \frac{(r+s_B)c_B}{s_B}\right)$, C1, C2 and C3 each hold with strict inequalities.

Now consider what happens to C1 as one moves along the line from $\left(\frac{(r+s_A)c_A}{s_A}, \frac{(r+s_B)c_B}{s_B}\right)$ to $(c_{A,} c_B)$ while setting the number of firms, *n*, such that C6 (zero profits) holds. For this purpose, it is helpful to rewrite C1 with total quantities rather than quantities per firm by multiplying the left and right-hand side of the inequality by the number of firms:

$$\frac{q_A(p_A'^* - c_A) + q_B(p_B'^* - c_B)}{r} \geq \frac{q_A p_A'^* + q_B p_B'^*}{(r + s_{AB})} \text{ (A1)}$$

For each set of prices on the line segment between $(c_{A,} c_B)$ and $\left(\frac{(r+s_A)c_A}{s_A}, \frac{(r+s_B)c_B}{s_B}\right)$, we can find a number of firms, n, such that C6 (zero profits) holds because of continuity. The right-hand side of C6 is a continuous function of *n*, because the right-hand side of C6 is just the left hand side of (A1) divided by *n*. We have assumed that total demand for A and B is sufficiently large to support competing firms, so there is some value of $n \geq 2$ for which the right-hand side of C6 at equals or exceeds the left-hand side. We also know that as *n* goes to infinity, the value of the right-hand side of C6 goes to zero. So there must be some value of *n* for which the equality in C6 is true.

Note that since total demand for A and total demand for B, q_A and q_B, are assumed to be continuous functions of the prices, (p_A, p_B), and since s_{AB} is strictly positive and a continuous function of q_A and q_B, which in turn are continuous functions of (p_A, p_B), then both sides of the inequality (A1) must also be continuous functions of prices.

Since the inequality in (A1) holds strictly at $\left(\frac{(r+s_A)c_A}{s_A}, \frac{(r+s_B)c_B}{s_B}\right)$ and is violated at $(c_{A,} c_B)$, and since the both sides vary continuously with p_A and p_B, there must be a place on the line between those two points where the left-hand and right-hand sides are equal. If there is only one such point, call that point (x_1, y_1). If there is more than one such point, denote as (x_1, y_1) the one with the highest value of p_A. Note that, for all points on the line between (x_1, y_1) and $\left(\frac{(r+s_A)c_A}{s_A}, \frac{(r+s_B)c_B}{s_B}\right)$, C1 holds with strict inequality, because if it held with equality that would violate the definition of (x_1, y_1), and if the inequality did not hold at all, then, because of continuity, there would have to be another point in that interval where C1 held with equality, which would also violate the definition of (x_1, y_1).

Now consider what happens to C2 as one moves along the line from $\left(\frac{(r+s_A)c_A}{s_A}, \frac{(r+s_B)c_B}{s_B}\right)$ to $\left(c_{A,}c_B\right)$ while also assuming that C6 (zero profits) holds. Following reasoning similar to that in the prior paragraph, there will be one or more points on the line segment between $\left(c_{A,}c_B\right)$ and $\left(\frac{(r+s_A)c_A}{s_A}, \frac{(r+s_B)c_B}{s_B}\right)$ where C2 holds with equality. If there is only one such point, call that point (x_2, y_2). If there is more than one such point, denote as (x_2, y_2) the one with the highest value of p_A. For every point on the line between (x_2, y_2) and $\left(\frac{(r+s_A)c_A}{s_A}, \frac{(r+s_B)c_B}{s_B}\right)$, C2 holds with strict inequality.

