University of Southern California Law School
Legal Studies Working Paper Series

Year 2015 *Paper* 167

Updating Priest and Klein

Yoon-Ho Alex Lee* Daniel M. Klerman†

*USC Gould School of Law, alee@law.usc.edu
†USC Law School, dklerman@law.usc.edu

Updating Priest and Klein

Yoon-Ho Alex Lee and Daniel M. Klerman

Abstract

In their 1984 article, "The Selection of Disputes for Litigation," Priest and Klein famously hypothesized a "tendency toward 50 percent plaintiff victories" among litigated cases. Nevertheless, many scholars doubt the validity of their conclusions, because the model they relied upon does not meet modern standards of rigor. This article updates the Priest-Klein model by considering three modifications. First, we raise a novel critique of the Priest-Klein model—that it is non-Bayesian—and show that most of the results of Priest and Klein (1984) pertaining to limits nevertheless remain valid under a modified model in which parties use Bayes' rule to refine their estimates of the plaintiff's probability of prevailing. Second, we show that even when an incentive-compatible mechanism is imposed, many of the results remain valid for symmetric Nash equilibria. Finally, we show how the Priest-Klein model can be modified to analyze asymmetric information, show that most results are false under this modification, and compare the modified Priest-Klein model to standard asymmetric information models.

Updating Priest and Klein

Yoon-Ho Alex Lee and Daniel Klerman*

Draft: August 12, 2015

Abstract

In their 1984 article, "The Selection of Disputes for Litigation," Priest and Klein famously hypothesized a "tendency toward 50 percent plaintiff victories" among litigated cases. Nevertheless, many scholars doubt the validity of their conclusions, because the model they relied upon does not meet modern standards of rigor. This article updates the Priest-Klein model by considering three modifications. First, we raise a novel critique of the Priest-Klein model—that it is non-Bayesian—and show that most of the results of Priest and Klein (1984) pertaining to limits nevertheless remain valid under a modified model in which parties use Bayes' rule to refine their estimates of the plaintiff's probability of prevailing. Second, we show that even when an incentive-compatible mechanism is imposed, many of the results remain valid for symmetric Nash equilibria. Finally, we show how the Priest-Klein model can be modified to analyze asymmetric information, show that most results are false under this modification, and compare the modified Priest-Klein model to standard asymmetric information models.

JEL Classification Codes: C11, C7, C78, D8, D81, D82, K, K4, K41,

* Yoon-Ho Alex Lee is Assistant Professor of Law, USC Gould School of Law. alee@law.usc.edu. Daniel Klerman is Charles L. and Ramona I. Hilliard Professor of Law and History, USC Gould School of Law. www.klerman.com. dklerman@law.usc.edu. The authors thank Ken Alexander, Andrew Daughety, Dan Erman, Jonah Gelbach, Richard Holden, Bart Kosko, Jennifer Reinganum, Neelesh Tiruviluamala for their helpful comments and suggestions. The authors also thank Yuxiao Li and Lawrence Liu for their excellent research assistance.

1. Introduction

Priest and Klein's 1984 article, "The Selection of Disputes for Litigation," famously hypothesized that there will be a "tendency toward 50 percent plaintiff victories" among litigated cases (p.20). Nevertheless, many scholars doubt the validity of their conclusions, because their model does not meet modern standards of rigor. This article updates the Priest-Klein model by considering three modifications. First, we alter the model so that parties use Bayes' rule to refine their estimates of the plaintiff's probability of prevailing, and show that most of Priest and Klein's predictions are still true. Second, we graft an incentive-compatible mechanism onto Priest and Klein's model and show that many of the results remain valid for symmetric Nash equilibria. Finally, we show how the Priest-Klein model can be used to analyze asymmetric information, demonstrate that most results are false under this modification, and compare the modified Priest-Klein model to standard asymmetric information models.

Priest and Klein's article has been one of the most influential legal publications, and its influence is growing as empirical work on law has become more common. Compare Shapiro and Pearse (2012) to Shapiro (1996). Even with the introduction of asymmetric information models of settlement, Priest and Klein's article continues to be cited by sophisticated empiricists and in respected peer-reviewed journals. See Hubbard (2013); Gelbach (2012); Atkinson (2009); Bernardo, Talley and Welch (2000); Waldfogel (1995); Siegelman and Donohue (1995).

It is helpful to distinguish six hypotheses plausibly attributable to the Priest and Klein (1984):

> THE TRIAL SELECTION HYPOTHESIS. "[D]isputes selected for litigation (as opposed to settlement) will constitute neither a random nor a representative sample of the set of all disputes" (p.4). This proposition is probably the most important contribution of their article.
>
> THE FIFTY-PERCENT LIMIT HYPOTHESIS. "[A]s the parties' error diminishes" there will be a "convergence towards 50 percent plaintiff victories" (pp.18). This hypothesis is often called *the* Priest-Klein hypothesis.
>
> THE ASYMMETRIC STAKES HYPOTHESIS. If the defendant would lose more from an adverse judgment than the plaintiff would gain, then the plaintiff will win less than fifty percent of the litigated cases. Conversely, if the plaintiff has more to gain, then the plaintiff will win more than fifty-percent (see pp. 24-26). This hypothesis is most plausibly, like the Fifty-Percent Limit Hypothesis, a statement about the limit percentage of plaintiff victories as the parties become increasingly accurate in predicting trial outcomes.

THE IRRELEVANCE OF THE DISPUTE DISTRIBUTION HYPOTHESIS. The plaintiff trial win rate will be "unrelated … to the shape of the distribution of disputes" (pp. 19 and 22). Like the two previous hypothesis, this hypothesis is about the limit as the parties become increasingly accurate in predicting trial outcomes. This hypothesis is closely related to the Fifty-Percent Limit Hypothesis, but more fundamental. It is also more general, because it also applies when the stakes are unequal.

THE NO INFERENCES HYPOTHESIS. Because selection effects are so strong, no inferences can be made about the law or legal decisionmakers from the plaintiff trial win rate. Rather, "the proportion of observed plaintiff victories will tend to remain constant over time regardless of changes in the underlying standards applied." (p. 31). Because this hypothesis was examined in great detail in Klerman and Lee (2014), it will not be analyzed further here.

THE FIFTY-PERCENT BIAS HYPOTHESIS. Regardless of the legal standard, the plaintiff trial win rate will exhibit "a strong bias toward . . . fifty percent" as compared to the percentage of cases plaintiff would have won if all cases went to trial (pp. 5 and 23). That is, the plaintiff trial win rate will be closer to fifty percent than the plaintiff win rate that would be observed if all cases went to trial.

Klerman and Lee (2014) shows that the No Inferences Hypothesis is false under both Priest and Klein's original model and more recent asymmetric information models. Lee and Klerman (2015) analyzes the mathematical validity of the other hypotheses under a formalization of Priest and Klein's original model and provides the first rigorous proofs that the other hypotheses, including the Fifty-Percent Limit Hypothesis, are largely true. This article moves beyond the original model and considers three extensions. The goal of these extensions is to retain as much as possible of Priest and Klein's original set up, while correcting problems with the original model and bringing it more in line with modern modeling standards.

First, we raise a novel critique of Priest and Klein's original model—that it is non-Bayesian. The original model assumes that parties estimate the plaintiff's probability of prevailing without taking into account information about the underlying distribution of all disputes. More precisely, Priest and Klein assume that the defendant's degree of fault can be represented by a real number, Y', and that the plaintiff and defendant estimate the defendant's fault with error. So the plaintiff's point estimate is $Y_p = Y' + \epsilon_p$, where ϵ_p has mean zero and standard deviation σ. Similarly, defendant's point estimate is $Y_d = Y' + \epsilon_d$, where ϵ_d also has mean zero and standard deviation σ. Parties then estimate the plaintiff's probability of prevailing by calculating the likelihood that the true Y' is greater than Y^* under the assumption that the true Y' is distributed with mean Y_p or Y_d and standard deviation σ. This means that the parties are not using Bayes' rule and the information they may have about the distribution of Y' to estimate the likelihood of plaintiff victory. Later scholars, including Waldfogel (1995), have adopted this problematic aspect of Priest and Klein's original set-up and effectively assumed that the parties are naïve and non-Bayesian. We explore a modification to the original model under which the parties are sophisticated Bayesians who take into account the underlying distribution of disputes. We find that most of the Priest-Klein hypotheses remain valid under this modified model.

Second, Priest and Klein's model has been criticized for lacking an incentive-compatible mechanism. This is, Priest and Klein assume that as long as the plaintiff's subjective expected net trial recovery is greater than the defendant's subjective expected net loss, the parties will settle. Nevertheless, much has been written about *ex post* inefficiency arising in strategic bargaining. See Myerson and Satterthwaite (1985). We address the possibility of *ex post* bargaining inefficiency by coupling Priest and Klein's model with an incentive-compatible mechanism. Our approach is similar to Friedman and Wittman (2007) in that we employ the Chatterjee-Samuelson mechanism, but different in that our model retains Priest and Klein's original set-up. By remaining faithful to Priest and Klein's model, we can identify the extent to which their hypotheses are robust to an incentive-compatible mechanism. Under this model, we show that there will always be at least one symmetric equilibrium in the limit that will yield a fifty-percent trial win rate for the plaintiff, even when stakes are slightly asymmetric. Moreover, this and other results continue to hold even under the Bayesian modification.

Third, we show how Priest and Klein's model can be relatively easily modified to analyze asymmetric information. In fact, the modified model is, in some ways, more flexible than standard asymmetric information models, Bebchuck (1984) and Reinganum & Wilde (1986). Whereas canonical asymmetric information models assume that either the plaintiff or defendant are perfectly informed, the modified Priest-Klein model can be used to analyze situations where the difference in information is a matter of degree and neither side is perfectly informed. Under the asymmetric-information version of the Priest-Klein model, only the Trial Selection and Irrelevance of Dispute Resolution Hypotheses are true. We also compare the modified Priest-Klein model to the canonical asymmetric information models in both their model structure and results.

Table 1 summarizes the results:

Table 1. Main Results

	Original Priest-Klein Model	Priest-Klein Model with Bayesian Correction	Priest-Klein Model with Incentive-Compatible Mechanism	Priest-Klein Model with Asymmetric Information
Trial Selection Hypothesis	True	True	True	True
Fifty-Percent Limit Hypothesis	True	True	True	False
Asymmetric Stakes Hypothesis	True	True	False	False
Irrelevance of Dispute Distribution Hypothesis	True	True	True	True
Fifty-Percent Bias Hypothesis	True under Some Conditions	True under Some Conditions	True under Some Conditions	False

Notes. "True" means true under a wide array of assumptions. "True under Some Conditions" means true under a narrow set of assumptions. "False" means not true with any meaningful generality. Results for the Priest-Klein Model with an Incentive-Compatible Bargaining Mechanism are for symmetric limit equilibria only. For asymmetric information models, the Fifty-Percent Limit Hypothesis means that the plaintiff trial win rate is Fifty-Percent when the stakes are symmetric. See Section 5 for more on the meaning of the middle three hypotheses under the asymmetric information models.

The No Inferences Hypothesis is omitted from the table, because it is the subject of Klerman and Lee (2014). Results in the first column ("Original Priest-Klein Model") are proved in Lee and Klerman (2015).

The rest of the article proceeds as follows. Section 2 presents a formalized version of Priest and Klein's original model. Section 3 analyzes the selection implications of a modified model under which parties use Bayes' rule to calculate the plaintiff's probability of prevailing. Section 4 explores the implications of grafting the Chatterjee-Samuelson mechanism onto Priest and Klein's original model. Section 5 briefly modifies the Priest-Klein model to take into account asymmetric information and compares the resulting model and results to canonical asymmetric information models. The Appendix contains technical proofs and additional results.

2. Formalization of Priest and Klein's Original Model

This section assumes familiarity with Priest and Klein (1984) and follows Waldfogel's (1995) formalization. Although there have been other attempts to formalize Priest and Klein's model (see Wittman (1985) and Hylton and Lin (2012)), Waldfogel offers the formalization that is most faithful to the model in Priest and Klein's original article. See Hylton and Lin (2012), n.5. Lee and Klerman (2015) used this formalization to prove most of the Priest-Klein hypotheses. We begin by first presenting the formalization in the most general manner possible and state the results under the original model in this section. We then introduce the extensions we consider in Sections 3 and 4.

The merits of a case are represented by a real number Y', and the decision standard is denoted Y^*, where the defendant prevails if $Y' \leq Y^*$, and the plaintiff prevails if $Y' > Y^*$. For example, in a negligence case, Y' might be the efficient level of precaution expenditures minus defendant's actual precaution, in which case $Y^* = 0$. Priest and Klein, for simulation purposes, assume Y' is distributed according to a standard normal distribution. We do not impose that restriction. Instead, we assume only that Y' is distributed according to a probability density function, $g(Y')$, that is bounded above everywhere and locally continuous and nonzero at Y^*. Note that if all disputes were litigated, the plaintiff win rate would be $\int_{Y^*}^{\infty} g(Y')dY'$. If $G(Y')$ is the corresponding cumulative distribution, then the plaintiff win rate can be rewritten as $1 - G(Y^*)$.

If a case goes to trial, the court observes the true Y' and gives judgment to the plaintiff if and if $Y' > Y^*$. The plaintiff and the defendant themselves make unbiased estimates of Y', $Y_p = Y' + \epsilon_p$ and $Y_d = Y' + \epsilon_d$, respectively, where ϵ_p and ϵ_d have mean zero, standard deviations σ_p and σ_d, respectively, and are distributed according to the joint probability distribution $f_{\sigma_p,\sigma_d}(\epsilon_p, \epsilon_d)$. Priest and Klein assume that ϵ_p and ϵ_d are independent and that $f_{\sigma_p,\sigma_d}(\epsilon_p, \epsilon_d)$ is bivariate normal with $\sigma_p = \sigma_d$. In other words, they assume $f_{\sigma_p,\sigma_d}(\epsilon_p, \epsilon_d) = f_{\sigma_p}(\epsilon_p) f_{\sigma_d}(\epsilon_d)$, where $f_\sigma(\cdot)$ is the normal distribution with standard deviation σ. Lee and Klerman (2015) have shown that these assumptions can be relaxed.

We assume instead that ϵ_p and ϵ_d are not necessarily independent, but are distributed with mean zero and standard deviations σ_p and σ_d, respectively, according to a joint probability density function $f_{\sigma_p,\sigma_d}(\epsilon_p, \epsilon_d)$ that may not be normal, but which has the following properties. First, $f_{\sigma_p,\sigma_d}(\epsilon_p, \epsilon_d)$ has full support over the entire $\boldsymbol{R}^2$. Second, $f_{\sigma_p,\sigma_d}(\epsilon_p, \epsilon_d)$ satisfies the following condition: $f_{1,1}(x, y) = \sigma_x \sigma_y f_{\sigma_x,\sigma_y}(\sigma_x x, \sigma_y y)$. Third, the corresponding univariate marginal distributions for ϵ_p and ϵ_d are $f_{p,\sigma_p}(\epsilon_p)$ and $f_{d,\sigma_d}(\epsilon_d)$ such that $f_{p,1}(x) = \sigma_x f_{p,\sigma_x}(\sigma_x x)$ and $f_{d,1}(y) = \sigma_y f_{y,\sigma_y}(\sigma_y y)$. Let $F_p[\cdot]$ and $F_d[\cdot]$ be the corresponding cumulative distributions when $\sigma_p = \sigma_d = 1$.

