Endogenous Compensation in a Firm with Disclosure and Moral Hazard

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Abstract

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September 30, 2009

Abstract

I model a firm where shareholders choose the manager’s compensation in light of the manager’s dual roles of exerting effort and making disclosures regarding the firm’s value. Because of limited contracting ability and the divergence of short-term interest between shareholder and manager, shareholders may be unable to obtain their first-best choices of effort and disclosure policy; where agency costs are too large, shareholders will be unwilling to award performance-based compensation, which induces both effort and fraudulent reporting. The principal findings are (1) fraud and effort are positively correlated, and given a poor outcome fraud is more likely to obtain when effort is exerted in equilibrium, (2) the incidence of fraud-inducing compensation increases as agency costs decrease, and (3) reductions in agency costs actually increase the incidence of fraud when agency costs are high.

1 Introduction

It is relatively rare that a complaint of securities fraud alleges that management’s motive for the fraud was to steal from the firm in a direct sense, such as actually running off with bags of loot. Rather, the self-serving incentives for fraud are typically thought to be more subtle: “managers hide bad news because they fear loss of their jobs... and they overstate favorable developments or inflate earnings in order to maximize the
value of their stock options and other equity compensation.” Coffee (2006 at 39).\(^1\) Put another way, the manager’s compensation contract is seen to be the principal motivation for committing fraud. That begs the question: why do shareholders choose to award such compensation contracts? And how does this fit with the claim that securities fraud is a product of agency costs, a typical conjecture in the academy?\(^2\)

This paper sheds some light on these questions by examining the relationship between corporate fraud, compensation, and the efficient exertion of managerial effort. The crux of the model is that shareholders designing the compensation of the manager may only be able to get some, not all, of what they want: given the limits of contractibility, shareholders can achieve their preferred level of effort or preferred disclosure policy, though not necessarily both. In particular, performance-based compensation – in the form of an equity share in the company – tends to induce managers not just to exert effort, but also to falsely report in the event that the firm does poorly (i.e., commit fraud).

The model incorporates agency costs in the form of managerial short term interests: the manager may be more likely to sell his equity share than are the shareholders. This captures the commonly expressed concern (for example, as in the Enron prosecution against Ken Lay and Jeffrey Skilling) that managers’ inflated reports of value are designed to increase the value of short term stock compensation. Shareholders are then faced with a tradeoff: while high equity compensation can induce managers to exert themselves, it also leads to a greater degree of fraud than shareholders would desire.

One result of this model is that effort and fraud tend to go together; indeed, fraud in the event of poor firm performance is in a sense more likely to occur when managerial effort has been exerted. The reason is twofold: (1) performance-based compensation rewards the faking of performance, and (2) there is more to lie about in an equilibrium where managers will exert effort and hence have a higher likelihood of high payoffs. Singlemindedly enacting policies, then, that may limit fraud may also have the effect of suboptimally limiting managerial effort. Conversely, singleminded policies that force performance-based compensation will have the effect of also inducing a greater degree of fraud than optimal.

Second, the degree of agency costs influences the compensation that shareholders pay. When the manager’s interests are such that she would commit much more fraud than shareholders desire, shareholders

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\(^1\) Arlen and Carney (1992 at 724-7) and Talley and Johnsen (2004) provide empirical support for this assertion.

\(^2\) Just a few examples are Arlen & Carney (1992), Alexander [ ], Coffee (2006), and the Paulson Committee Report [ ].
may choose not to award equity compensation, which eliminates fraud at the expense of managerial effort. As agency costs decrease, shareholders will eventually choose to award equity compensation, which induces effort and may also induce fraud. The compensation contract offered to the manager, whether or not it induces fraud, maximizes shareholder payoffs given the limited contracting context of the manager-shareholder relationship. Hence, unless regulatory intervention is somehow able to increase the degree of contractibility between manager and shareholder, such interventions are likely reduce shareholder welfare.

Third, contrary to what is sometimes stated in the literature on securities fraud, a reduction in agency costs does not necessarily lead to a reduction in fraud. To the contrary, when agency costs are high, marginal reductions in agency costs will lead to an increase in fraud. This is because shareholders control compensation, and will rein in a manager too predisposed to commit fraud by limiting or eliminating the manager’s equity compensation, which reduces or eliminates the manager’s incentive to commit fraud. If agency costs are relatively low, shareholders will award equity compensation to achieve approximately their preferred policies on effort and disclosure. If fraud is seen as a major problem in and of itself, such as by producing significant externalities, reducing agency costs may be undesirable.