By similar reasoning, let (x_3, y_3) be the sole point where C3 holds with equality on the line segment between $\left(c_{A,}c_B\right)$ and $\left(\frac{(r+s_A)c_A}{s_A}, \frac{(r+s_B)c_B}{s_B}\right)$ or the one with the highest value of p_A. C3 will hold with strict inequality for every point (x_3, y_3) and $\left(\frac{(r+s_A)c_A}{s_A}, \frac{(r+s_B)c_B}{s_B}\right)$.

$$\text{Let } (p_A'^*, p_B'^*) = \begin{cases} (x_1, y_1) \text{ if } x_1 \geq x_2 \text{ and } x_1 \geq x_3 \\ (x_2, y_2) \text{ if } x_2 > x_1 \text{ and } x_2 \geq x_3 \\ (x_3, y_3) \text{ if } x_3 > x_1 \text{ and } x_3 > x_2 \end{cases}$$

By construction, all 6 constraints – C1 through C6 -- are satisfied at these prices, so there exists at least one pair of quality-assuring prices consistent with competition in which the two-good firms have prices lower than the one-good firms. Q.E.D.

Note that there are an infinite number of ordered pairs $(p_A'^*, p_B'^*)$, satisfying constraints C1 through C6. Their existence can be proved by considering the family of curves (e.g. increasingly concave or convex bowed-out lines) passing through $\left(c_{A,}c_B\right)$ and $\left(\frac{(r+s_A)c_A}{s_A}, \frac{(r+s_B)c_B}{s_B}\right)$, but overlapping or intersecting the straight line between those points only at the endpoints. By reasoning similar to the above for straight lines, each of those curves will also have a point where all six constraints are satisfied, and that point will not be on the straight line. For the purposes of this paper, it is sufficient to prove that there is just one such ordered pairs. Consideration of additional pairs satisfying the constraints introduces complications, such as how consumers coordinate on one pair of prices. By just considering the pair whose existence was proved first, such complications can be avoided.

Appendix B. No Mixing of High and Low Quality

The main text (and other articles in the literature) assume that a manufacturer that produces a single good produces either all high quality or all low quality of that good. Similarly, it is generally assumed that a manufacturer who produces two goods, A and B, produces uniform quality of A and uniform quality of B. That is, the text and literature assume that, although a firm may produce high quality A and low quality B, or low quality A and high quality B, the firm does *not* produce some high quality A and some low quality A and/or some high quality B and some low quality B. Given the assumption of fixed costs, *F*, that assumption may be reasonable. Perhaps producing some high and some low quality would require duplication of the fixed costs (or at least additional fixed costs). On the other hand, given the assumption that the marginal cost, *c*, is constant, no matter how many units are produced, it is also reasonable to consider the possibility that the manufacturer would choose to produce some high and some low quality goods.

First consider the umbrella branding case, where there are two goods. To show that mixing is not rational, one must prove the following inequality:

$$\frac{q_{A_i}(p_A'^{*}-c_A)+q_{B_i}(p_B'^{*}-c_B)}{r} \geq \frac{(q_{A_i}-k_{A_i})(p_A'^{*}-c_A)+k_{A_i}p_A'^{*}+(q_{B_i}-k_B)(p_B'^{*}-c_B)+k_{B_i}p_B'^{*}}{r+s_{AB}(k_{A_i},k_{B_i})} \quad \text{(A2)}$$

Note that if $k_{A_i} = k_{B_i} = 0$ that the left and right sides of the above expression would be equal. Note also that, given conditions C1, C2, and C3, if $k_{A_i} = q_{A_i}$ and/or $k_{B_i} = q_{B_i}$, the left and right sides of the above expression are either equal or the right side is smaller. The inequality can be rewritten as:

$$0 \geq r(k_{A_i} + c_A + k_B c_B) - s_{AB}(k_{A_i}, k_{B_i})\left[q_{A_i}(p_A'^{*} - c_A) + q_{B_i}(p_B'^{*} - c_B)\right] \quad (A3)$$

Like A2, the right and left sides of A3 are equal when $k_{A_i} = k_{B_i} = 0$. Similarly, if $k_{A_i} = q_{A_i}$ and/or $k_{B_i} = q_{B_i}$, the left and right sides of the above expression are either equal or the right side is smaller. As a result, it is sufficient to prove that the second partial second-derivatives of the right-hand side with respect to k_{A_i} and k_{B_i} are positive. Since the expressions are identical with respect to k_{A_i} and k_{B_i}, it sufficient to prove the second derivative with respect to k_{A_i} is positive. The second derivative with respect to k_{A_i} is:

$$-\frac{d^2 s_{AB}}{d^2 k_{A_i}}\left[q_{A_i}(p_A'^{*} - c_A) + q_{B_i}(p_B'^{*} - c_B)\right]$$

Since the first term (the second derivative of s_{AB}) is negative, and the second term (in square brackets) is positive, the negative of the product of the two terms is positive. Q.E.D.