The second assumption, in particular, indicates that $f_{\sigma_p,\sigma_d}(\epsilon_p, \epsilon_d)$ belongs to a family of mean-zero probability density functions that vary parametrically with σ_p and σ_d and can be standardized with proper scaling. The reason for making this assumption is that some of the hypotheses require taking the limit as σ_p and σ_d approach zero. Therefore, there needs to be a well-defined family of distributions as the standard deviation parameter varies. A bivariate normal distribution with mean zero certainly satisfies all these conditions, but a host of other distributions also satisfy these conditions.[1]

In Section 5, where we explore asymmetric information in Section 5, we allow σ_p and σ_d to be different. For example, σ_p may be greater than σ_d if the defendant is systematically superior in estimating the true merit of the case than the plaintiff. Therefore, we let $\sigma_p = \sigma_d/\beta = \sigma$ for a fixed $\beta > 0$. For sections 3 and 4, we will assume that the parties are, on average, equally well informed ($\beta = 1$).

In order to estimate the probability with which the plaintiff will prevail, both the plaintiff and the defendant need to take into account the fact that their estimates of case merit, $Y' + \epsilon_p$ and $Y' + \epsilon_d$, are not wholly accurate. Therefore, they must estimate both the mean and standard deviation of their estimates of Y'. Priest and Klein (1984) assume that plaintiff estimates the mean of sampling distribution of Y' to be $Y' + \epsilon_p$ and the standard deviation to be σ_p. Waldfogel (1995) notes that under this set-up the plaintiff's subjective estimate of the probability it will prevail, $P_p = P(Y' \geq Y^* | Y' + \epsilon_p)$, will simply be $P_p = F_p\left[\frac{Y' + \epsilon_p - Y^*}{\sigma_p}\right]$. Similarly, the defendant estimates the mean of Y' to be $Y' + \epsilon_d$, and the standard deviation to be σ_d. So the defendant estimates the probability that the plaintiff prevails to be $P_d = F_d\left[\frac{Y' + \epsilon_d - Y^*}{\sigma_d}\right]$. We reconsider this assumption in Section 3, where we modify the model and assume the parties take the underlying distribution of disputes into consideration in calculating their subjective beliefs.

Priest and Klein assume that the parties go to trial[2] if $P_p J - C_p + S_p > P_d J + C_d - S_d$, where $J > 0$ is the damages that the defendant pays the plaintiff if the case is litigated and the plaintiff prevails, C_p and C_d are litigation costs for the plaintiff and the defendant, respectively, and S_p and S_d are settlement costs for the plaintiff and the defendant, respectively. This condition for litigation makes sense, because settlement can only happen if both parties perceive the payoffs to settlement to be higher than the payoffs to litigation. The litigation condition can be rewritten as $(P_p - P_d)J > C - S$, where $C = C_p + C_d$ and $S = S_p + S_d$. $(P_p - P_d)J > C - S$

[1] A partial list of distributions (with full support over $\boldsymbol{R}^2$) that satisfy this condition include bivariate distributions composed of generalized normal distributions, Laplace distributions, and logistic distributions. Given any univariate probability density function $f(x)$ with mean zero and standard deviation 1, one can always construct such a family of bivariate density function by setting $f_{\sigma_p,\sigma_d}(\epsilon_p, \epsilon_d) = \frac{f\left(\frac{\epsilon_p}{\sigma_p}\right)f\left(\frac{\epsilon_d}{\sigma_d}\right)}{\sigma_p \sigma_d}$.

[2] Priest and Klein and much of the later literature assume that "litigate" and "go to trial" are synonymous, because they assume that all cases either settle or go to trial. More recent work explores the fact that many cases are resolved by motions to dismiss or summary judgment. Gelbach (2012); Hubbard (2013). Cases resolved by such motions are litigated, but did not go to trial. This article, however, retains the simplifying assumption that all litigated cases go to trial. The term "disputes" or "all disputes" means both cases that settle and cases that are litigated.

is known as the Landes-Posner-Gould condition for litigation, after the three scholars who formulated it. Priest and Klein simulate their results with $\frac{C-S}{J} = 1/3$. We assume $0 < (C - S)/J \leq 1$. Priest and Klein assume that the plaintiff always has a credible threat to go to trial and thus can litigate or settle even when $P_p J < C_p$. We retain that assumption, even though it is unrealistic. Relaxing it would complicate the math, but have little effect on the main conclusions.

Priest and Klein are silent about how the parties bargain to arrive at a settlement. Technically, the Landes-Posner-Gould condition is merely a sufficient condition for litigation, not a necessary one. Litigation might happen even if the condition is violated, because parties might not be able to agree on the settlement amount, even if there is a range of settlement amounts that would be in their perceived mutual interest. As modern mechanism design research has shown, bargaining is frequently inefficient. See Myerson and Satterthwaite (1983); but see McAfee and Reny (1992). Nevertheless, Priest and Klein (1984) and others using the divergent expectations model have assumed that the Landes-Posner-Gould condition is necessary as well as sufficient for litigation. In this section and in Section 3, we proceed with this assumption, but we relax it in Section 4.

Priest and Klein allow for the possibility that parties may have asymmetric stakes and suggest there will be a deviation from fifty-percent in such cases. For example, the defendant may be more concerned about its reputation or an adverse precedent, so it may lose more from an adverse judgment than the plaintiff gains from prevailing. As Priest and Klein point out, asymmetric stakes can be formalized by assuming that the plaintiff would win αJ if it prevailed and the defendant would lose J if the plaintiff won. If α is greater than 1, then the plaintiff faces a greater stake in the litigation than the defendant, and vice versa. Taking into account the possibility of asymmetric stakes, the trial condition becomes $\alpha P_p - P_d > (C - S)/J$.

Let $P_{\alpha,\sigma_p,\sigma_d}(Y'; Y^*)$ denote the probability that a dispute Y' goes to trial when the decision standard is Y^* and where the parties predict case merit with errors ϵ_p and ϵ_d that are distributed with mean zero and standard deviations σ_p and σ_d. We shall call this the "litigation probability function." When $\sigma_p = \sigma_d = \sigma$, we will simply denote this probability as $P_{\alpha,\sigma}(Y'; Y^*)$. $P_{\alpha,\sigma_p,\sigma_d}(Y'; Y^*)$ can be written as the probability that

$$\alpha F_p \left[\frac{Y' + \epsilon_p - Y^*}{\sigma_p}\right] - F_d \left[\frac{Y' + \epsilon_d - Y^*}{\sigma_d}\right] > \frac{C - S}{J}.$$

In other words, $P_{\alpha,\sigma_p,\sigma_d}(Y'; Y^*) = \iint_{R_{\alpha,\sigma_p,\sigma_d}(Y';Y^*)} f_{\sigma_p,\sigma_d}(\epsilon_p, \epsilon_d) d\epsilon_p d\epsilon_d$, where $R_{\alpha,\sigma_p,\sigma_d}(Y'; Y^*) = \left\{(\epsilon_p, \epsilon_d) \in R^2 \middle| \alpha F_p \left[\frac{Y' + \epsilon_p - Y^*}{\sigma_p}\right] - F_d \left[\frac{Y' + \epsilon_d - Y^*}{\sigma_d}\right] > \frac{C-S}{J}\right\}$. Therefore, $P_{\alpha,\sigma_p,\sigma_d}(Y'; Y^*)$ can be expressed as a double integral over a region of integration that is implicitly defined by the inequality.

The fraction of cases litigated is $\int_{-\infty}^{\infty} P_{\alpha,\sigma_p,\sigma_d}(Y';Y^*)g(Y')dY'$. This value approaches zero as σ_p and σ_p approach zero. The plaintiff trial win rate is thus

$$W_{\alpha,\sigma_p,\sigma_d}(Y^*) = \frac{\int_{Y^*}^{\infty} P_{\alpha,\sigma_p,\sigma_d}(Y';Y^*)g(Y')dY'}{\int_{-\infty}^{\infty} P_{\alpha,\sigma_p,\sigma_d}(Y';Y^*)g(Y')dY'}.$$

When $\sigma_p = \sigma_d = \sigma$, we denote the plaintiff trial win rate as simply $W_{\alpha,\sigma}(Y^*)$. Since $W_{\alpha,\sigma_p,\sigma_d}(Y^*)$ is mathematically different from $\int_{Y^*}^{\infty} g(Y')dY'$, the plaintiff trial win rate if all disputes were litigated, we can readily see that *the set of litigated cases is not simply a random set of all disputes*. The Trial Selection Hypothesis is therefore clearly correct.

At this point, we introduce a useful change of variables. Let $\sigma_p = \sigma_d/\beta = \sigma$ for a fixed $\beta > 0$. Therefore, as σ_p approaches zero, σ_d will necessarily approach zero as well. For a given $\sigma > 0$, let $u = \frac{Y'+\epsilon_p-Y^*}{\sigma_p} = \frac{Y'+\epsilon_p-Y^*}{\sigma}, v = \frac{Y'+\epsilon_d-Y^*}{\sigma_d} = \frac{Y'+\epsilon_d-Y^*}{\beta\sigma}, z = \frac{Y'-Y^*}{\sigma}$. Then we have

$$f_{\sigma_p,\sigma_d}(\epsilon_p,\epsilon_d)d\epsilon_p d\epsilon_d = f_{\sigma_p,\sigma_d}\left(\sigma_p(u-z), \sigma_d\left(v-\frac{z}{\beta}\right)\right)\sigma_p\sigma_d dudv = f_{1,\beta}\left(u-z, v-\frac{z}{\beta}\right)dudv.$$

Meanwhile, $R_{\alpha,\sigma_p,\sigma_d}(Y';Y^*) = \left\{(u,v)\middle|\alpha F_p[u] - F_d[v] > \frac{C-S}{J}\right\} = R_\alpha(u,v)$. Therefore, for each $\sigma > 0$,

$$P_{\alpha,\sigma_p,\sigma_d}(Y';Y^*) = \iint\limits_{R_\alpha(u,v)} f_{1,\beta}\left(u-z, v-\frac{z}{\beta}\right)dudv$$

This normalization of the variables is useful. Prior to normalization, $P_{\alpha,\sigma_p,\sigma_d}(Y';Y^*)$, as a function of Y', was a double integral of a fixed bivariate distribution over a region of integration in the $\epsilon_p\epsilon_d$–plane that depended on three parameters: Y', σ_p, and σ_d. After the change of variables, $P_{\alpha,\sigma_p,\sigma_d}(Y';Y^*) = P_{\alpha,\sigma,\beta\sigma}(\sigma z + Y^*;Y^*)$, as a function of z, is a double integral of a bivariate distribution over a region of integration in the uv–plane that depends on only one parameter: z. Indeed, the key insight from Lee and Klerman (2015) was that, when $\alpha = 1$, the region of integration, $R_\alpha(u,v)$, is invariant under σ, and is symmetric around the line $v = -u$.[3] This fact, together with Chebyshev's inequality, allowed for a construction of a Lebesgue-integrable dominating function. This allowed us to take the limits under the integral using Lebesgue's Dominated Convergence Theorem. In addition, the symmetry of $R_\alpha(u,v)$ around the line $v = -u$ led to the result that the litigation probability function, too, must be symmetric around Y^* when $\alpha = 1$.

Figures 1a and 1b depict examples of litigation probability functions, $P_{\alpha,\sigma_p,\sigma_d}(Y';Y^*)$ for large and small σ where $\alpha = 1$, $\sigma_p = \sigma_d = \sigma$ ($\beta = 1$), $Y^* = 1$, and F is the cumulative normal distribution. As these figures show, when $\alpha = 1$ the probability of litigation is single-peaked and

[3] The region of integration labeled as $R_1(u,v)$ in Figure 3 illustrates this point.

symmetric around Y^*. Therefore, disputes close to Y^* are the most likely to be litigated. In addition, as σ becomes smaller, the probability of litigation becomes highly concentrated near Y^*.

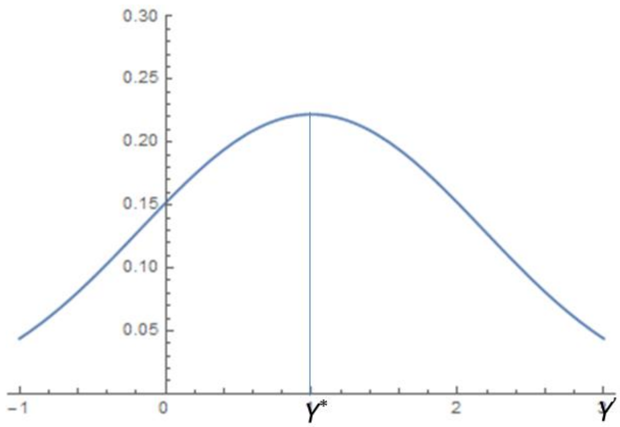


Figure 1a. $P_{1,\sigma}(Y'; 1)$ for $\sigma = 1$.

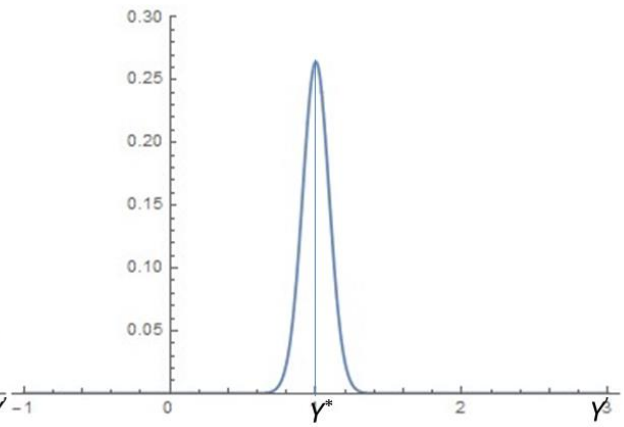


Figure 1b. $P_{1,\sigma}(Y'; 1)$ for $\sigma = 0.1$.

The following is a list of some of the major results under the original model from Lee and Klerman (2015). These results provide a point of comparison for the modified models explored in the rest of this article.

- IRRELEVANCE OF THE DISPUTE DISTRIBUTION HYPOTHESIS. *Suppose $g(Y')$ is bounded above everywhere and locally continuous and nonzero at Y^*, and ϵ_p and ϵ_d are distributed with mean zero according to $f_{\sigma_p,\sigma_d}(\epsilon_p, \epsilon_d)$ such that $f_{1,1}(x, y) = \sigma_x \sigma_y f_{\sigma_x,\sigma_y}(\sigma_x x, \sigma_y y)$ with full support over R^2. Then for $\alpha > \frac{C-S}{J}$, the limit of the plaintiff trial win rate as σ_p* and *σ_d approach 0 reduces to an expression independent of $g(Y')$. For $\alpha \leq \frac{C-S}{J}$, all cases settle, so the plaintiff trial win rate is undefined.*

- FIFTY-PERCENT LIMIT HYPOTHESIS. *Suppose $g(Y')$ is bounded above everywhere and locally continuous and nonzero at Y^*, the stakes are equal (i.e., $\alpha = 1$), and the parties' prediction errors, ϵ_p and ϵ_d, are distributed according to the same probability density function (i.e., $f_{p,\sigma} = f_{d,\sigma}$) with a common standard deviation (i.e., $\beta = 1$) and according to a joint probability density function that is symmetric around 0 and symmetric with respect to each other, then the limit value of the plaintiff trial win rate is fifty percent.*

- ASYMMETRIC STAKES HYPOTHESIS. *Under the assumptions set out for the Fifty-Percent Limit Hypothesis, except the stakes are unequal, if $\alpha > \frac{C-S}{J}$, the limit value of plaintiff trial win rate will be greater than fifty percent for $\alpha > 1$ and less than fifty percent for $\alpha < 1$.*

- FIFTY-PERCENT BIAS HYPOTHESIS. *Under the assumptions set out for the Fifty-Percent Limit Hypothesis, the Fifty-Percent Bias Hypothesis is true for sufficiently small values of* σ. *That is, the plaintiff trial win rate will be closer to fifty percent than the plaintiff win rate among all disputes. In other words,* $\left|W_{1,\sigma}(Y^*) - 1/2\right| \leq \left|\int_{Y^*}^{\infty} g(Y')dY' - 1/2\right|$ *for all* $Y^* \in R$. *For large values of* σ, *the Fifty-Percent Bias Hypothesis will be true if* $g(Y')$ *is symmetric and logarithmically concave.*

We are now ready to consider the extensions.