Finally, an additional, though less detailed result, bears on the usage of corporate fines (often referred to as "vicarious liability" in the legal literature, since the firm is vicariously liable for the actions of its managers) as opposed to personal sanctions levied against the manager. To the extent that shareholders desire fraud – which they do because of their short term interests – personal sanctions on the manager are ineffective deterrents because the shareholders can always make it worth the manager’s while by heaping on more and more stock compensation. In contrast, fines against the corporation actually decrease equity compensation’s efficacy in encouraging fraud.

The paper proceeds as follows. Section 2 presents the basic model, styled as a one shot game between strategic players: shareholders, the manager, and purchasers. Section 3 presents a solution in the form of each player’s optimal strategies at each stage of the game. Section 4 analyzes the solution and presents results. Section 5 concludes.
2 The model

Here I present a model of agency costs in a firm with endogenous compensation. There are three aspects to agency costs in this model. First, the manager’s effort, which positively impacts the expected value of the firm, is not observable or verifiable. Second, the manager may face costs from securities fraud that shareholders do not; these costs may be reputational, moral, or legal. Third, to the extent that the manager owns shares of the firm, the manager may have shorter term interests with regard to the stock price of the firm than do the shareholders. This assumption is common in the legal literature on securities fraud: it is presumed that managers are not in it for the long term (e.g., Bebchuk and Fried (2009)), and may maximize short term stock price at the expense of long term stock price and performance.

Shareholders can, to an extent, remedy this conflict of interest via contract. Contracting in this model is imperfect, in that shareholders have only one contractual instrument at their disposal: they can award to the manager a number of shares of the company. If the shareholders award zero stock, the manager’s preference for sloth and truth telling prevails. As shareholders award more stock, the manager will be incentivized to exert effort, but also to falsely inflate her report regarding the firm’s value.

2.1 The economy

The economy in this model consists of five types of entities:

1. The firm, which has a production technology and $N$ shares of stock outstanding

2. $N$ identical shareholders, who each own a share of stock of the firm and choose the manager’s compensation contract

3. The manager, who may exert effort to increases the firm’s likelihood of a good outcome, and who also makes a disclosure to the marketplace concerning the firm’s value

4. Purchasers, who stand ready to purchase the firm’s shares for their conditional expected value, given the manager’s report of value

5. A regulator, who assesses a fine of $l$ against the firm in the event that the manager reports falsely
The Firm

The firm can be one of two types: \( \eta \in \{H, L\} \). High type firms have cash flows per share of \( H \). Low type firms have cash flows of \( L \), which I let equal 0 without loss of generality. While type is completely deterministic of cash flows, a firm’s type is influenced by managerial effort \( e \in \{0, 1\} \). I write the general probability of the firm’s type as \( \gamma_e \equiv \Pr(H|e) \), where \( \gamma_1 = \Pr(H|e = 1) \) and \( \gamma_0 = \Pr(H|e = 0) \), where \( 1 \geq \gamma_1 \geq \gamma_0 \geq 0 \), and \( \Delta \gamma \equiv \gamma_1 - \gamma_0 \). The unconditional expected value per share of a firm whose manager exerts effort is \( \gamma_1 H \); if the manager does not exert effort, the expected value is \( \gamma_0 H \).

Shareholders

There are \( N \) shareholders who each own a share of the firm, and are entitled to all of the firm’s cash flows. In order to affect the manager’s choice of effort and disclosure decisions, the shareholders may choose to award a number \( \alpha \) of shares of the firm to the manager; each shareholder then contributes \( \alpha/N \) to the manager’s equity compensation. I assume that this compensation level \( \alpha \) is observable to shareholders and the manager only; this tracks the reality that shareholders can always choose, ex post, to reward the manager, and they cannot commit not to. After choosing the manager’s compensation contract, some exogenous proportion \( \pi \) of shareholders will wish to sell their shares, which occurs after the manager chooses effort and makes a disclosure about the firm’s type; \( \pi \) is then the degree of shareholders’ short term interest, while \( (1 - \pi) \) is the shareholder’s long term interest in the firm’s performance and stock price. The market price \( p(\eta') \) that shareholders receive for selling their shares is a function of the manager’s publicly observable report. If the firm is found by the regulator to have committed fraud, the shareholders who have not sold each bear a fine of \( l \) per share, while those who did sell simply keep their sale proceeds; this functions similarly to actual corporate penalties in the U.S. securities antifraud regime.