The one good case, where the manufacturer produces only one good, follows easily from the proof above. To prove that a firm would not produce some high and some low quality, just remove any terms with B subscripts and then remove all the A and AB subscripts. The proof then follows in exactly the same way.

Appendix C. Proof of Reputational Economies of Scale for Any Number of Periods in the Finite Horizon Game

There are two cases to consider: (1) the firm produced low quality in the first period of the game of $n-1$ periods, and (2) the firm produced high quality in the first period of the game of $n-1$ periods. Because the games of three or fewer periods were considered in the main text, this Appendix will assume $n \geq 3$.

If the firm produced low quality in the first period of the game of $n-1$ periods, the goal is to show (a) that it might produce high quality in the game of n periods and (b) that it is more likely to do so if large (e.g. if s is higher). Since the firm would produce low quality in the first period of the game of $n-1$ periods, as shown below, it would produce low quality in all periods of the game of $n-1$ periods, so for the firm to produce high quality in the first period of the game of n periods, the following inequality must be satisfied

$$1 - c + \sum_{i=1}^{n-1} \delta^i (1-s)^{i-1} \emptyset \geq 1 + \sum_{i=1}^{n-1} \delta^i (1-s)^i \emptyset$$

Solving for $\emptyset$:

$$\emptyset \geq \frac{c}{\delta s \sum_{i=1}^{n-1} \delta^{i-1} (1-s)^{i-1}} \ (A4)$$

Note that, because all terms are positive, all else equal, as n increases, the denominator increase, to the right hand side of the expression decreases, which means that high quality in the first period becomes rational for a wider array of parameters as the number of periods increases. So it is possible that a firm might produce low quality in all periods of the game of $n-1$ periods, but produce high quality in the first period of the game of n periods.

Reputational economies of scale can be proved by differentiating the right side of (A4) with respect to s and showing that the derivative is always negative, which means that higher quality in the first period is rational for a wider array of parameters as quantity and thus s increases. By the quotient rule, it is sufficient to show that the derivative of the denominator of (A4) is positive. It is helpful then to rewrite the denominator using the formula for the sum of a geometric series:

$$\delta s \frac{1 - \delta^{n-2}(1-s)^{n-2}}{1 - \delta(1-s)}$$

Differentiating that with respect to s is a bit messy and involves using of the product rule, quotient rule, and composite function rule and yields:

$$\delta\frac{1-\delta^{n-2}(1-s)^{n-2}}{1-\delta(1-s)}+\partial s\frac{(n-2)\delta^{n-2}(1-s)^{n-3}[1-\delta(1-s)]+\delta[1-\delta^{n-2}(1-s)^{n-2}]}{[1-\delta(1-s)]^2}$$

Because $0<\delta<1$ and $0<s<1$, and because all of terms connected by plus signs are positive, and because the denominators are positive, the overall derivative is positive, which proves reputational economies of scale. That is, a high quantity firm will be trustworthy and produce high quality under a wider array of parameters than a smaller firm.

Now consider the second case, where the firm produced high quality in the first period of the game of $n-1$ periods. We need only prove that, under the same parameters, the firm will produce high quality in the first period of the n-period game of. Because the 2nd period of the game of n periods is identical to the 1st period of the game of $n-1$ periods, that will be sufficient to prove that the firm will continue to do whatever it did in the game of $n-1$ periods. That is, if in the game of $n-1$ periods, the firm produced low quality in the last m periods, $1\leq m<n-1$ and high quality in the first periods $n-m-1$ periods, the firm will produce high quality in the first $n-m$ periods of the n-period game and low quality in the last m periods.