3. Priest-Klein Model with Bayesian Correction

As mentioned above, Priest and Klein's original model assumes (implicitly) that the parties estimate the plaintiff's probability of prevailing without using Bayes' rule or information about the underlying distribution of all disputes. Consequently, Priest and Klein's model assumed that $P_p = P\left(Y' \geq Y^* \middle| Y' + \epsilon_p\right) = F_p\left[\frac{Y\prime + \epsilon_p - Y^*}{\sigma_p}\right]$ and $P_d = P(Y' \geq Y^* | Y' + \epsilon_d) = F_d\left[\frac{Y\prime + \epsilon_d - Y^*}{\sigma_d}\right]$. This means that the parties are not taking into consideration the actual distribution of Y' in estimating the likelihood of the plaintiff's victory. It is as if the parties were assuming—for the purpose of making their estimates—that $g(Y')$ is flat *and* has full support, which is not possible.[4] It also means that the parties are assuming that $\sigma_p$and σ_d, the standard deviations of ϵ_p and ϵ_d, are also the standard deviations of the parties' estimates Y'.

These are problematic assumptions. Consider a simple discrete case where Y' can take only integer values between 1 and 10, with each integer value equally likely. Suppose each party observes Y' within an error that is zero, +1, or -1, each with probability one third. In this case, if a party observes 5, he would be correct to presume that the true Y' will be 4, 5, or 6, each with probability one third. As a result, it would be rational for the party to assume that the mean of the sampling distribution of Y' is 5 and that the same error distribution (0, +1, -1) that generated the party's observations will also describe the sampling distribution. Suppose, on the other hand, that it is known that Y' never takes the values 5 or 6. That is, $Y' \in \{1,2,3,4,7,8,9,10\}$, with each value equally likely. As before, assume that each party observes Y' with an error that is zero, +1, or -1, each with probability one third. In this situation, if a party observes 5 and knows the underlying distribution of Y', he would know with certainty that the true Y' is exactly 4. Given that 5 and 6 are never observed, it would be irrational for the party to assume that the mean of the sampling distribution was his observation, 5, or that the true value of $Y'$was distributed around 5 with

[4] One could approximate a completely flat distribution by assuming that the distribution of disputes had very large standard deviation. Nevertheless, this approach is implausible, because it implies that close cases are extremely rare and that the outcome of nearly all cases can be predicted with close to absolute certainty. That would imply settlement rates even higher than observed today.

errors zero, +1, or -1 with equal probability. This example suggests that a rational party would use information about the distribution of disputes both in calculating the mean of its estimate of Y' and in calculating the standard deviation of Y'. Although litigating parties may lack precise information about the distribution of disputes, experienced lawyers probably have a rough sense of the distribution of disputes.

The most plausible way of modifying the model to incorporate party knowledge of the distribution of disputes is to interpret Y_p and Y_d not as the parties' own estimates of Y', the defendant's degree of fault, but as *informative signals* the parties receive about Y'. The parties then use the signals to make their estimates of Y' and its standard deviation. This modification brings the model closer to the literature on settlement under two-sided asymmetric information. Schweizer (1989); Sobel (1989); Daughety & Reinganum (1994); Friedman & Wittman (2006). Under the modified model the parties' estimates would not be $P_p = P\left(Y' \geq Y^* \middle| Y' + \epsilon_p\right) = F_p\left[\frac{Y'+\epsilon_p-Y^*}{\sigma_p}\right]$ and $P_d = P(Y' \geq Y^*|Y' + \epsilon_d) = F_d\left[\frac{Y'+\epsilon_d-Y^*}{\sigma_d}\right]$, as before. Instead, they would be

$$P_p = P\left(Y' \geq Y^* \middle| Y' + \epsilon_p, g(Y')\right) = \frac{\int_0^\infty f_{p,\sigma_p}(Y'+\epsilon_p-Y^*-W')g(Y^*+W')dW'}{\int_{-\infty}^\infty f_{p,\sigma_p}(Y'+\epsilon_p-Y^*-W')g(Y^*+W')dW'}, \text{ and}$$

$$P_d = P(Y' \geq Y^*|Y' + \epsilon_d, g(Y')) = \frac{\int_0^\infty f_{d,\sigma_d}(Y'+\epsilon_p-Y^*-W')g(Y^*+W')dW'}{\int_{-\infty}^\infty f_{d,\sigma_d}(Y'+\epsilon_p-Y^*-W')g(Y^*+W')dW'}.$$

Likewise, the trial condition would be determined by the following inequality:

$$\alpha\left(\frac{\int_0^\infty f_{p,\sigma_p}\left(Y' + \epsilon_p - Y^* - W'\right)g(Y^* + W')dW'}{\int_{-\infty}^\infty f_{p,\sigma_p}\left(Y' + \epsilon_p - Y^* - W'\right)g(Y^* + W')dW'}\right) - \left(\frac{\int_0^\infty f_{d,\sigma_d}\left(Y' + \epsilon_p - Y^* - W'\right)g(Y^* + W')dW'}{\int_{-\infty}^\infty f_{d,\sigma_d}\left(Y' + \epsilon_p - Y^* - W'\right)g(Y^* + W')dW'}\right) > \frac{C - S}{J}$$

Note that in this case, even after effecting a same change of variables as in Section 2, the region of integration will continue to depend on σ and Y^* (see Appendix). Despite the added layer of complexity, it turns out that this modification does not make a significant difference when it comes to results pertaining to limits. For this reason, all the limit hypotheses remain valid under the Priest-Klein model with Bayesian correction. Intuitively, this makes sense, because as a party's information (signal) becomes more accurate (as σ approaches zero), it will rely more heavily on its signal about this particular dispute and less on information about disputes more generally.

PROPOSITION 1: PRIEST-KLEIN HYPOTHESES UNDER THE BAYESIAN MODEL. *Suppose the Priest-Klein model is modified to include the Bayesian correction, the Irrelevance of Dispute Distribution, Fifty-Percent Limit and Asymmetric Stakes Hypotheses are valid as stated in Section 2, and the Fifty-Percent Bias Hypothesis is valid for sufficiently small σ.*

Although this modified approach corrects a weakness in Priest and Klein's original model, it has some implications that may be unappealing. Under the modified model, the parties' estimates are no longer unbiased. For example, suppose the distribution of disputes is standard normal and true case merit is $Y' = 1.5$. Although the parties receive signals that average 1.5 (because the signals are unbiased), they will take into account the fact that the standard normal is centered at zero, so their estimates will, on average, be somewhat less than 1.5. This assumption differs from nearly all the literature on suit and settlement, including Priest and Klein's model. In addition, it seems unrealistic to think that experienced litigators would be unable to formulate unbiased estimates of the plaintiff's probability of prevailing. Furthermore, it seems odd that both parties' estimates would deviate from the true value in the same direction.

A consequence of these biased estimates is that the most heavily litigated disputes will not be distributed around the decision standard, Y^*. For example, in the case of normal distributions centered at 0, given a decision standard Y^*, the disputes most heavily litigated will be centered around $(1 + \sigma^2)Y^*$ rather than Y^*. This contradicts an aspect of Priest and Klein's model that many readers found intuitive and realistic—that disputes lying closest to the decision standard would be most heavily litigated. The underlying logic was that the uncertainty as to who would win would be greatest for such disputes and that differences in the parties' estimates of their probability of prevailing would also be largest there. Under the modified approach, however, litigated disputes will tend to be farther away from the mean than the decision standard. For large σ values, the center of litigated disputes will in fact be far from the decision standard. This means that, when the legal standard favors the plaintiff, $Y^* < 0$, litigated cases may more often be those the defendant will win. Conversely, when the legal standard favors the defendant, $Y^* > 0$, litigated cases may more often be those the plaintiff will win. Finally, for large σ values, there are still other results, that seem implausible. Simulation results using normal distributions reveal that, for sufficiently large (but plausible) σ values, the plaintiff's win rate will be monotonically *increasing* as the decision standard increases (and thus becomes more defendant-friendly). This is in tension Klerman and Lee (2014), which found that, under plausible conditions and parameter assumptions, inferences regarding the legal standard of liability are generally possible under Priest and Klein's original model as well as under screening and signaling models.

4. Priest-Klein Model with an Incentive-Compatible Mechanism

Priest and Klein's model has been criticized because it does not include an incentive-compatible bargaining mechanism. Instead, Priest and Klein assume that whenever settlement would be in the parties' perceived mutual interest, they will successfully bargain to a settlement. That is, they assume that the Landes-Posner-Gould condition, $(P_p - P_d)J > C - S$, is a necessary as well as sufficient condition for litigation. In doing so, they follow the lead of other early analyses of settlement, which, although they generally recognized that the Landes-Posner-Gould condition was only a sufficient condition, often assumed for the purposes of analysis that it was both necessary and sufficient. (Landes 1971, p. 66; Posner 1973, pp. 417-18; Gould 1973, p. 284) Put another way, like many other pioneers of law and economics, Priest and Klein implicitly assume the existence of an *ex post* efficient[5] bargaining mechanism through which the parties would always be able to settle when doing so was in their perceived mutual best interest.

Nevertheless, modern research in bargaining and mechanism design has reached the pessimistic conclusion that, when there is asymmetric information, such an efficient mechanism may not exist. Myerson and Satterthwaite (1983). On the other hand, the Myerson and Satterthwaite theorem does not apply to the Priest-Klein model, because the parties' estimates are not independent and type spaces are infinite. McAfee and Reny (1992) suggest that in such cases an incentive-compatible *ex post* efficient trading may be possible if there is an outside broker—a budget balancer. Nevertheless, settlement negotiations seldom if ever employ an outside broker who contributes his or her own money, nor has anyone proposed or implemented any other kind of efficient mechanism for settlement. Consequently, it is worth investigating whether the validity of the Priest-Klein hypotheses would be affected by relaxing the assumption that the Landes-Posner-Gould litigation condition is both necessary and sufficient, and instead assuming a plausible (albeit inefficient) bargaining mechanism.

Like Friedman and Wittman (2007), we investigate the implications of the Chatterjee-Samuelson mechanism. Under that mechanism, plaintiff and defendant each submit secret offers to a neutral party (or computer). If the plaintiff's offer is greater than the defendant's offer, the case goes to trial. If the plaintiff's offer is less than or equal to the defendant's offer, then the case settles for the average of the two offers. Although the Chatterjee-Samuelson mechanism is seldom used in actual litigation, it can be seen as a "reduced form of a more complicated but unspecified haggling between the plaintiff and defendant lawyers." (Friedman and Wittman (2007), p. 110). We diverge from Friedman and Wittman (2007) in that we graft the Chaterjee-Samuelson mechanism onto the Priest-Klein model rather than constructing a completely new model.

We begin by making two simplifying assumptions. First, as in asymmetric information models, Bechuk's (1984) and Reinganum and Wilde's (1986), we assume settlement costs are zero, $S = S_p = S_d = 0$. Second, we assume ϵ_p and ϵ_d are distributed independently and according to joint probability distribution $f_{\sigma,\sigma}(\epsilon_p, \epsilon_d) = f_\sigma(\epsilon_p) f_\sigma(\epsilon_d)$ such that $f(x) = \sigma f_\sigma(\sigma x)$, $f(x)$ is symmetric around 0, and $F[x]$ is the corresponding standard cumulative distribution.

The mechanism proceeds as follows. The plaintiff receives a signal $Y_p = Y' + \epsilon_p$ and the defendant receives a signal $Y_d = Y' + \epsilon_d$. Then, the plaintiff makes a secret settlement demand,

[5] The literature uses "*ex post* inefficiency" to refer to cases where there is a mutually beneficial agreement but the parties fail to reach it. In the litigation-settlement model, there is always *ex post* inefficiency, because the parties would always be better off settling for the judgment amount and thus saving all litigation costs. Thus, the relevant inefficiency must be relative to the parties' *ex ante* evaluation of the situation. That is, there is inefficiency if a settlement would have made both parties think they were better off, *given their ex ante evaluations of the case*.

$p(Y_p)$, and the defendant makes a secret settlement offer, $d(Y_d)$. If $p \leq d$, the parties settle at $\frac{p+d}{2}$; otherwise, parties litigate.

Like Friedman and Wittman (2007), we consider pure strategies contingent on the realized signal. Thus, a plaintiff's strategy is a measurable function $p(\cdot)\colon \boldsymbol{R} \to \boldsymbol{R}^{+}$ that assigns the demand $p = p(Y_p) \in [0, \infty)$ when it observes signal Y_p. Similarly, a defendant's strategy is a measurable function $d(\cdot)\colon \boldsymbol{R} \to \boldsymbol{R}$ that assigns the demand $d = d(Y_d) \in (-\infty, \infty)$ when it observes signal Y_d. The objective of the plaintiff is to maximize expected net payments, conditioned on its realized signal Y_p and the defendant's strategy $d(\cdot)$. The defendant's object is to minimize expected net payments.