The expected value of the shareholder’s payoff is then

\[
EU_s = (1 - \alpha/N)[\pi p(\eta') + (1 - \pi) [\gamma_e(H - l(\eta', H)) + (1 - \gamma_e)(L - l(\eta', L))]]
\]
I assume that \( N \) is very large relative to the manager’s compensation, making the value of compensation paid to the manager insignificant for the shareholder’s purposes (i.e., \( \alpha/N \approx 0 \)).\(^3\) This allows the shareholder’s payoff function to be written more simply without the \( \alpha/N \) term as:

\[
EU_s = \pi p(\eta') + (1 - \pi) [\gamma_e (H - l(\eta', H)) + (1 - \gamma_e)(L - l(\eta', L))] 
\]

**The Manager**

The manager has two sets of actions in this game. He gets to choose whether to exert effort or not \( e \in \{0, 1\} \), at personal cost to himself of \( c(0) = 0, \ c(1) = c \). Effort increases the likelihood of the firm achieving high cash flows.

The manager observes the firm’s type \( \eta \), and then makes a report \( \eta' \) to the marketplace of the firm’s type. In this report, the manager may choose to tell the truth \( (\eta = \eta') \), or lie \( (\eta \neq \eta') \). After making this report, the manager sells exogenous proportion \( \pi_m \) of his shares, and retains proportion \( 1 - \pi_m \). If the manager does not sell his stock, he also bears a fine of \( l \) per share for fraud. The proportion of stock sold \( \pi_m \) is thus the manager’s short term interest, while \( 1 - \pi_m \) is her long term interest.

In general, I assume that the manager has a slight preference for not overreporting the firm’s value. This reflects, perhaps, reputational capital of the manager or the possibility for individual sanctions. Formally, I denote this manager-specific cost of fraud as \( R(\eta', \eta) \), where \( R(H, L) = R > 0 \), while \( R(H, H) = R(L, H) = R(L, L) = 0 \).

The expected value of the manager’s payoff manager’s payoff is then:

\[
EU_m = \alpha[\pi_m p(\eta') + (1 - \pi_m) [\gamma_e (H - l(\eta', H)) + (1 - \gamma_e)(L - l(\eta', L))]] - c(e) - R(\eta', \eta)
\]

**Purchasers**

\(^3\)This is generally true for large public corporations. For instance, the top 5 employees of Exxon received compensation equivalent to 0.014% of net revenue in 2008, Microsoft 0.048%, Johnson & Johnson 0.122%, JP MorganChase 0.197%, and Apple 0.339%. 

http://law.bepress.com/usclwps-lewps/art107
Purchasers observe the manager’s signal and then pay a price \( p(\eta') \) such that, in expectation, they will break even – that is, they are individually rational. Purchasers are strategic in the sense that they take into account the incentives of both shareholders in choosing \( \alpha \) and the manager in choosing \( e \) and \( \eta' \). The purchaser’s individual rationality (IR) constraint is then

\[
\text{(IR): } U_p = \sum_{\eta \in \{H, L\}} \Pr(\eta|\eta') \eta - p(\eta') = 0
\]

**The regulator**

The regulator imposes a fine against the firm if the regulator determines that there has been fraud. Because the fine is levied against the firm, it is effectively borne by the shareholders of the firm. I assume, for simplicity, that the regulator ensures that purchasers are left unharmed by the fines (that is, the regulator only fines the non-selling shareholders, which is a fairly accurate approximation of how the current securities fraud class action system works (see Spindler 2009)). This assumption keeps \( l \) from affecting the purchaser’s IR constraint, but does not affect the overall analysis. I assume that there is perfect enforcement of fraudulent overstatements of value: \( l(H', L) = l > 0 \), while \( l(H', H) = l(L', H) = l(L', L) = 0 \).\(^4\)

Because enforcement is perfect in the sense of zero type 1 and type 2 error, one might suppose that the best enforcement strategy is always a fine of infinity. This is trivially true. However, I assume that the regulator is unable to do that, perhaps because of heterogeneity of firms, judgment proofness, concerns about fairness, deadweight losses of regulatory sanctions (such as forcing a firm into bankruptcy), or because of concerns about type 1 error that are not explicitly incorporated into the model. One useful concept that this captures is that firms may have different costs and detectability of fraud, value of projects, and managerial predispositions, which means that a set level of fine will not be appropriate for all firms in all situations.

In addition, for simplicity, I assume that only the old shareholders of the firm bear the cost of the fine, and that new purchasers do not. This reflects the real-world right of purchasers to recover against the firm for fraud and a policy preference that defrauded purchasers not be further harmed by punitive action against the firm, and simplifies the purchasers’ IR constraint.