Proof by induction. Assume that the firm rationally produced high quality in just the first period of a game of $m+1$ periods and produced low quality in the other $m\geq 1$ periods. Under that assumption:

$$(1-c)+\sum_{i=1}^{m}\delta^i(1-s)^{i-1}\emptyset\ \geq 1+\sum_{i=1}^{m}\delta^i(1-s)^i\emptyset\ (A5)$$

The first step is to prove that, if $\emptyset$ satisfies A5, then in the game of $m+2$ periods, it will be rational for the firm to produce high quality in the first two periods and low quality in the last m periods:

$$(1-c)+\delta(1-c)+\sum_{i=1}^{m}\delta^{i+1}(1-s)^{i-1}\emptyset\ \geq 1+\delta(1-s)+\sum_{i=1}^{m}\delta^{i+1}(1-s)^{i+1}\emptyset\ (A6)$$

First note that A5 implies that:

$$-c\ \geq -s\sum_{i=1}^{m}\delta^i(1-s)^{i-1}\emptyset\ (A7)$$

Which implies that

$$\sum_{i=1}^{m}\delta^{i-1}(1-s)^{i-1}\emptyset\geq\frac{c}{\delta s}\ (A8)$$

If we denote the payoff from producing high quality in the first a periods and low quality in the last b periods as π_b^a, then A6 can be rewritten as:

$$(1-c)+\delta\pi_m^1 \geq 1+\delta(1-s)\pi_{m+1}^0(A9)$$

Since A5 can be rewritten as $\pi_m^1 \geq \pi_{m+1}^0$, it is sufficient to prove that

$$-c \geq -\delta s\pi_{m+1}^0(A10)$$

A10 is equivalent to:

$$\pi_{m+1}^0 \geq \frac{c}{\delta s}(A11)$$

Note that, the left side of A8 is less than the right side of A5, and, as noted above the right side of A5 can be rewritten as π_{m+1}^0, so $\pi_{m+1}^0 \geq \sum_{i=1}^{m}\delta^{i-1}(1-s)^{i-1}\emptyset \geq \frac{c}{\delta s}$. So if parameters are such that A5 is satisfied, A11 will also be satisfied, which means that if parameters are such that $\emptyset$ satisfies A5, then in the game of $m+2$ periods, it will be rational for the firm to produce high quality in the first two periods and low quality in the last m periods.

By similar logic, it is easy to prove that if the firm produced high quality in the first $n-m-1 \geq 2$ periods (and low quality in the last m periods) of the game of $n-1$ periods, it will produce high quality in the first $n-m$ periods of the game of n periods. The assumption that the firm will produce high quality in the first $n-m-1 \geq 2$ periods (and low quality in the last m periods) of the game of $n-1$ periods can be rewritten as:

$$\pi_m^{n-m-1} \geq \pi_{n-1}^0\ (A12)$$

We need to prove that:

$$1-c+\delta\pi_m^{n-m-1} \geq 1+\delta(1-s)\pi_{n-1}^0(A13)$$

Because of (A12) it is sufficient to prove:

$$-c \geq -\delta s\pi_{n-1}^0(\text{A14})$$

A14 can be rewritten as:

$$\pi_{n-1}^0 \geq \frac{c}{\partial s}(\text{A15})$$

Note that $\pi_{n-1}^0 > \pi_{m+1}^0$ because $n-1 > m+1$, and because the first $m+1$ terms are identical and π_{n-1}^0 just has additional positive terms. As noted above, $\pi_{m+1}^0 \geq \frac{c}{\delta s}$, which means that $\pi_{n-1}^0 > \frac{c}{\delta s}$, which is stronger than (A15), which is what we needed to prove. So, if in the game of $n-1$ periods, the bad firm produced high quality in the first $n-m-1 \geq 2$ periods, it will produce high quality in at least the $n-m$ periods of the n-period game.

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