There are several important differences, however, from Friedman and Wittman (2006). First, the support for the strategy functions is the entire real line. This means that we cannot work with uniform distributions, and consequently, we do not limit our attention to piecewise linear continuous equilibria. Second, the signals Y_p and Y_d are not independent. Instead, the plaintiff makes inferences about the distribution of Y_d based on Y_p, and likewise for the defendant. This complicates the analysis. The plaintiff estimates Y_d using a compound distribution: it first figures out the conditional distribution of Y' given Y_p, and then conditions the expected distribution of Y_d on its expected conditional distribution of Y'. The defendant does likewise. Third, if the parties cannot settle, the parties' respective payoffs at trial are determined by the true value of Y'. In contrast, Friedman and Wittman assume the trial judgment lies halfway between the plaintiff's demand and the defendant's offer

Since the plaintiff and the defendant are considered to be playing a different game for each $\sigma > 0$, we index their strategies by σ and refer to each game as the "σ-game." The payoff function for the plaintiff in the σ-game is:

$$\begin{aligned}
&\Pi^{p}\big(p, Y_p, d(Y_d;\sigma);\sigma\big) \\
&\qquad = \int_{\{Y_d \mid p \leq d(Y_d;\sigma)\}} \left(\frac{d(Y_d;\sigma) + p}{2}\right) f_\sigma\big(Y_d \big| Y_p\big) dY_d \\
&\qquad + \int_{\{Y_d \mid p > d(Y_d;\sigma)\}} \big(\alpha J P\big(Y' \geq Y^* | Y_p, d(Y_d;\sigma)\big) - C_p\big) f_\sigma\big(Y_d \big| Y_p\big) dY_d
\end{aligned}$$

The first-term in the right-hand side is the expected value of settling, and the second-term is the expected value of litigating. $f_\sigma\big(Y_d \big| Y_p\big)$ and $f_\sigma\big(Y_p \big| Y_d\big)$ represent the conditional distribution of the other party's signal given the party's own signal. Likewise, the payoff for the defendant is:

$$\Pi^d\big(d, Y_d, p(Y_p;\sigma);\sigma\big) = \int_{\{Y_p | p(Y_p;\sigma) \le d\}} \left(\frac{d + p(Y_p;\sigma)}{2}\right) f_\sigma(Y_p | Y_d) dY_p + \int_{\{Y_p | p(Y_p;\sigma) > d\}} \big(JP(Y' \ge Y^* | Y_d, p(Y_p;\sigma)) + C_d\big) f_\sigma(Y_p | Y_d) dY_p$$

A Nash equilibrium of the σ-game is defined as follows:

Definition. A *Nash equilibrium* (NE) of the σ-game is a strategy pair $\big(p(Y_p;\sigma), d(Y_d;\sigma)\big)$ such that $p(Y_p;\sigma) = \text{argmax}_p \Pi^p\big(p, Y_p, d(Y_d;\sigma);\sigma\big)$ and $d(Y_d;\sigma) = \text{argmin}_d \Pi^d\big(d, Y_d, p(Y_p;\sigma);\sigma\big)$.

For hypotheses that pertain to results in the limit, we consider *continuous families* of Nash equilibria $\big(p(Y_p;\sigma), d(Y_d;\sigma)\big)$ as σ approaches zero.

Definition. A *continuous family* of Nash equilibria is a set of Nash equilibrium strategy pairs $\big(p(Y_p;\sigma), d(Y_d;\sigma)\big)$ defined for each $\sigma \in (0, \bar{\sigma})$ for some $\bar{\sigma} > 0$ such that $p(Y_p;\sigma)$ and $d(Y_d;\sigma)$ are both continuous in σ. Given a continuous family, a *limit equilibrium* is a pair of strategies, $\big(p(Y_p;0), d(Y_d;0)\big)$, such that the following conditions hold true:

- $p(Y_p;0) = \lim_{\sigma\to 0} p(Y_p;\sigma)$,
- $d(Y_d;0) = \lim_{\sigma\to 0} d(Y_d;\sigma)$,
- $p(Y_p;0) = \text{argmax}_p \lim_{\sigma\to 0} \Pi^p\big(p, Y_p, d(Y_d;\sigma);\sigma\big) = \text{argmax}_p \lim_{\sigma\to 0} \Pi^p\big(p, Y_p, d(Y_d;0);\sigma\big)$, and
- $d(Y_d;0) = \text{argmin}_d \lim_{\sigma\to 0} \Pi^d\big(d, Y_d, p(Y_p;\sigma);\sigma\big) = \text{argmin}_d \lim_{\sigma\to 0} \Pi^d\big(d, Y_d, p(Y_p;0);\sigma\big)$.

Therefore, a *limit equilibrium* is the limit (as σ approaches 0) of a continuous family of Nash equilibria defined over $\sigma \in (0, \bar{\sigma})$, and is itself a Nash equilibrium of the σ-game in the limit. Note first that, as Friedman and Wittman (2007) observed, existence of a Nash equilibrium is not guaranteed even for a fixed σ value because we are restricting our analysis to pure strategy Nash equilibria. Even less obvious is the existence of a limit equilibrium. Nevertheless, in the analysis contained in this Section and the Appendix, we show that regardless of the shape of the dispute distributions, $g(Y')$, there are always at least four distinct classes of limit equilibria, each one consisting of an infinite number of limit equilibria. Given a limit equilibrium, we can readily calculate the plaintiff trial win rate in the limit.

Friedman and Wittman (2007) limit their substantive analysis to *symmetric* Nash equilibria, and we do so as well in this Section.[6] Their definition of symmetry would translate to our set-up as follows:

> *Definition*. The strategies $p(Y_p;\sigma)$ and $d(Y_d;\sigma)$ are *symmetric around* Y^* if there exists some $K > 0$ such that, for all Y, we have $p(Y;\sigma) = K - d(2Y^* - Y;\sigma)$, or equivalently, $d(Y;\sigma) = K - p(2Y^* - Y;\sigma)$.

It turns out that for all *limit equilibria* that are symmetric around Y^*, the Fifty Percent Limit Hypothesis will hold true. In fact, if a limit equilibrium is symmetric, the win rate will approach fifty percent regardless of α. As a result, the Asymmetric Stakes Hypothesis does not hold for symmetric limit equilibria.

The Proposition below summarizes our results for symmetric limit equilibria:

PROPOSITION 2: PRIEST-KLEIN HYPOTHESES UNDER A MODEL WITH AN INCENTIVE-COMPATIBLE MECHANISM AND SYMMETRIC LIMIT EQUILIBRIA. *Under the assumptions for the hypotheses set out at the end of Section 2, and in addition assuming* $S = S_p = S_d = 0$ *and* ϵ_p *and* ϵ_d *are distributed with mean zero according to* $f_{\sigma,\sigma}(\epsilon_p, \epsilon_d) = f_\sigma(\epsilon_p) f_\sigma(\epsilon_p)$ *such that* $f(x) = f_\sigma(\sigma x)$ *with full support over* $\boldsymbol{R}^2$, *and* $f(x)$ *is symmetric around 0 and is continuously differentiable*, *then, under the Priest-Klein model with Chatterjee-Samuelson bargaining, we have the following results:*

- *PRIEST-KLEIN HYPOTHESES FOR SYMMETRIC LIMIT EQUILIBRIA. For all families of Nash equilibria that are symmetric in the limit, (i) the Trial Selection Hypothesis, the Irrelevance of Dispute Distribution Hypothesis, and the Fifty-Percent Limit Hypothesis are valid, (ii) the Fifty-Percent Bias Hypothesis is valid for sufficiently small* σ, *and (iii) the Asymmetric Stakes Hypothesis is false. In addition, these results remain valid even when parties use Bayes' rule and their knowledge of the distribution of disputes to estimate the plaintiff's probability of prevailing and to formulate their offers.*

- *EXISTENCE OF A SYMMETRIC LIMIT EQUILIBRIUM. There exists at least one symmetric limit equilibrium for* α *sufficiently close to 1. In particular, there exists a symmetric 2-step limit equilibrium in which the plaintiff and the defendant both offer the same two (high and low) settlement values, but at two different thresholds, which are symmetric around 0 (when scaled by* σ *and normalized by* Y^**). Specifically, there exists* $\varepsilon > 0$ *and some suitable value* $P(\alpha) \in (0,1)$ *such that for each* $\alpha \in (1-\varepsilon, 1+\varepsilon)$ *the following condition*

[6] We explore asymmetric equilibria in the Appendix.

holds: there exists $\sigma_0 > 0$ *such that for all* $\sigma \in (0, \sigma_0)$, *there will generally*[7] *exist a pair of continuous functions* $\left(\gamma_p(\sigma), \gamma_d(\sigma)\right): \boldsymbol{R}^+ \to \boldsymbol{R}^2$ *such that* $\lim_{\sigma \to 0^+} \gamma_p(\sigma) = \gamma = \lim_{\sigma \to 0^+} \gamma_d(\sigma)$ *and for each* $\sigma \in (0, \sigma_0)$, *the following* $\left(p(Y_p; \sigma), d(Y_d; \sigma)\right)$ *is a Nash equilibrium in the* σ*-game and its limit is a limit equilibrium:*[8]

$$p(Y_p; \sigma) = \begin{cases} J(\alpha + (\alpha - 1)P(\alpha)) - C_p + C_d \text{ for } Y_p \geq Y^* - \sigma\gamma_p(\sigma) \\ 0 \text{ for } Y_p < Y^* - \sigma\gamma_p(\sigma) \end{cases}$$

$$d(Y_d; \sigma) = \begin{cases} J(\alpha + (\alpha - 1)P(\alpha)) - C_p + C_d \text{ for } Y_d \geq Y^* + \sigma\gamma_d(\sigma) \\ 0 \text{ for } Y_d < Y^* + \sigma\gamma_d(\sigma) \end{cases}$$

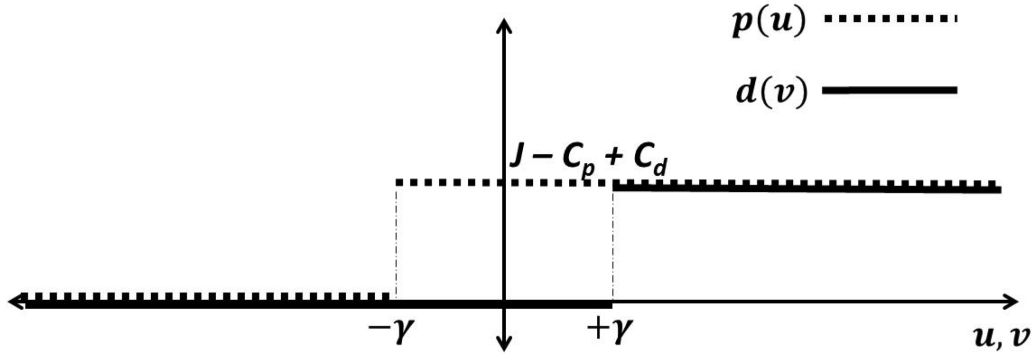
In addition, this symmetric limit equilibrium exists even when parties use Bayes' rule.

The proof of Proposition 2 is in the Appendix. The general result can be seen by applying the same change of variables as before and looking at the symmetry of the region of integration (along the line $v = -u$) when the parties use symmetric strategies. Here we provide a very brief sketch as to how to construct a two-step symmetric limit equilibrium. We begin by establishing the existence of a suitable γ value for $\alpha = 1$. This is given by first writing down the condition under which the plaintiff's best response condition and the defendant's best response condition would collapse to one condition, and then applying the Intermediate Value Theorem to it. Second, by continuity of the equations in α, we can also conclude that there is some neighborhood around $\alpha = 1$ for which we can similarly find appropriate $\gamma(\alpha)$ values that serve as a symmetric threshold in the limit even for $\alpha \neq 1$. Third, the existence of a pair of continuous functions $\gamma_p(\sigma)$ and $\gamma_d(\sigma)$, each converging to γ, is guaranteed by the Implicit Function Theorem as long as the Jacobian does not vanish at $\sigma = 0$. The Jacobian does not vanish and is indeed nonzero when simulated with normal distributions. Finally, given this region of integration, we can easily derive results for all hypotheses pertaining to limit results.

Figure 2 depicts the symmetric limit equilibrium strategies for $\alpha = 1$.

[7] We say "generally" because the proof makes use of the Implicit Function Theorem and thus will depend on a particular Jacobian not taking on the value of zero at the particular equilibrium value. Because the particular Jacobian is not identically zero, this will generally be the case, although it may be possible to *construct* an example in which the Jacobian can take on the value of zero at the particular equilibrium point. Calculation using *Mathematica* confirmed that the Jacobian is indeed nonzero for normal distributions.

[8] The stability of this Nash equilibrium (over thresholds) was checked for normal distributions using *Mathematica* simulations.



Plaintiff demands and defendant offers zero, when the case is below the decision standard by more than γ. Similarly, plaintiff demands and defendant offers $J - C_p + C_d$ when the case above the decision standard by more than γ. The plaintiff demand exceeds the defendant offer only near the decision standard. Thus, the parties settle cases far from the decision standard and litigate only cases close to the decision standard. The validity of Fifty-Percent Limit Hypothesis follows.

Figure 3 depicts the corresponding litigation set.

Figure 2. Symmetric Limit Equilibrium ($\boldsymbol{\alpha = 1}$)[9]

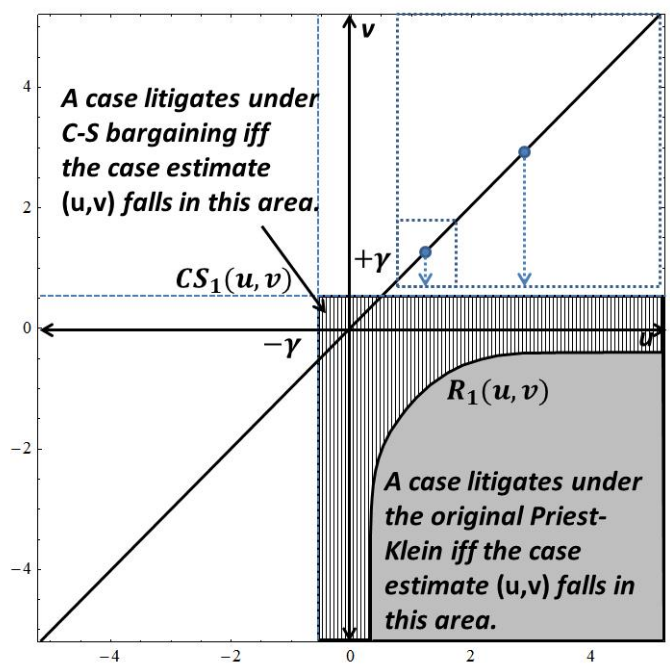


Figure 3. Litigation Set under Bargaining ($\boldsymbol{\alpha = 1}$)

[9] Note that, even though for legibility, the graph makes it look like $p(u)$ is slightly higher than $d(v)$ to the left of $-\gamma$ and to the right of $+\gamma$, in fact the two are at exactly the same height in these regions.

$CS_1(u,v) = \{(u,v)|u > -\gamma, v < \gamma\}$ is the set of cases litigated in the uv -plane when the parties use the Chatterjee-Samuelson mechanism as σ approaches zero. In the case depicted above, $R_1(u,v) = \left\{(u,v)\middle|\alpha F[u] - F[v] > \frac{C}{J}\right\}$, the set representing litigated disputes under the original model, is properly contained in $CS_1(u,v)$,[10] and the set $CS_1(u,v) \cap R_1(u,v)^C$ can be considered the region of *ex post* inefficiency.

It is somewhat surprising that, under the Chatterjee-Samuelson mechanism, the plaintiff trial win rate will be fifty-percent in the limit, even with asymmetric stakes ($\alpha \neq 1$). The intuition behind this result is that for $\alpha \neq 1$ but sufficiently close to 1, even though $R_\alpha(u,v)$ will be asymmetric with respect to the line $v = -u$, we will be able to find a symmetric limit equilibrium such that $CS_\alpha(u,v) = \{(u,v)|u > -\gamma(\alpha), v < \gamma(\alpha)\}$ will be symmetric with respect to the line $v = -u$.

It is important to note that the symmetric equilibria explored above are not the only equilibria. For example, there is the trivial class of equilibria in which all cases go to trial because plaintiff's demand is absurdly high and defendant's offer is unreasonably low. Friedman and Wittman (2007). In addition, the Appendix discusses two additional classes of asymmetric equilibria (Propositions A1 and A2). Most of the Priest-Klein hypotheses are false under these equilibria.