\(^4\)The absence of a penalty for underreporting does not affect the analysis because, given the assumption of perfect enforcement, underreporting of value will never occur. Underreporting could occur when the regulator makes errors in observing firm type \( \eta \), but I do not consider that here.
2.2 The sequence of play

Putting the action sequence of the game in chronological order:

1. The regulator randomly chooses a level of fine per share for fraud, \( l > 0 \), which is observed by all.

2. Shareholders may choose to award share \( \alpha \) shares of the firm to the manager, \( \alpha \geq 0 \).

3. The manager has the choice to exert effort \( e \in \{0, 1\} \), where effort is costly, \( c(0) = 0 \), \( c(1) = c \).

4. The manager observes the firm’s type, \( \eta \in \{H, L\} \).

5. The manager makes a disclosure to the marketplace of the firm’s type: \( \eta' \in \{H, L\} \). The manager may choose to tell the truth about the firm’s type (\( \eta' = \eta \)) or lie (\( \eta' \neq \eta \)).

6. Proportion \( \pi \) of shareholders sell their shares, and the manager sells proportion \( \pi_m \) of the manager’s shares. Purchasers break even in expectation; that is, the price is determined subject to the purchasers’ individual rationality (IR) constraint.

7. A fine of \( l \) is assessed against each of the firm’s non-selling shareholders if the regulator finds that there has been fraud.

The formal model is then:

\[
\text{Obj} : \max_{\alpha, e, \eta' | \eta} (1 - \alpha)[\pi p(\eta') + (1 - \pi) [\gamma_e (H - l(\eta', H)) + (1 - \gamma_e)(L - l(\eta', L)))]
\]

subject to

\[
\text{IC1} : e = \arg \max_{\epsilon \in \{0, 1\}} \alpha[\pi_m p(\eta') + (1 - \pi_m)[\gamma_e (H - l(\eta', H)) + (1 - \gamma_e)(L - l(\eta', L))]] - c(e) - R(\eta', \eta)
\]

\[
\text{IC2} : \eta' = \arg \max_{\eta' \in \{H, L\}} (\alpha[\pi_m p(\eta') + (1 - \pi_m)[\eta - l(\eta', \eta)])] - R(\eta', \eta)|\eta)
\]

\[
\text{IR} : \sum_{\eta \in \{H, L\}} \Pr(\eta | \eta') \eta - p(\eta') = 0
\]
An equilibrium consists of a price given the manager’s signal, the manager’s choice of disclosure signal given type and share grant \( \alpha \), the manager’s choice of effort given \( \alpha \), and the shareholder’s choice of \( \alpha \).

3 Solving for equilibrium

I solve for the shareholders’ and manager’s choices by backward induction. Proceeding from the last stage of the game, the purchaser prices the firm’s shares so as to break even in expectation. In the penultimate stage, the manager makes a decision whether to report truthfully or not to maximize her expected payoffs given the firm’s type. In the second stage, the manager decides whether to exert effort or not. Finally, in the first stage, the shareholders choose the manager’s compensation.

3.1 The purchaser’s pricing decision

The purchaser’s IR constraint determines the price \( p \) for which shareholders and the manager can sell their shares. Given the manager’s signal, the price is the conditional expected value of the share – i.e., the purchasers break even in conditional expectation.

While purchasers do not observe share compensation \( \alpha \), the manager’s choice of effort, or the manager’s disclosure policy, they do anticipate these factors in equilibrium. In pure strategies, the manager either always discloses truthfully or else will disclose a falsely inflated value (i.e., \((\eta', \eta) = (H, L))\). There is also the possibility that managers will employ a mixed strategy, sometimes disclosing falsely and sometimes disclosing truthfully. In both the pooling and the mixing equilibria, the equilibrium price paid will vary depending upon whether the purchasers believe that the manager would have exerted effort.

In a separating equilibrium,

\[
p(H) = H \\
p(L) = 0
\]
In a pure strategy pooling equilibrium,

\[ p(H) = \gamma_c H \equiv p_e \]
\[ p(L) = 0 \]

Note that I assume that purchasers interpret a disclosure of \( L \) to be informative and hence \( p(L) = 0 \) (otherwise a pooling equilibrium is problematic since low type managers would prefer to disclose \( L \) to avoid the fine of \( l \) for disclosing high.)

In a mixed strategy equilibrium, utilizing Bayes’ law, we have

\[ p(H) = \frac{\gamma_c H}{\gamma_c + x(1 - \gamma_c)} \equiv p_{x,e} \] \hspace{1cm} (1)
\[ p(L) = 0 \] \hspace{1cm} (2)

where \( x \) is the proportion (or likelihood) of managers of low type firms that choose to falsely disclose high.