5. Asymmetric Information

A. Priest-Klein Model and Asymmetric Information

Although Priest and Klein (1984) did not discuss asymmetric information, their model can be used to explore asymmetric information. This can be done by varying, β, the parameter that indicates how much more or less accurate the plaintiff is than the defendant in estimating Y'. That is, $\beta = \sigma_d/\sigma_p$, where the parties predict case merit, Y', with errors ϵ_p and ϵ_d that are distributed with mean zero and standard deviations σ_p and σ_d. If $\beta > 1$, then the plaintiff has an informational advantage over the defendant; conversely, if $\beta < 1$, then the defendant has superior information. In some ways, this version of the Priest-Klein model is a more flexible way of modeling asymmetric information than the standard screening and signaling models, because under this version of the Priest-Klein model, information is a continuous variable. Whereas under standard asymmetric information models, one part is fully informed while the other knows only the distribution of all disputes, under the asymmetric information version of the Priest-Klein model, informedness can be varied continuously by adjusting σ_p and σ_d. Large

[10] It seems likely that this is true generally, because it would be odd for cases to settle under the Chatterjee-Samuelson mechanism that would not settle under an *ex post* efficient mechanism (as implicitly assumed by Priest and Klein). Nevertheless, we were not able to prove this result in greater generality. Because we are assuming settlement costs are zero in this section, we plotted $R_1(u,v)$ under the assumption that $\frac{C}{J} = \frac{2}{3}$.

σ_p means the plaintiff has practically no information, very small σ_p means that the plaintiff is very well informed, and intermediate values mean intermediate levels of information. σ_d similarly indicates the degree to which the defendant is informed.

If plaintiff and defendant differ in their ability to predict the merit of the case, then, even when $f_{\sigma_p,\sigma_d}(\epsilon_p, \epsilon_d)$ is bivariate normal, the joint probability function will not be symmetric in ϵ_p and ϵ_p and thus the limit value will not equal to fifty percent (except by coincidence). Instead, the plaintiff trial win rate will vary predictably with the information asymmetry. When the stakes are equal, (i.e. α=1), the party that can predict trial outcomes more accurately will win more often at trial. This leads to the following Proposition.

PROPOSITION 3: PRIEST-KLEIN MODEL AND ASYMMETRIC INFORMATION. *Under the assumptions for the hypotheses set out at the end of Section 2 except $\beta \neq 1$:*

- *If $\frac{C-S}{J} < \alpha < 1 + \frac{C-S}{J}$, the plaintiff trial win rate is (comparatively) higher the more (comparatively) accurately plaintiff can estimate the true Y', and vice versa. In other words, given $\beta_1 < \beta_2$, $\lim_{\sigma \to 0^+} W_{\alpha,\sigma,\beta_1\sigma}(Y^*) < \lim_{\sigma \to 0^+} W_{\alpha,\sigma,\beta_2\sigma}(Y^*)$. Furthermore, as β approaches infinity (that is, plaintiff has full-knowledge), the plaintiff trial win rate will approach one; and as β approaches zero (that is, defendant has full-knowledge), the plaintiff trial win rate will approach zero. If $\alpha > 1 + \frac{C-S}{J}$, the plaintiff trial win rate is one, so it does not vary with β. When $\alpha \leq \frac{C-S}{J}$, all cases settle, so the plaintiff trial win rate is undefined. If $\alpha = 1 + \frac{C-S}{J}$, $\lim_{\sigma \to 0^+} W_{\alpha,\sigma,\beta_1\sigma}(Y^*) < \lim_{\sigma \to 0^+} W_{\alpha,\sigma,\beta_2\sigma}(Y^*)$* or $\lim_{\sigma \to 0^+} W_{\alpha,\sigma,\beta_1\sigma}(Y^*) = \lim_{\sigma \to 0^+} W_{\alpha,\sigma,\beta_2\sigma}(Y^*) = 1$.

- *The Irrelevance of Dispute Distribution Hypothesis is true, but the Fifty-Percent Limit, Asymmetric Stakes, and Fifty-Percent Bias Hypotheses are false.*

The first part of the proposition is proved in the Appendix. The validity of the Irrelevance of Dispute Distribution Hypothesis for asymmetric information ($\beta \neq 1$) was proved in Lee and Klerman (2015), because the proof in that paper did not assume $\beta = 1$. The falsity of the Fifty-Percent Limit, Asymmetric Stakes, and Fifty-Percent Bias Hypotheses follows from the first part of the proposition. If the plaintiff trial win rate is not fifty-percent in the limit, the Fifty-Percent Limit Hypothesis is obviously false. To see the falsity of the Asymmetric Stakes hypothesis, consider α a little less than one, but β as it approaches infinity. According to the first part of Proposition 3, the plaintiff trial win rate approaches one, even though according to the Asymmetric Stakes Hypothesis, the plaintiff trial win rate should be less than fifty percent. The falsity of the Fifty-Percent Bias hypothesis is also apparent when one considers small σ as β approaches infinity. Under these circumstances, if β is large enough, the plaintiff trial win rate can be arbitrarily close to one, so the plaintiff trial can always be farther from fifty-percent than the plaintiff trial win rate if all cases were litigated (unless, of course, the full distribution of cases contained only cases plaintiff would win or only cases the plaintiff would lose).

B. Priest-Klein Hypotheses under Standard Screening and Signaling Models

Proposition 3 is similar to results under the standard screening and signaling models. To establish that similarity we explore selection effects under the standard asymmetric information models and establish which of the Priest-Klein hypotheses are valid under the canonical screening and signaling models. Although no one has systematically explored all of the Priest-Klein hypotheses under the standard asymmetric information models, it is relatively easy to show that all but the Trial Selection Hypothesis are false.

The canonical asymmetric information models are Bechuk's (1984) screening model and Reinganum and Wilde's (1986) signaling model. There has been surprisingly little work on selection under these models. Hylton (1993) and Shavell (1996) showed that Priest and Klein's prediction that plaintiffs will generally win fifty-percent of litigated cases is false under the screening model. Wicklengren (2013) showed that, under both the screening and signaling models, the informed party wins a greater fraction of the litigated cases than if all cases had gone to trial. Klerman and Lee (2014) showed that the No Inferences Hypothesis is false under both the screening and signaling models.

To explore selection effects under the signaling model, Reinganum and Wilde's (1986) model must be modified so that the parties disagree about the probability that the plaintiff wins rather than about damages. See Klerman and Lee (2014); Wickelgren (2013 p. 342). Under Bebchuk's (1984) screening model and the modified Reinganum and Wilde (1986) signaling model, the proportion of litigated cases won by the party with the informational advantage will be larger than the proportion of cases it would have won if all cases were litigated. See Wickelgren (2015, p. 344). The prediction of Proposition 2, however, is more extreme, because it predicts that a party that knows its type (the true Y'), will win 100 percent of the time. This will not generally be true under standard asymmetric information models. This extreme prediction follows from the fact that, in Priest and Klein's model, a person's type is a real-number representation of the merits of the case, and the real line is divided into two halves: in one part the plaintiff wins with certainty, and in the other, the defendant wins with certainty. Thus, if a party knows his type, he can predict the outcome with certainty and thus will refrain from litigation if the outcome is a certain loss for him. In contrast, in the standard asymmetric information models, party type is only the *probability* that the plaintiff will prevail, which ranges from zero to one and can take on all intermediate values. As a result, for such models, even if a party knows its type, it knows only the probability with which it will prevail, which is in most cases neither zero nor one.

Other than the extreme implications of the Priest-Klein model discussed in in the previous paragraph, the selection implications of the modified Priest-Klein model and canonical asymmetric information models are similar. The fact noted above, that the informed party wins more often in litigated cases than if all cases were litigated, is sufficient to show that the Trial Selection Hypothesis is true under standard asymmetric information models.

The falsity of the Fifty Percent Bias Hypothesis under these models also follows from the fact that selection favors the informed party. That is, it is not true that the plaintiff trial win rate will be closer to fifty percent than the plaintiff win rate if no cases settled. When defendants have superior information, the plaintiff trial win rate will be lower than the no-settlement plaintiff win rate. That is, the bias is toward zero. Thus, if the no-settlement plaintiff win rate were less than fifty percent, the plaintiff trial win rate would be *farther* from fifty percent than the no-settlement plaintiff win rate. If the no-settlement plaintiff win rate were greater than fifty percent, the plaintiff trial win rate might be closer to fifty percent than the distribution of all disputes, but it might also be so close to zero that it would be farther from fifty percent than the no-settlement win rate. Conversely, when the plaintiff has superior information, the plaintiff trial win rate will be higher than the no-settlement plaintiff win rate. That is, the bias would be toward 100 percent.

The remaining three hypotheses – the Irrelevance of Dispute Distribution Hypothesis, the Fifty-Percent Limit Hypothesis, and the Asymmetric Stakes Hypothesis – are all hypotheses about the limit of the plaintiff trial win rate as both parties become increasingly accurate in predicting trial outcomes. Limit hypotheses are not directly applicable to the canonical asymmetric information models, because in such models one party has full information and the other party knows only the distribution of types. Nevertheless, the limit hypotheses can be meaningfully reinterpreted as hypotheses about the plaintiff trial win rate generally, without reference to limits. That is, the Irrelevance of Dispute Distribution Hypotheses could be interpreted to mean that the plaintiff trial win rate would not vary with the distribution of all disputes, the Fifty-Percent Limit Hypothesis could be interpreted to mean that the plaintiff trial win rate would be fifty percent when the stakes are symmetric, and the Asymmetric Stakes Hypothesis could be interpreted to mean that the plaintiff trial win rate would be greater or less than fifty percent depending on whether the plaintiff' or defendant's payoff were more affected by a trial victory. All three of these hypotheses are false. Because limits are irrelevant to the asymmetric information models, the rest of this section will call the Fifty Percent Limit Hypothesis simply the Fifty-Percent Hypothesis.

The falsity of the Irrelevance of Dispute Distribution Hypothesis under canonical asymmetric information models was proved in Klerman and Lee (2014). That article explored the implications of changing the legal standard. It interpreted a change in the legal standard as a change in the distribution of disputes and showed that a change in the distribution of disputes led to changes in the plaintiff trial win rate. It proved this result under both the signaling and screening models and for situations both when the plaintiff had superior information and when the defendant had superior information.   The falsity of the Irrelevance of Dispute Distribution Hypothesis under the canonical model contrasts with its validity under the Priest-Klein model with asymmetric information.

The falsity of the Fifty-Percent Hypothesis under canonical asymmetric information models follows directly from the falsity of the Irrelevance of Dispute Distribution Hypothesis. If the percentage of plaintiff trial victories varies with the distribution of all disputes, the plaintiff trial win rate cannot always be fifty percent, even when stakes are equal. This is consistent with Hylton (1993) and Shavell (1996).

The falsity of the Asymmetric Stakes Hypothesis follows from the invalidity of the Fifty-Percent Hypothesis. It is easy to construct examples where the Asymmetric Stakes Hypothesis is false. Suppose, for example, that the defendant has slightly more at stake than the plaintiff, so a judgment for the plaintiff would cost the defendant \$100 but benefit the plaintiff only \$99. Suppose further that defendants have superior information, that half of defendants have a 90 percent chance of losing, while half of defendants have a 60 percent chance of losing. If litigation costs are \$10 for each party, the plaintiff's optimal strategy is to offer \$100 to all defendants. The defendants with a 90 percent chance of losing would accept the settlement offer, but the other defendants would reject. As a result, the plaintiff trial win rate would be 60 percent. This contradicts the Asymmetric Stakes Hypothesis, because that hypothesis would predict that the plaintiff trial win rate would be less than fifty percent, because the defendant had more at stake. It is easy to construct similar examples for the screening model when plaintiff has the informational advantage and for the signaling model.

6. Conclusion

This article updates Priest and Klein's model to correct two problems. We show that most of Priest and Klein's results remain valid when the model is modified to be consistent with Bayes' rule and/or to include an incentive-compatible mechanism. In particular, the Trial Selection Hypothesis, Fifty-Percent Limit Hypothesis, Irrelevance of Dispute Distribution, and Asymmetric Stakes Hypothesis remain valid even if the parties use Bayes' rule to calculate the mean and distribution of case merit. In addition, with the exception of the Asymmetric Stakes Hypothesis, these hypotheses remain valid for symmetric equilibria if the parties use the Chatterjee-Samuelson mechanism. Finally, Priest and Klein's model can be used to explore asymmetric information and to show that, even when parties differ in the accuracy with which they can predict outcomes, only the Trial Selection and Irrelevance of Dispute Distribution Hypotheses remain valid. Asymmetric information results are similar to results under standard screening and signaling models, except that under those models the Irrelevance of Dispute Distribution Hypothesis is false.

Appendix

A.1. Priest-Klein Model with Bayesian Correction

PROOF OF PROPOSITION 1. The proof of this and other propositions build on proofs of the original Priest and Klein hypotheses that are included in Lee and Klerman (2015). We note only that the key insight from Lee and Klerman (2015) was that the region of integration under the changed coordinate, $R_{\alpha}(u,v)$, was invariant under σ. This fact, together with Chebyshev's inequality and the convergence of an infinite series, allowed for a construction of a Lebesgue-integrable dominating function. This then allowed us to take the limits under the integral under Lebesgue's Dominated Convergence Theorem, and the result of the Irrelevance of the Distribution Dispute hypothesis immediately followed. For the Fifty-Percent Limit Hypothesis, we needed only that that the region of integration was symmetric around the line $v=-u$, which turned out to be true when $\alpha=1$ and $F_p[\cdot]=F_d[\cdot]$. For the Asymmetric Stakes Hypothesis, we needed to show only that the region of integration was asymmetric around the line $v=-u$ in the correct direction, so as to yield the limit greater than or less than fifth percent according to whether α was greater than or less than 1.

For the insight behind the proof of Proposition 1, we need to note only that all limit results will go through as long as we can take the limits under the integral pursuant to Lebesgue's Dominated Convergence Theorem. Although the region of integration is no longer invariant under σ in this case, constructing a Lebesgue-integrable dominating function does not actually require the *actual* region of integration to be invariant under σ, but only that the region of integration, for sufficiently small values of σ, can be *contained* in another region of integration that is in fact invariant under σ.

As explained in the main text, the trial condition will be determined by the following inequality:

$$\alpha\left(\frac{\int_0^\infty f_{p,\sigma_p}\left(Y'+\epsilon_p-Y^*-W'\right)g(Y^*+W')dW'}{\int_{-\infty}^\infty f_{p,\sigma_p}\left(Y'+\epsilon_p-Y^*-W'\right)g(Y^*+W')dW'}\right)-\left(\frac{\int_0^\infty f_{d,\sigma_d}(Y'+\epsilon_d-Y^*-W')g(Y^*+W')dW'}{\int_{-\infty}^\infty f_{d,\sigma_d}(Y'+\epsilon_d-Y^*-W')g(Y^*+W')dW'}\right)>\frac{C-S}{J}$$

For a given $\sigma>0$, we employ the following change of variables: $u=\frac{Y\prime+\epsilon_p-Y^*}{\sigma_p}=\frac{Y\prime+\epsilon_p-Y^*}{\sigma}, v=\frac{Y\prime+\epsilon_d-Y^*}{\sigma_d}=\frac{Y\prime+\epsilon_d-Y^*}{\beta\sigma}$. Then by setting the dummy variable ω appropriately, we can rewrite $R_{\alpha,\sigma_p,\sigma_d}(Y';Y^*)$ as $R_{\alpha,\sigma_p,\sigma_d}(Y';Y^*)=\left\{(u,v)\middle|\alpha\varphi_p(u,\sigma)-\varphi_d(v,\beta\sigma)>\frac{C-S}{J}\right\}=R_{\alpha,\sigma,\beta\sigma}(u,v;Y^*)$ where $\varphi_p(u,\sigma)=\frac{\int_{-\infty}^{u} f_{p,1}(\omega)g(Y^*+\sigma(w-\omega))d\omega}{\int_{-\infty}^{\infty} f_{p,1}(\omega)g(Y^*+\sigma(w-\omega))d\omega}$ and $\varphi_d(v,\beta\sigma)=\frac{\int_{-\infty}^{v} f_{d,1}(\omega)g(Y^*+\beta\sigma(w-\omega))d\omega}{\int_{-\infty}^{\infty} f_{d,1}(\omega)g(Y^*+\beta\sigma(w-\omega))d\omega}$.

Therefore, for each $\sigma>0$,

$$P_{\alpha,\sigma_p,\sigma_d}(Y';Y^*)=\iint\limits_{R_{\alpha,\sigma,\beta\sigma}(u,v;Y^*)} f_{1,\beta}\left(u-z,v-\frac{z}{\beta}\right)dudv$$

Now the region of integration in the uv–plane will continue to depend on σ and Y^*. But still, we have

$$\frac{\int_{Y^*}^\infty P_{\alpha,\sigma_p,\sigma_d}(Y';Y^*)g(Y')dY'}{\int_{-\infty}^{Y^*} P_{\alpha,\sigma_p,\sigma_d}(Y';Y^*)g(Y')dY'}=\frac{\int_0^\infty P_{\alpha,\sigma,\beta\sigma}(\sigma z+Y^*;Y^*)g(\sigma z+Y^*)dz}{\int_{-\infty}^0 P_{\alpha,\sigma,\beta\sigma}(\sigma z+Y^*;Y^*)g(\sigma z+Y^*)dz}$$

Notice that there is an absolute lower limit $\underline{u}$ and an absolute upper limit $\bar{v}$ such that, for *each* $\sigma>0$ (where σ can be assumed to be sufficiently small) and for each $Y^*\in\boldsymbol{R}$, $R_{\alpha,\sigma,\beta\sigma}(u,v;Y^*)$ is bounded above by $v=\bar{v}$ and bounded on the left side by $u=\underline{u}$. In this case,

$P_{\alpha,\sigma,\beta\sigma}(\sigma z + Y^*; Y^*) g(\sigma z + Y^*) < g_U \iint_{\{(u,v)|u\geq \underline{u}, v\leq \overline{v}\}} f_{1,\beta}\left(u - z, v - \frac{z}{\beta}\right) dudv$, which will eventually decrease at least as fast as in $|z|^{-2}$ according to Chebyshev's inequality. In this case, we can take the limit inside the integral. Then

$$\lim_{\sigma\to 0^+} \frac{\int_0^\infty P_{\alpha,\sigma,\beta\sigma}(\sigma z + Y^*; Y^*) g(\sigma z + Y^*) dz}{\int_{-\infty}^0 P_{\alpha,\sigma,\beta\sigma}(\sigma z + Y^*; Y^*) g(\sigma z + Y^*) dz} = \frac{\int_0^\infty \lim_{\sigma\to 0^+} P_{\alpha,\sigma,\beta\sigma}(\sigma z + Y^*; Y^*) dz}{\int_{-\infty}^0 \lim_{\sigma\to 0^+} P_{\alpha,\sigma,\beta\sigma}(\sigma z + Y^*; Y^*) dz}$$

since $g(Y')$ is locally continuous at Y^* and $g(Y^*) \neq 0$. Meanwhile

$$\lim_{\sigma\to 0^+} P_{\alpha,\sigma,\beta\sigma}(\sigma z + Y^*; Y^*) = \lim_{\sigma\to 0^+} \iint_{R_{\alpha,\sigma,\beta\sigma}(u,v;Y^*)} f_{1,\beta}\left(u - z, v - \frac{z}{\beta}\right) dudv$$

$$= \iint_{\lim_{\sigma\to 0^+} R_{\alpha,\sigma,\beta\sigma}(u,v;Y^*)} f_{1,\beta}\left(u - z, v - \frac{z}{\beta}\right) dudv$$

Since

$\lim_{\sigma\to 0^+} R_{\alpha,\sigma,\beta\sigma}(u, v; Y^*) =$

$\left\{(u,v) \middle| \alpha \frac{\lim_{\sigma\to 0^+} \int_{-\infty}^u f_{p,1}(\omega) g(Y^*+\sigma(u-\omega)) d\omega}{\lim_{\sigma\to 0^+} \int_{-\infty}^\infty f_{p,1}(\omega) g(Y^*+\sigma(u-\omega)) d\omega} - \frac{\lim_{\sigma\to 0^+} \int_{-\infty}^v f_{d,1}(\omega) g(Y^*+\beta\sigma(v-\omega)) d\omega}{\lim_{\sigma\to 0^+} \int_{-\infty}^\infty f_{d,1}(\omega) g(Y^*+\beta\sigma(v-\omega)) d\omega} > \frac{C-S}{J} \right\}$, and since each integrand is bounded above by $f_1(\omega) g_U$, which is clearly Lebesgue-integrable, we can take the limits inside the integral once again and factor out $g(Y^*)$. And therefore,

$$\lim_{\sigma\to 0^+} R_{\alpha,\sigma,\beta\sigma}(u, v; Y^*) = \left\{(u,v) \middle| \alpha \frac{\int_{-\infty}^u f_{p,1}(\omega) d\omega}{\int_{-\infty}^\infty f_{p,1}(\omega) d\omega} - \frac{\int_{-\infty}^v f_{d,1}(\omega) d\omega}{\int_{-\infty}^\infty f_{d,1}(\omega) d\omega} > \frac{C-S}{J} \right\}$$

$$= \left\{(u,v) \middle| \alpha F_p[u] - F_d[v] > \frac{C-S}{J} \right\} = R_\alpha(u,v)$$

It thus suffices to show that there is an absolute upper limit $\overline{v} > 0$ and an absolute lower limit $\underline{u} < 0$ for $R_{\alpha,\sigma,\beta\sigma}(u, v; Y^*)$, where σ is sufficiently small. We need show that, for sufficiently small σ, as u approaches infinity, the boundary of $R_{\alpha,\sigma,\beta\sigma}(u, v; Y^*)$ does not go to infinity, and as v approaches negative infinity, the boundary does not go to negative infinity. But this is obvious since $\frac{\int_{-\infty}^u f_{p,1}(\omega) g(Y^*+\sigma(u-\omega)) d\omega}{\int_{-\infty}^\infty f_{p,1}(\omega) g(Y^*+\sigma(u-\omega)) d\omega}$ and $\frac{\int_{-\infty}^v f_{d,1}(\omega) g(Y^*+\beta\sigma(v-\omega)) d\omega}{\int_{-\infty}^\infty f_{d,1}(\omega) g(Y^*+\beta\sigma(v-\omega)) d\omega}$, purely as functions defined in terms of variable σ, are continuous in σ at $\sigma = 0$. Since limits can be taken under the integrals, this establishes the results of Propositions 1-3 from Lee and Klerman (2015) – namely, the Irrelevance of Dispute Distribution Hypothesis, the Fifty-Percent Limit Hypothesis, and the Asymmetric Stakes Hypothesis. The Fifty-Percent Bias 4 will also go through for sufficiently small σ, since it is a corollary of the Fifty-Percent Limit Hypothesis when σ is sufficiently small. *Q.E.D.*

A.2. Priest-Klein Model with an Incentive-Compatible Mechanism

PROOF OF PROPOSITION 2. To see the general result for all symmetric limit equilibria, notice first that the region of integration is defined as follows: $CS_1(u,v) = \{(u,v) | \lim_{\sigma\to 0^+} p(\sigma u + Y^*; \sigma) > \lim_{\sigma\to 0^+} d(\sigma v + Y^*; \sigma)\}$. This region will be symmetric around the line $v = -u$ if and only if we can show that whenever $(u,v) \in CS_1(u,v)$, we must also have $(-v,-u) \in CS_1(u,v)$. But if the strategies are symmetric around Y^*, we must have $p(\sigma u + Y^*; \sigma) = K - d(-\sigma u + Y^*; \sigma)$ and $d(\sigma v + Y^*; \sigma) = K - p(-\sigma v + Y^*; \sigma)$. Therefore, we must have $\lim_{\sigma\to 0^+} p(\sigma u + Y^*; \sigma) > \lim_{\sigma\to 0^+} d(\sigma v + Y^*; \sigma)$ if and only if we have $\lim_{\sigma\to 0^+} p(-\sigma v + Y^*; \sigma) > \lim_{\sigma\to 0^+} d(-\sigma u + Y^*; \sigma)$. Therefore, the region of integration is symmetric in the limit, and the logic of Proposition 1 applies. To see that the result is robust to Bayesian correction, we need to recognize only that the Bayesian correction at most changes the region of integration. But in this case, the region of integration is determined not directly by the parties' estimates of the probability that the plaintiff will prevail, but by the (limit) equilibrium strategy. As long as the equilibrium is symmetric in the limit, the region of integration will be symmetric as well.

To understand the symmetric limit equilibrium exhibited in the third part of Proposition 2, we begin by making the usual change of variables. Rewrite the plaintiff's and the defendant's strategies as follows: $p(Y_p;\sigma) = p(u)$ and $d(Y_d;\sigma) = D(v)$. Our strategy is to look for Nash equilibrium strategies in which both parties employ a step function strategy with the same offer amounts but with different cutoff points. Consider the following type of Nash equilibrium. When the case estimate is very low (u and v are both highly negative), then both parties offer 0, so they settle for 0. When the case estimate is very high (u and v are both highly positive), then both parties offer some $H \in (\alpha J - C_p, J + C_d)$, and they settle for Has well. Notice that such H value to exist, we must have $\alpha < 1 + \frac{C}{J}$. We will show the result for the case with the Bayesian correction. The non-Bayesian case will then follow by plugging in $g(Y') = 1$ where appropriate.

Although the plaintiff and the defendant both converge on either 0 or at H, they differ in their cutoff points, which we write as $-\gamma_p(\sigma) < 0$ for the plaintiff and $\gamma_d(\sigma) > 0$ for the defendant. For this pair of strategies to be an equilibrium for a fixed $\sigma > 0$, the plaintiff must be indifferent between demanding 0 and demanding H at $u = -\gamma_p(\sigma)$, and the defendant must be indifferent between offering 0 and offering H at $v = \gamma_d(\sigma)$. It is easy to check that under this set-up, these two indifference conditions will be the necessary and sufficient conditions for the stated pair of strategies to be a Nash equilibrium.

For a given $\sigma > 0$, the defendant is indifferent at $v = \gamma_d(\sigma)$ if and only if

$$\begin{aligned}
&P\left(u < -\gamma_p(\sigma) | v = \gamma_d(\sigma)\right)\left(\frac{H}{2}\right) + P\left(u \geq -\gamma_p(\sigma) | v = \gamma_d(\sigma)\right)(H) \\
&\qquad = P\left(u < -\gamma_p(\sigma) | v = \gamma_d(\sigma)\right)(0) \\
&\qquad + P\left(u \geq -\gamma_p(\sigma) | v = \gamma_d(\sigma)\right)\left(P(Y' \geq Y^* | v = \gamma_d(\sigma), u \geq -\gamma_p(\sigma))J + C_d\right)
\end{aligned}$$

Rewrite this as

$$\begin{aligned}
&P\left(u < -\gamma_p(\sigma) | v = \gamma_d(\sigma)\right)\left(\frac{H}{2}\right) \\
&\qquad + P\left(u \geq -\gamma_p(\sigma) | v = \gamma_d(\sigma)\right)\left(H - P\left(Y' \geq Y^* \middle| v = \gamma_d(\sigma), u \geq -\gamma_p(\sigma)\right)J - C_d\right) = 0
\end{aligned}$$

and call this Condition X_1. For the plaintiff, we have

$$\begin{aligned}
&P\left(v < \gamma_d(\sigma) | u = -\gamma_p(\sigma)\right)(0) + P\left(v \geq \gamma_d(\sigma) | u = -\gamma_p(\sigma)\right)\left(\frac{H}{2}\right) \\
&\qquad = P\left(v < \gamma_d(\sigma) | u = -\gamma_p(\sigma)\right)\left(\alpha P\left(Y' \geq Y^* \middle| u = -\gamma_p(\sigma), v < \gamma_d(\sigma)\right)J - C_p\right) \\
&\qquad + P\left(v \geq \gamma_d(\sigma) | u = -\gamma_p(\sigma)\right)(H)
\end{aligned}$$

Rewrite this as

$$\begin{aligned}
&P\left(v \geq \gamma_d(\sigma) | u = -\gamma_p(\sigma)\right)\left(\frac{H}{2}\right) \\
&\qquad + P\left(v < \gamma_d(\sigma) | u = -\gamma_p(\sigma)\right)\left(\alpha P\left(Y' \geq Y^* \middle| u = -\gamma_p(\sigma), v < \gamma_d(\sigma)\right)J - C_p\right) = 0
\end{aligned}$$

and call it Condition X_2.

At this point, our strategy to constructing this continuous family of Nash equilibria is as follows. We first find out what the limit equilibrium must be if such a continuous family exists. Then we show that such the limit is indeed a Nash equilibrium of the σ-game in the limit. Then we appeal to the Implicit Function Theorem to conclude that there must indeed be continuous families in the small neighborhood around $\sigma = 0$ that satisfy the two indifference conditions.