Finally, there can arise a case where shareholders will mix compensation strategies, although I defer discussion of this to the section on shareholder choice of compensation.

### 3.2 The manager’s disclosure decision

At the time that the manager decides which signal to send to the market, she knows the type of the firm \( (\eta \in \{H, L\}) \) as well as her share of the firm’s ownership, \( \alpha \). Things are relatively simple where equity compensation is not awarded: if \( \alpha = 0 \), the manager’s personal reputational concerns \( R \) dominate, and she always reports truthfully. If the fine \( l \) is low enough relative to the manager’s short term interest \( \pi_m \), and the equity grant \( \alpha \) is high enough, the manager will prefer fraudulent disclosure in the low state \( (\eta = L) \).

As the fine gets higher, the manager will commit less fraud: as \( l \) increases relative to \( \pi_m \), the manager goes from always lying when equity compensation is high, to sometimes lying when equity compensation is low and always lying when it is high, to sometimes lying at all equity compensation levels greater than zero, to
never lying when the fine \( l \) is very high.

Restating these results more formally, we have the following propositions:

**Proposition 1** Depending upon the fine \( l \), there exist up to two cutoff levels of equity compensation, \( \alpha_x = \frac{R}{\pi_m H - (1-\pi_m)l} \) and \( \alpha_p = \frac{R}{\pi_m \gamma_e H - (1-\pi_m)l} \), where the manager (i) always tells the truth when \( \alpha < \alpha_x \lor \alpha_x < 0 \), (ii) plays a mixed disclosure strategy (lying in the low state with probability \( x \)) when \( \alpha \in [\alpha_x, \alpha_p) \land \alpha_x, \alpha_p > 0 \) or \( \alpha > \alpha_x > 0 \land \alpha_p < 0 \), and (iii) always lies in the low state when \( \alpha > \alpha_p \land \alpha_p > 0 \).

**Proposition 2** Making the simplifying assumption that \( R \to 0^+ \), without loss of generality the manager’s behavior varies across 4 zones, zone M1 through M4, demarcated by the level of fine \( l \). The manager’s behavior in each zone is as follows:

- **Zone M1**: For \( l \in (0, \frac{\pi_m}{1-\pi_m} \gamma_0 H) \), the manager will always tell the truth if \( \alpha = 0 \) and always lie in the low state when \( \alpha > 0 \).

- **Zone M2**: For \( l \in [\frac{\pi_m}{1-\pi_m} \gamma_0 H, \frac{\pi_m}{1-\pi_m} \gamma_1 H) \), the manager reports truthfully if \( \alpha = 0 \), and for \( \alpha > 0 \) always lies in the low state where \( e = 1 \) and mixes disclosure in the low state when \( e = 0 \).

- **Zone M3**: For \( l \in [\frac{\pi_m}{1-\pi_m} \gamma_1 H, \frac{\pi_m}{1-\pi_m} H) \), the manager reports truthfully if \( \alpha = 0 \), and for \( \alpha > 0 \) always mixes disclosure in the low state.

- **Zone M4**: For \( l \geq \frac{\pi_m}{1-\pi_m} H \), the manager always reports truthfully.

The proof follows.

### 3.2.1 Separating equilibrium

In order for a separating equilibrium to result, it must be the case that IC2 is satisfied in that the manager of a low type firm must prefer to disclosure truthfully.\(^5\)

\[
\alpha [\pi_m L + (1-\pi_m)L] \geq \alpha [\pi_m H + (1-\pi)(L - l)] - R
\]

\(^5\)A manager of a high type firm always prefers to disclose truthfully, since there is no type 1 error in the model.
In the case where $\alpha = 0$ (the shareholders grant the manager no shares of stock), the manager always prefers to tell the truth because $R$ is strictly positive.

If $\alpha > 0$, rearranging yields a lower bound for $l$:

$$l \geq \frac{\pi_m}{1 - \pi_m} H - \frac{1}{\alpha (1 - \pi_m)} R$$

Note that if $l \geq \frac{\pi_m}{1 - \pi_m} H$, no level of compensation can induce non-separating behavior.

This can be rearranged to given an upper bound for $\alpha$ that maintains separation (assuming $l < \frac{\pi_m}{1 - \pi_m} H$):

$$\alpha \leq \frac{R}{\pi_m H - (1 - \pi_m) l} \equiv \alpha_x$$

The term $\alpha_x$ denotes the level of equity compensation at which a manager will switch from always telling the truth (separation) to sometimes lying (mixing at rate $x$).

I assume that $R$ is small both in relation to $c$ and the firm’s overall value, $N \gamma c H$. There are two reasons for making this assumption. First, it simplifies things. In this case, it allows one to write the separating condition as:

$${\text{Separation:}} \; l \geq \frac{\pi_m}{1 - \pi_m} H \text{ or } \alpha = 0$$

The second reason is that it makes it easier to capture the dynamic that equity compensation both encourages fraud and encourages effort, and that encouraging more effort encourages more fraud. This assumption ensures that shareholders cannot, for some values of $l$, induce effort without inducing fraud. This assumption is not necessary to capture the dynamic, as I show in Appendix A, though it does make it easier to describe.

[See appendix for more detail on this assumption.]

### 3.2.2 Pooling equilibrium

In order for pooling to take place, it must be that managers of low type firms prefer to disclose high rather than to tell the truth and receive a payoff of zero:

$$\alpha [\pi_m \gamma c H - (1 - \pi_m) l] - R > 0$$
In the case where $\alpha = 0$, the manager’s personal cost of fraud $R$ makes truthful disclosure a dominant strategy, and no pooling equilibrium will exist. Following the earlier assumption that $R$ is relatively very small (reflecting only a weak preference on the manager’s part), then even a relatively trivial stock grant makes truthful disclosure no longer a dominant strategy. We then obtain the following condition.

$$l < \frac{\pi_m}{1 - \pi_m} \gamma e H - \frac{1}{\alpha(1 - \pi_m)} R$$

Or, equivalently

$$\alpha > \frac{R}{\pi_m \gamma e H - (1 - \pi_m) l} \equiv \alpha_p$$

Again with the assumption that $R$ is small, this is more simply written as:

$$\text{Pooling: } l < \frac{\pi_m}{1 - \pi_m} \gamma e H, \alpha > 0$$

### 3.2.3 Mixed strategy equilibrium

For a certain range of $l$, namely $l + \frac{1}{\alpha(1 - \pi_m)} R \in (\frac{\pi_m}{1 - \pi_m} \gamma e H, \frac{\pi_m}{1 - \pi_m} H)$ and $\alpha > 0$, there exists no pure strategy equilibrium. In such a case, managers will pursue a mixed strategy where they lie with probability $x$ in the low state, and tell the truth with probability $1 - x$. In order for managers to be willing to mix, managers of low type firms must be indifferent between disclosing high (which entails receiving the mixed strategy price $p_x$ and bearing the risk of liability $l$) and disclosing low (and receiving zero). Formally,

$$\alpha (\pi_m p_x - (1 - \pi_m) l) - R = 0$$

$$\Leftrightarrow p_x = \frac{R}{\alpha \pi_m} + \frac{1 - \pi_m}{\pi_m} l$$

The purchaser’s pricing decision in the mixed strategy case (Eq. 1) gives the condition that, to break
even, the price must be \( p_x = \gamma_x H(\gamma_x + x(1 - \gamma_x))^{-1} \). Combing these yields a solution for \( x \):

\[
x = \frac{\gamma_x}{1 - \gamma_x} \left( \frac{\alpha (\pi_m H - (1 - \pi_m) l) - R}{\alpha (1 - \pi_m) l + R} \right)
\]

(5)

There are multiple equilibria, since there are potentially infinite \((\alpha, x)\) pairs that solve Eq.5. As \(R/\alpha \to 0\), which I generally assume to be the case, \(\lim x = \frac{\gamma_x}{1 - \gamma_x} \frac{\pi_m H - (1 - \pi_m) l}{(1 - \pi_m) l} \), which has only one solution.

**Remark 3** The likelihood of disclosing falsely in the low state, \( x \), is increasing in the effort level \( \gamma_x \). This is because the gains from fraud are higher: where the equilibrium strategy is to exert effort, the pooling/mixing price will consequently be higher.

While \( p_x \) is, in a sense, nailed down by the exogenous parameters \( \pi_m \) and \( l \), it is still a function of the level of effort \( e \) and consequent probability of success \( \gamma_e \) in that the range of \( l \) in which the manager will mix depends upon whether the manager had earlier undertaken effort (in the sense that it was subgame perfect for her to do so). The range of fine \( l \) in which mixing behavior is possible can be further subdivided: for \( l + \frac{1}{\alpha (1 - \pi_m)} R \in \left( \frac{\pi_m}{1 - \pi_m} \frac{\gamma_0 H}{1 - \gamma_0}, \frac{\pi_m}{1 - \pi_m} \frac{\gamma_1 H}{1 - \gamma_1} \right) \), the manager will mix only if it was optimal for her to undertake effort in the prior stage of the game; otherwise, she will play a pure strategy of falsely reporting in the low state. In the higher subdivision of the range, i.e. for \( \left( l + \frac{1}{\alpha (1 - \pi_m)} R \right) \in \left( \frac{\pi_m}{1 - \pi_m} \gamma_1 H, \frac{\pi_m}{1 - \pi_m} H \right) \), the manager will always mix, even if it was not optimal to undertake effort in the prior subgame.