Therefore, we take the limits of Condition X_1 and Condition X_2 as σ goes to zero. In the limit, we must have the following

$$P(u<-\gamma|v=\gamma)\left(\frac{H}{2}\right)+P(u\geq-\gamma|v=\gamma)(H-P(Y'\geq Y^*|v=\gamma,u\geq-\gamma)J-C_d)=0=$$
$$P(v\geq\gamma|u=-\gamma)\left(\frac{H}{2}\right)+P(v<\gamma|u=-\gamma)\big(\alpha P(Y'\geq Y^*|u=-\gamma,v<\gamma)J-C_p\big).$$

Notice

$$P(u<-\gamma|v=\gamma)=\lim_{\sigma\to0^+}P\left(u<-\gamma_p(\sigma)|v=\gamma_d(\sigma)\right)$$
$$=\lim_{\sigma\to0^+}\left(\frac{\int_{-\infty}^{\infty}g(\sigma z+Y^*)f\big(z-\gamma_d(\sigma)\big)F\big(-\gamma_p(\sigma)-z\big)dz}{\int_{-\infty}^{\infty}g(\sigma z+Y^*)f\big(z-\gamma_d(\sigma)\big)dz}\right)$$
$$=\int_{-\infty}^{\infty}f(z-\gamma)F(-\gamma-z)dz$$

The last equality comes from Lebesgue's Dominated Convergence Theorem since $|g(\sigma z+Y^*)f(z-\gamma_d(\sigma))F(-\gamma_d(\sigma)-z)|\leq g_u f(z-\gamma_d(\sigma))$ which must integrate to $g_u$over all z. Likewise,

$$P(v\geq\gamma|u=-\gamma)=\lim_{\sigma\to0^+}P\left(v\geq\gamma_d(\sigma)|u=-\gamma_p(\sigma)\right)$$
$$=\lim_{\sigma\to0^+}\left(\frac{\int_{-\infty}^{\infty}g(\sigma z+Y^*)f\left(z+\gamma_p(\sigma)\right)F\big(z-\gamma_d(\sigma)\big)dz}{\int_{-\infty}^{\infty}g(\sigma z+Y^*)f\left(z+\gamma_p(\sigma)\right)dz}\right)$$
$$=\int_{-\infty}^{\infty}f(z+\gamma)F(z-\gamma)dz=\int_{-\infty}^{\infty}f(-z+\gamma)F(-z-\gamma)dz=\int_{-\infty}^{\infty}f(z-\gamma)F(-\gamma-z)dz$$
$$=P(u<-\gamma|v=\gamma)$$

In other words, given the symmetry of f, it is clear that $P(u<-\gamma|v=\gamma)=P(v\geq\gamma|u=-\gamma)$ and likewise $P(u\geq-\gamma|v=\gamma)=P(v<\gamma|u=-\gamma)$ (since we have no probability masses). Furthermore, in the limit we will also have $P(Y'\geq Y^*|u=-\gamma,v<\gamma)=1-P(Y'\geq Y^*|v=\gamma,u\geq-\gamma)$. This can be seen as follows:

$$\lim_{\sigma\to0^+}P\left(Y'\geq Y^*\middle|u=-\gamma_p(\sigma),v<\gamma_d(\sigma)\right)=\lim_{\sigma\to0^+}\frac{\int_0^{\infty}g(\sigma z+Y^*)f(z+\gamma_p(\sigma))F(\gamma_d(\sigma)-z)dz}{\int_{-\infty}^{\infty}g(\sigma z+Y^*)f(z+\gamma_p(\sigma))F(\gamma_d(\sigma)-z)dz}$$
$$=\frac{\int_0^{\infty}\lim_{\sigma\to0^+}g(\sigma z+Y^*)f(z+\gamma_p(\sigma))F(\gamma_d(\sigma)-z)dz}{\int_{-\infty}^{\infty}\lim_{\sigma\to0^+}g(\sigma z+Y^*)f(z+\gamma_p(\sigma))F(\gamma_d(\sigma)-z)dz}=\frac{\int_0^{\infty}f(z+\gamma)F(\gamma-z)dz}{\int_{-\infty}^{\infty}f(z+\gamma)F(\gamma-z)dz}$$

Similarly, we have

$$\lim_{\sigma\to0^+}P\left(Y'\geq Y^*\middle|v=\gamma_d(\sigma),u\geq-\gamma_p(\sigma)\right)=\lim_{\sigma\to0^+}\frac{\int_0^{\infty}g(\sigma z+Y^*)f\big(z-\gamma_d(\sigma)\big)F(z+\gamma_p(\sigma))dz}{\int_{-\infty}^{\infty}g(\sigma z+Y^*)f(z-\gamma_d(\sigma))F(z+\gamma_p(\sigma))dz}$$
$$=\frac{\int_0^{\infty}f(z-\gamma)F(z+\gamma)dz}{\int_{-\infty}^{\infty}f(z-\gamma)F(z+\gamma)dz}=\frac{\int_{-\infty}^{0}f(-w-\gamma)F(-w+\gamma)dw}{\int_{-\infty}^{\infty}f(-w-\gamma)F(-w+\gamma)dw}$$
$$=\frac{\int_{-\infty}^{0}f(w+\gamma)F(\gamma-w)dw}{\int_{-\infty}^{\infty}f(w+\gamma)F(\gamma-w)dw}=1-\frac{\int_0^{\infty}f(z+\gamma)F(\gamma-z)dz}{\int_{-\infty}^{\infty}f(z+\gamma)F(\gamma-z)dz}$$

Therefore, in the limit we have $P(Y'\geq Y^*|u=-\gamma,v<\gamma)=1-P(Y'\geq Y^*|v=\gamma,u\geq-\gamma)$. This means that if we choose H such that $H-\left(1-\frac{\int_0^{\infty}f(z+\gamma)F(\gamma-z)dz}{\int_{-\infty}^{\infty}f(z+\gamma)F(\gamma-z)dz}\right)J-C_d=\alpha\left(\frac{\int_0^{\infty}f(z+\gamma)F(\gamma-z)dz}{\int_{-\infty}^{\infty}f(z+\gamma)F(\gamma-z)dz}\right)J-C_p$, then the two conditions collapse into one in the limit. This will be true if $H=J\big(\alpha+(1-\alpha)P(\alpha)\big)+$

$C_d - C_p$ where $P(\alpha) = 1 - \frac{\int_0^\infty f(z+\gamma)F(\gamma-z)dz}{\int_{-\infty}^\infty f(z+\gamma)F(\gamma-z)dz}$. At this point, we show that a suitable $\gamma > 0$ value does exist such that the two conditions (now one) are satisfied:

$$P(u < -\gamma | v = \gamma)\left(\frac{H}{2}\right) + P(v < \gamma | u = -\gamma)\big(\alpha P(Y' \geq Y^* | u = -\gamma, v < \gamma)J - C_p\big)$$

Suppose $\alpha = 1$ so that $H = J + C_d - C_p$. At $\gamma = 0$, this equation is greater than $\left(\frac{1}{2}\right)\left(\frac{J+C_d-C_p}{2}\right) - \left(\frac{1}{2}\right)C_p$, which is positive for reasonable values of C_p and C_d. As γ approaches infinity, the first term approaches 0, and the second term approaches $-C_p$, which is strictly negative. Therefore, by the Intermediate Value Theorem, there must be at least one value of γ for which the equation is true. By continuity, for α close to 1, the same argument shows that such γ must also exist generally.

To establish that this is indeed a Nash equilibrium in the limit, we must also show that for all values of Y_p below some threshold, the plaintiff cannot do better than demanding 0, and for all values of Y_p above the threshold the plaintiff cannot do better than demanding H, and similarly for the defendant. By symmetry, we need only show one party's case. Notice first that, given the plaintiff's strategy, it is never optimal at any Y_d for the defendant to make an offer strictly below 0 because this strategy is strictly dominated by the offer of 0. For all cases that would have litigated had the defendant offered 0, the outcome is the same; but all cases that would have settled had the defendant offered 0, the defendant will incur a minimum loss of C_d. Second, it is also never optimal at any Y_d for the defendant to make a settlement offer that is strictly between 0 and H. This is because the plaintiff is playing by the discontinuous 2-step strategy of playing either 0 or H himself. Therefore, if the defendant were to make a settlement offer strictly between 0 and H, he will end up (i) litigating all the cases he would have litigated had he offered 0 instead (that is, those cases in which the plaintiff observes Y_p above the threshold) but (ii) will also be settling all other cases (that is, those cases in which the plaintiff observes Y_p below the threshold) at a higher settlement value than had he simply offered 0. Therefore he's better off offering 0 than any intermediate value. Third, it is never optimal for the defendant to make a settlement offer strictly greater than H, since that is dominated by an offer of H. Finally, the indifference condition shows that after some threshold, it is superior still for the defendant to offer H than to offer 0. Therefore, the defendant's best response to plaintiff's strategy is the specified 2-step function, and likewise for the plaintiff.

Now we show that there exist a pair of continuous functions $\left(\gamma_p(\sigma), \gamma_d(\sigma)\right): \boldsymbol{R}^+ \to \boldsymbol{R}^2$ such that $\lim_{\sigma\to 0^+} \gamma_p(\sigma) = \gamma = \lim_{\sigma\to 0^+} \gamma_d(\sigma)$ and $\left(p\left(Y_p; \sigma\right), d(Y_d; \sigma)\right)$ is a Nash equilibrium for each σ. Again, by continuity, we need only show these are true for $\alpha = 1$. We rewrite Conditions X_1 and X_2 as follows:

$$X_1(\sigma, x_1, x_2) = \left(\frac{\int_{-\infty}^\infty g(\sigma z + Y^*)f(z - x_2)F(-x_1 - z)dz}{\int_{-\infty}^\infty g(\sigma z + Y^*)f(z - x_2)dz}\right)\left(\frac{J - C_p + C_d}{2}\right)$$
$$+ \left(\frac{\int_{-\infty}^\infty g(\sigma z + Y^*)f(z - x_2)F(x_1 + z)dz}{\int_{-\infty}^\infty g(\sigma z + Y^*)f(z - x_2)dz}\right)\left(\alpha J \frac{\int_0^\infty g(\sigma z + Y^*)f(z - x_2)F(x_1 + z)dz}{\int_{-\infty}^\infty g(\sigma z + Y^*)f(z - x_2)F(x_1 + z)dz}\right.$$
$$\left. - C_p\right) = 0$$

$$X_2(\sigma, x_1, x_2) = \left(\frac{\int_{-\infty}^{\infty} g(\sigma z + Y^*) f(z + x_1) F(x_2 - z) dz}{\int_{-\infty}^{\infty} g(\sigma z + Y^*) f(z + x_1) dz}\right)\left(\frac{J - C_p + C_d}{2}\right) + \left(\frac{\int_{-\infty}^{\infty} g(\sigma z + Y^*) f(z + x_1) F(z - x_2) dz}{\int_{-\infty}^{\infty} g(\sigma z + Y^*) f(z + x_1) dz}\right)\left(\alpha J \frac{\int_{0}^{\infty} g(\sigma z + Y^*) f(z + x_1) F(z - x_2) dz}{\int_{-\infty}^{\infty} g(\sigma z + Y^*) f(z + x_1) F(z - x_2) dz} - C_p\right) = 0$$

Then $K(\sigma, x_1, x_2) = \big(X_1(\sigma, x_1, x_2), X_2(\sigma, x_1, x_2)\big)$ is a continuously differentiable function from $\boldsymbol{R}^3$ to $\boldsymbol{R}^2$ such that $K(0, \gamma, \gamma) = (0,0)$. Then by the Implicit Function Theorem,[11] as long as the Jacobian matrix is invertible at $\sigma = 0$, there is a small neighborhood around $\sigma = 0$[12] for which we can find a unique $\big(\gamma_p(\sigma), \gamma_d(\sigma)\big)$ for each σ such that $K\left(\sigma, \gamma_p(\sigma), \gamma_d(\sigma)\right) = (0,0)$ and $\lim_{\sigma \to 0^+} \gamma_p(\sigma) = \gamma = \lim_{\sigma \to 0^+} \gamma_d(\sigma)$. Thus, we need only check that the Jabobian matrix is invertible at $\sigma = 0$. Looking at the equation, we see that $X_1(\sigma, x_1, x_2) = X_2(\sigma, -x_2, -x_1)$ and the determinant cannot be identically zero since $\left(\frac{\partial X_1(0,x_1,x_2)}{\partial x_1}\right)\left(\frac{\partial X_2(0,x_1,x_2)}{\partial x_2}\right) \neq \left(\frac{\partial X_1(0,x_1,x_2)}{\partial x_2}\right)\left(\frac{\partial X_2(0,x_1,x_2)}{\partial x_1}\right)$. The left-hand side has terms involving mostly $f(x)$'s while the right-hand side has terms involving $F(x)$'s and $f'(x)$'s. Calculation using *Mathematica* confirmed that the Jacobian was indeed not zero when working with normal distributions. Hence, the proposition is proved.

Similar logic can be applied to show that when $\alpha \neq 1$ but close to 1, for a suitable P value, there will continue to exist a similar equilibrium in which they settle at 0 for u and v far less than 0, settle at $J\big(\alpha + (\alpha - 1)P(\alpha)\big) + C_d - C_p$ for u and v far greater than 0, and seek to litigate when the signals are sufficiently closer to 0. Notice this equilibrium settlement values equal the first equilibrium when $\alpha = 1$. Because under the set-up, the two strategies converge to a pair of points symmetric around 0, in this case even with asymmetric stakes, the Fifty-Percent Limit Hypothesis will go through. Meanwhile, since the region of integration in the limit is $CS_1(u, v) = \{(u, v) | u \geq -\gamma, v \leq \gamma\}$, which is symmetric around the line $v = -u$, even for $\alpha \neq 1$, the limit value will be fifty-percent even for such $\alpha \neq 1$. Thus, the Asymmetric Stakes Hypothesis will be false. *Q.E.D.*

PROPOSITION A1: EXISTENCE OF ASYMMETRIC TWO-STEP LIMIT EQUILIBRIA. *Given $\alpha \in (1 - \varepsilon, 1 + C/J)$ such that the symmetric 2-step limit equilibrium exists from Proposition 2, there (generally) exists a family of 2-step limit equilibria (which contains the identified symmetric limit equilibrium) in which the plaintiff and the employ a 2-step strategy of converging at either 0 or at $J(\alpha + (\alpha - 1)P(\alpha)) - C_p + C_d \pm \tau$ where τ is sufficiently small. All other limit equilibria in this family, however, are asymmetric in the limit.*

PROOF OF PROPOSITION A1. By the reasoning of the proof of Proposition 2, it suffices to show the result for $\alpha = 1$. Given the limit equilibrium from Proposition 2, there exists a family of limit equilibria that converge around 0 and $J(\alpha + (\alpha - 1)P(\alpha)) - C_p + C_d \pm \tau$ where τ is sufficiently small. Let

[11] We thank Ken Alexander for suggesting the use of the Implicit Function Theorem to complete this argument.