### 3.2.4 Summing up

The attached figure summarizes the manager’s potential disclosure strategies as the fine \( l \) changes. With the simplification that \( R \to 0^+ \), we can characterize the manager’s behavior as a function of \( l \) and \( \alpha \). Where \( \alpha = 0 \) the manager always tells the truth. For \( \alpha > 0 \), the manager’s behavior depends upon \( l \) and the equilibrium effort level \( \gamma_e \). Along the continuum of possible fines, the leftmost region (where \( l \) is the lowest), denoted as zone M1, is one of pure pooling behavior, regardless of prior effort level. Moving rightward, the next zone of behavior (zone M2) is where the manager pools if it was not optimal to undertake effort in the prior stage, and mixes if the opposite is true. Moving again rightward, in zone M3, the manager chooses
3.3 The manager’s choice of effort

The manager chooses effort after the fine $l$ and compensation level $\alpha$ are set, but before she has observed the firm’s type and before she makes a disclosure to the marketplace. However, the manager can backward induce both her own disclosure decision and the market’s equilibrium response to it in the subsequent stages of the game. The manager, then, will make her choice of effort taking these reactions into account.

The manager’s effort exertion behavior may be described in the following proposition:

**Proposition 4** If $\alpha$ and $l$ are such that the manager will separate (i.e., zone M1), the manager exerts effort $e = 1$ if

$$\frac{c}{\Delta \gamma H} = \bar{a}$$
and chooses \( e = 0 \) if \( \alpha < \bar{\alpha}_s \). Where \( \alpha \) and \( l \) are such that the manager will not engage in pure separating behavior (i.e., in zones M2, M3, and M4), the manager exerts effort if

\[
\alpha \geq \frac{c - \Delta \gamma RL}{\Delta \gamma (1 - \pi_m) (H + l)} = \bar{\alpha}_p
\]

and chooses \( e = 0 \) if \( \alpha < \bar{\alpha}_p \).

The proof follows.

### 3.3.1 Effort in the separating case

If the fine \( l \) is high enough that the manager will choose to disclose truthfully regardless of the firm’s type and regardless of the optimal effort level (Zone M4 in Figure 1), we ask simply whether the manager would do better off incurring the cost of effort and enjoying a higher probability that her equity share is valuable, as opposed to slacking. Formally, the condition for the exertion of effort is:

\[
\alpha [\gamma_1 (\pi_m H + (1 - \pi_m) H) + (1 - \gamma_1) (\pi_m L + (1 - \pi_m) L)] - c \\
\geq \alpha [\gamma_0 (\pi_m H + (1 - \pi_m) H) + (1 - \gamma_0) (\pi_m L + (1 - \pi_m) L)]
\]

Solving this for \( \alpha \) pins down the minimum equity share for which the manager will exert effort in the separating case:

\[
\bar{\alpha}_s = \frac{c}{\Delta \gamma H}
\]

### 3.3.2 Effort in the pooling case

Where the fine \( l \) is very low (\( l \leq \frac{\pi_m}{1 - \pi_m} \gamma H \) in Zone 1 in Figure 1), pooling is always the optimal strategy regardless of the effort choice. Thus, a defection from exerting (or not exerting) effort does not affect the manager’s subsequent disclosure behavior. The effect, then, of a defection from a given level of effort is the altered chance of success and whether the cost of effort \( c \) is incurred.
The manager will exert effort $e = 1$ if the following is true:

$$\gamma_1 \alpha [\pi_m p_e + (1 - \pi_m) H] + (1 - \gamma_1) [\alpha [\pi_m p_e - (1 - \pi) l] - R] - c$$

$$\geq \gamma_0 \alpha [\pi_m p_e + (1 - \pi_m) H] + (1 - \gamma_0) [\alpha [\pi_m p_e - (1 - \pi) l] - R]$$

This may be rewritten as:

$$\Delta \gamma \alpha (1 - \pi_m) (H + l) + \Delta \gamma R \geq c$$

The first term on the left hand side is the manager’s expected increased share payoff from exerting effort: $\Delta \gamma$ is the change in likelihood that the firm’s project will be successful due to the manager’s effort, $\alpha$ is the manager’s share of the firm, $(1 - \pi_m)$ is the likelihood that the manager will retain her shares and receive the firm’s cash flows and any liability, and finally $(H + l)$ is the marginal pecuniary benefit to a shareholder of the project’s success ($H$ is the cash flow received, while $l$ is the liability avoided). The second term on the left hand side is the expected personal benefit to the manager from exerting effort: with increased probability of $\Delta \gamma$, the project will be successful, meaning the manager will not lie and will not incur personal costs of $R$. On the right hand side, $c$ is the cost of the manager’s effort.