[12] Although in this model, σ, as standard deviation, is necessarily positive, both $X_1(\sigma, x_1, x_2)$ and $X_2(\sigma, x_1, x_2)$, simply as mathematical functions in three variables σ, x_1, and x_2, are continuous in σ around $\sigma = 0$ and well-defined for $\sigma < 0$ as well.

$$Y_1(H,x_1,x_2) = \left(\int_{-\infty}^{\infty} f(z-x_2)F(-x_1-z)dz\right)\left(\frac{H}{2}\right) + \left(\int_{-\infty}^{\infty} f(z-x_2)F(x_1+z)dz\right)\left(\alpha J \frac{\int_0^{\infty} f(z-x_2)F(x_1+z)dz}{\int_{-\infty}^{\infty} f(z-x_2)F(x_1+z)dz} - C_p\right) = 0$$

$$Y_2(H,x_1,x_2) = \left(\int_{-\infty}^{\infty} f(z+x_1)F(x_2-z)dz\right)\left(\frac{H}{2}\right) + \left(\int_{-\infty}^{\infty} f(z+x_1)F(z-x_2)dz\right)\left(\alpha J \frac{\int_0^{\infty} f(z+x_1)F(z-x_2)dz}{\int_{-\infty}^{\infty} f(z+x_1)F(z-x_2)dz} - C_p\right) = 0$$

Then $M(H,x_1,x_2) = \big(Y_1(H,x_1,x_2), Y_2(H,x_1,x_2)\big)$ is a continuously differentiable function from $\boldsymbol{R}^3$ to $\boldsymbol{R}^2$ such that $K\big(J(\alpha+(\alpha-1)P(\alpha)) - C_p + C_d, \gamma, \gamma\big) = (0,0)$ and the function is defined for all values of H. Then by the Implicit Function Theorem, as long as the Jacobian matrix is invertible at $H = J(\alpha+(\alpha-1)P(\alpha)) - C_p + C_d$, there is a small neighborhood around $H = J(\alpha+(\alpha-1)P(\alpha)) - C_p + C_d$ for which we can find a unique $\big(\gamma_p(H), \gamma_d(H)\big)$ for each H such that $M\left(H, \gamma_p(H), \gamma_d(H)\right) = (0,0)$ and $\lim_{H\to J(\alpha+(\alpha-1)P(\alpha))-C_p+C_d} \gamma_p\big(J(\alpha+(\alpha-1)P(\alpha)) - C_p + C_d\big) = \gamma = \lim_{H\to J(\alpha+(\alpha-1)P(\alpha))-C_p+C_d} \gamma_d\big(J(\alpha+(\alpha-1)P(\alpha)) - C_p + C_d\big)$. Thus, we need only check that the Jabobian matrix is invertible at $H = J(\alpha+(\alpha-1)P(\alpha)) - C_p + C_d$. Looking at the equation, we see that $Y_1(H,x_1,x_2) = Y_2(H,-x_2,-x_1)$ and the determinant, once again, is not *identically zero* since $\left(\frac{\partial Y_1(H,x_1,x_2)}{\partial x_1}\right)\left(\frac{\partial Y_2(H,x_1,x_2)}{\partial x_2}\right)\Big|_{H=J(\alpha+(\alpha-1)P(\alpha))-C_p+C_d} \neq \left(\frac{\partial Y_1(H,x_1,x_2)}{\partial x_2}\right)\left(\frac{\partial Y_2(H,x_1,x_2)}{\partial x_1}\right)\Big|_{H=J(\alpha+(\alpha-1)P(\alpha))-C_p+C_d}$. As before, the left-hand side has terms involving mostly $f(x)$'s while the right-hand side has terms involving $F(x)$'s and $f'(x)$'s. Hence, for each H near $J(\alpha+(\alpha-1)P(\alpha)) - C_p + C_d$, we can find a suitable $\left(\gamma_p(H), \gamma_d(H)\right)$. Finally, for each such triple, $\left(H, \gamma_p(H), \gamma_d(H)\right)$, we can increase σ infinitesimally as well, as in Proposition 2. *Q.E.D.*

PROPOSITION A2: EXISTENCE OF OBSTINATE LIMIT EQUILIBRIA. *For each* $\in \left(\frac{C_p}{J}, 1+\frac{C}{J}\right)$, *there exist an infinite number of obstinate plaintiff and obstinate defendant limit equilibria. Specifically, suppose* $g(Y')$ *is bounded above everywhere and locally continuous and nonzero at* Y^* *and* ϵ_p *and* ϵ_d *are distributed with mean zero according to* $f_{\sigma,\sigma}(\epsilon_p,\epsilon_d) = f_\sigma(\epsilon_p)f_\sigma(\epsilon_p)$ *such that* $f(x) = f_\sigma(\sigma x)$ *with full support over* R^2 *and* $f(x)$ *is symmetric around 0 and is continuously differentiable. Then for* $\alpha \in \left(\frac{C_p}{J}, 1+\frac{C}{J}\right)$, *where* $C = C_p + C_d$, *the following two classes of continuous families of Nash equilibria exist: (i) the "obstinate plaintiff" equilibrium under which,* $p\big(Y_p;\sigma\big) = s_p$ *for all* Y_p *and* σ *where* $\max\{\alpha J - C_p, C_d\} < s_p < J + C_d$, *and* $d(Y_d;\sigma) = -C_p$ *for* $Y_d < {Y_d}^*(s_p;\sigma)$ *and* $d(Y_d;\sigma) = s_p$ *for* $Y_d \geq {Y_d}^*(s_p;\sigma)$ *for some* ${Y_d}^*(s_p;\sigma) \in \boldsymbol{R}$; *and (ii) the "obstinate defendant" equilibrium under which,* $d(Y_d;\sigma) = s_d$ *for all* Y_p *and* σ *where* $0 < s_d < \min\{\alpha J - C_p, C_d\}$, *and* $p\big(Y_p;\sigma\big) = s_d$ *for* $Y_p < {Y_p}^*(s_p;\sigma)$ *and* $p\big(Y_p;\sigma\big) = J + C_d$ *for* $Y_p \geq {Y_p}^*(s_p;\sigma)$ *for some* ${Y_p}^*(s_p;\sigma) \in \boldsymbol{R}$. *For each family,* $p\big(Y_p;\sigma\big)$ *and* $d(Y_d;\sigma)$ *are continuous in* σ, *and* $\left(p\big(Y_p;0\big), d(Y_d;0)\right)$ *is a limit equilibrium. Furthermore, the plaintiff trial win rate is 0 for the obstinate plaintiff equilibrium and 1 for the obstinate defendant equilibrium.*

PROOF OF PROPOSITION A2. Consider the first pair of strategies. $p\big(Y_p;\sigma\big) = s_p$ for all Y_p and σ where $\max\{\alpha J - C_p, C_d\} < s_p < J + C_d$, and $d(Y_d;\sigma) = -C_p$ for $Y_d < {Y_d}^*(s_p;\sigma)$ and $d(Y_d;\sigma) = s_p$ for $Y_d \geq {Y_d}^*(s_p;\sigma)$ for some ${Y_d}^*(s_p;\sigma) \in \boldsymbol{R}$. Notice first that if there exists ${Y_d}^*(s_p;\sigma) \in \boldsymbol{R}$ such that at observing such value as his estimate of the case merit, the defendant is indifferent between playing $-C_p$ or s_p, then the defendant will strictly prefer to play s_p to $-C_p$ for all $Y_d > Y_d{}^*(s_p;\sigma)$ and will likewise strictly prefer to play $-C_p$ over s_p for all $Y_d < Y_d{}^*(s_p;\sigma)$. Second, the defendant's strategy of playing s_p dominates playing any value above s_p since the plaintiff is never asking more than this amount and the defendant in that case has no desire to settle for any value higher than s_p. Third, if the defendant is not playing any value lower than s_p, he is indifferent between playing that value or $-C_p$, since litigation is sure to ensue. Therefore, given the plaintiff's strategy, if there exists a point of indifference $Y_d{}^*(s_p;\sigma)$, then the defendant's strategy is a best-response. Existence of $Y_d{}^*(s_p;\sigma)$ can be seen as follows. Consider the following equation:

$$P(Y' \geq Y^*|Y_d = x)J + C_d = s_p < J + C_d$$

Regardless of σ, as x goes from negative infinity to positive infinity, $P(Y' \geq Y^*|Y_d = x)$ will go from 0 to 1 and there must therefore be a unique x value at which the equality will hold. We can let $Y_d{}^*(s_p;\sigma)$ be that value. Now, we need to prove that the plaintiff's strategy is also a best response to the defendant's strategy. Notice that playing s_p dominates playing any value higher, since the plaintiff can never settle for any amount higher than s_p (given the defendant's strategy), and the plaintiff's litigation value is maximized at $\alpha J - C_p$, which is lower than s_p. But if the plaintiff were to play any value lower than s_p, his chance of litigating remains the same (as if he were to play s_p) but his settlement value, where feasible, will be reduced to the average of his value and s_p. Therefore, the plaintiff's best response is to always play s_p. Since $P(Y' \geq Y^*|Y_d = x)J + C_d$ is continuous in σ, we therefore have a continuous family of Nash equilibria.

Furthermore, if we make a change of variables, we can rewrite the condition as follows:

$$P(Y' \geq Y^*|v = v') = \frac{\int_0^\infty g(\sigma z + Y^*)f(z - v')dz}{\int_{-\infty}^\infty g(\sigma z + Y^*)f(z - v')dz}$$

where $Y' = \sigma x + Y^*$. Then as σ goes to zero, $P(Y' \geq Y^*|v = v')$ approaches $\int_0^\infty f(z - v')dz$, which can take on any value between 0 and 1 depending on v'. Therefore, the corresponding v' that satisfies $P(Y' \geq Y^*|v = v')J + C_d = s_p$ will be uniquely determined, and this pair of strategies will be an equilibrium for the σ-game in the limit. Since $\frac{\int_0^\infty g(\sigma z+Y^*)f(z-v')dz}{\int_{-\infty}^\infty g(\sigma z+Y^*)f(z-v')dz}$ is continuous in σ and for each σ, $Y_d{}^*(s_p;\sigma)$ value determined uniquely, this equilibrium will be a limit of a continuous family of Nash equilibria. It is easy to see that the plaintiff trial win rate in the limit will be 0 in this case, because the region of integration is defined as the area to the left of a vertical line in the uv-plane. A similar argument shows that there is a class of limit equilibria in which the defendant remains obstinate, and the rest follows. *Q.E.D.*

PROOF OF FIRST PART OF PROPOSITION 3. For $\frac{C-S}{J} < \alpha < 1 + \frac{C-S}{J}$, $R_\alpha(u,v)$ is properly contained by a translated fourth quadrant in the uv –plane (with the origin at $(F_p{}^{-1}\left(\frac{C-S}{\alpha J}\right), F_d{}^{-1}(\alpha - \frac{C-S}{J}))$. β affects the slope of the graph $v = u/\beta$ along which to take the integral over v. As β increases, plaintiff is comparatively more accurate in assessing the merit of the case than the defendant. But as β increases, the slope of the line decreases and the portion of the graph $v = u/\beta$ in the first quadrant gets closer to $R_\alpha(u,v)$ and the portion in the fourth quadrant gets farther away from $R_\alpha(u,v)$. Thus, the plaintiff trial win rate increases. As β approaches infinity, the line $v = u/\beta$ will approximate the u–axis and will become parallel to the horizontal asymptote for $R_\alpha(u,v)$. If $\alpha - \frac{C-S}{J} \geq \frac{1}{2}$, then this line will eventually be contained in $R_\alpha(u,v)$ for sufficiently large u, and hence $\int_0^\infty P_{\alpha,1,\beta}(z;0)dz$ will be infinite. Even if the line will not be contained in $R_\alpha(u,v)$, such as when $\alpha - \frac{C-S}{J} < \frac{1}{2}$, the u –axis will be parallel to the asymptote, and $P_{\alpha,1,\infty}(z;0)$ converges a finite value other than zero as z approaches infinity. As z approaches negative infinity, however, $P_{\alpha,1,\infty}(z;0)$ wil approach zero at a rate at least as fast as inverse quadratic. And thus, the plaintiff trial win rate will approach 1. Similarly, as β approaches 0, plaintiff's win rate will approach zero. As discussed in Lee and Klerman (2015)'s Proposition 1, if $\alpha > 1 + \frac{C-S}{J}$, the plaintiff trial win rate is one, so it does not vary with β. Likewise, when $\alpha \leq \frac{C-S}{J}$, all cases settle, so the plaintiff trial win rate is undefined. Finally, if $\alpha = 1 + \frac{C-S}{J}$, the general logic as above goes through, except the path of integration may already be eventually contained in the region of integration since the region of integration will have no horizontal asymptote. The boundary will be increasing along u. So even for finite β values, the limit may already become 1. *Q.E.D.*

References

Atkinson, Scott. 2009. The Economics of a Centralized Judiciary: Uniformity, Forum Shopping, and the Federal Circuit. *Journal of Law and Economics* 52:411-43.

Bagnoli, Mark, and Ted Bergstrom. 2004. Log-concave Probability and Its Applications. *Economic Theory* 26:445-69.

Bebchuck, Lucian Arye. 1984. Litigation and Settlement under Imperfect Information. *RAND Journal of Economics* 15:404-15.

Bernardo, Antonio, Eric Talley, and Ivo Welch. 2000. A Theory of Legal Presumptions. *Journal of Law, Economics and Organization* 15:1-48.

Daughety, Andrew and Jennifer Reinganum. 1994. Settlement Negotiations with Two-Sided Information: Model Duality, Information Distribution, and Efficiency. *International Review of Law and Economics* 14: 283-296.

Friedman, Daniel and Donald Wittman. 2007. Litigation with Symmetric Bargaining and Two-Sided Incomplete Information. *The Journal of Law, Economics, and Organization* 23:98-126.

Gelbach, Jonah. 2012. Locking the Doors to Discovery? Assessing the Effects of Twombly and Iqbal on Access to Discovery. *Yale Law Journal* 121: 2270-2346.

Gould, John P. 1973. The Economics of Legal Conflicts. *Journal of Legal Studies* 2:279-300.

Hubbard, William. 2013. Testing for Change in Procedural Standards, with Application to *Bell Atlantic* v. *Twombly*. *Journal of Legal Studies* 42:35-68.

Hylton, Keith. 1993. Asymmetric Information and the Selection of Disputes for Litigation. *Journal of Legal Studies* 22:187-210

Hylton, Keith N. and Haizhen Lin, 2012. A Formalization and Extension of the Priest-Klein Hypothesis. Unpublished. Available at SSRN: http://ssrn.com/abstract=1864728 or http://dx.doi.org/10.2139/ssrn.1864728.

Klerman, Daniel and Yoon-Ho Alex Lee. 2014. Inferences from Litigated Cases. *Journal of Legal Studies* 43:209-248.

Landes, William M. 1971. An Economic Analysis of the Courts. *Journal of Law and Economics* 14:61-107.

Lee, Yoon-Ho Alex and Daniel Klerman. 2015. The Priest-Klein Hypotheses: Proofs and Generality. Available at SSRN: http://ssrn.com/abstract=2538854.

McAfee, R. Preston and Philip J. Reny. 1992. Correlated Information and Mechanism Design. *Econometrica* 60:395-421.

Myerson, Roger B. and Satterthwaite, Mark A. 1983. Efficient Mechanisms for Bilateral Trading. *Journal of Economic Theory* 29:265-81.

Posner, Richard A. 1973. An Economic Approach to Legal Procedure and Judicial Administration. *Journal of Legal Studies* 2:399-458.

Priest, George and Benjamin Klein. 1984. The Selection of Disputes for Litigation. *Journal of Legal Studies* 13:1-55.

Reinganum, Jennifer F, and Louis L. Wilde. 1986. Settlement, Litigation, and the Allocation of Litigation Costs. *RAND Journal of Economics* 17:557-566.

Schweizer, Urs. 1989. Litigation and Settlement under Two-Sided Incomplete Information. *Review of Economic Studies* 56:163-177.

Shavell, Steven. 1996. Any Frequency of Plaintiff Victory at Trial is Possible, *Journal of Legal Studies* 26:493-501.

Shapiro, Fred and Michelle Pearse. 2012. The Most-Cited Law Review Articles of All Time. *Michigan Law Review* 110:1483-1520.

Siegelman, Peter and John J. Donohue, III. 1995. The Selection of Employment Discrimination Suits for Litigation: Using Business Cycle Effects to Test the Priest-Klein Hypothesis. *Journal of Legal Studies* 24:427-62.

Shapiro, Fred. 1996. The Most-Cited Law Review Articles Revised. *Chicago-Kent Law Review* 71:751-59.

Sobel, Joel. 1989. The Analysis of Discovery Rules. *Law and Contemporary Problems* 52: 133-159.

Waldfogel, Joel. 1995. The Selection Hypothesis and the Relationship between Trial and Plaintiff Victory. *Journal of Political Economy* 103:229-60.

Waldfogel, Joel. 1998. Reconciling Asymmetric Information and Divergent Expectations Theories of Litigation. *Journal of Law and Economics* 41:451-76.

Wickelgren, Abraham L. 2013. Law and Economics of Settlement. Jennifer Arlen, ed., *Research Handbook of Torts*, 330-59. Edward Elgar.

Wittman, Donald. 1985. Is the Selection of Cases for Trial Biased? *Journal of Legal Studies* 14:185-214.