Since price as a function of effort, $p_e$, drops out, there is no need to check separately whether the manager would choose to defect from a state in which managers are not exerting effort, as the conditions will be the same.

Rearranging, this condition also pins down the level of share ownership $\alpha$ necessary for the manager to exert effort in the pooling case (denoted as $\bar{\alpha}_p$):

$$\bar{\alpha}_p \equiv \frac{c - \Delta \gamma R}{\Delta \gamma (1 - \pi_m) (H + l)}$$

3.3.3 Effort in the mixing case (Zone 3)
The following equation defines the condition for the manager to exert effort in Zone 3, with the left hand side giving equilibrium behavior of $e = 1$, and the right hand side giving the payoff for a defection to $e = 0$:

$$
\gamma_1 \alpha (\pi_m p_x + (1 - \pi_m) H) + (1 - \gamma_1) (x (\alpha (\pi_m p_x - (1 - \pi_m) l) - R)) - c
\geq
\gamma_0 \alpha (\pi_m p_x + (1 - \pi_m) H) + (1 - \gamma_0) (x (\alpha (\pi_m p_x - (1 - \pi_m) l) - R))
$$

From the manager’s mixing condition, Eq. 4, $\alpha (\pi_m p_x - (1 - \pi_m) l) - R = 0$, and the above then reduces to:

$$
\Delta \gamma \alpha (1 - \pi_m) (H + l) + \Delta \gamma R \geq c
$$

which is the same as in the pooling case. The level of equity compensation necessary to induce effort is therefore the same as well:

$$
\bar{\alpha}_x = \bar{\alpha}_p = \frac{c - \Delta \gamma R}{\Delta \gamma (1 - \pi_m) (H + l)}
$$

3.3.4 Effort in the mixing/pooling case (Zone 2)

In Zone 2 of Figure 1, where $l \in \left(\frac{\pi_m}{1 - \pi_m} \gamma H, \frac{\pi_m}{1 - \pi_m} \gamma H\right)$, the manager will pool if the equilibrium outcome is to exert effort ($e = 1$) and mix if the equilibrium outcome is to slack ($e = 0$).

Taking first the pooling case, in order for a manager not to defect from exerting effort, the following must be true:

$$
\gamma_1 \alpha [\pi_m p_1 + (1 - \pi_m) H] + (1 - \gamma_1) [\alpha [\pi_m p_1 - (1 - \pi) l] - R] - c
\geq
\gamma_0 \alpha [\pi_m p_1 + (1 - \pi_m) H] + (1 - \gamma_0) [\alpha [\pi_m p_1 - (1 - \pi) l] - R]
$$

This is identical to the pure pooling equilibrium (Zone 1), and yields the same minimum level of equity compensation $\alpha$ to induce effort:
\[ \bar{\alpha}_p = \frac{c - \Delta \gamma R}{\Delta \gamma (1 - \pi_m)(H + l)} \]

In the case where the manager would not exert effort \( (e = 0) \) and mix her disclosure strategy, the manager will choose not to defect if:

\[
\gamma \alpha (\pi_m p_x + (1 - \pi_m) H) \\
+ (1 - \gamma) \left( x (\alpha (\pi_m p_x - (1 - \pi_m) l) - R) \right) \\
\geq \gamma \alpha (\pi_m p_x + (1 - \pi_m) H) \\
+ (1 - \gamma) \left( x (\alpha (\pi_m p_x - (1 - \pi_m) l) - R) \right) - c
\]

Solving for the maximum bound of equity compensation to induce no effort,

\[
\alpha \leq \frac{c - \Delta \gamma R}{\Delta \gamma (1 - \pi_m)(H + l)}
\]

which again yields the same threshold level of compensation \( \bar{\alpha}_p \).

### 3.4 Shareholder’s choice of compensation

In this section, the final step of the backward induction problem is reached: the shareholder’s choice of compensation for the manager. Compensation has a dual role: a sufficient grant of equity can induce effort, which we might suppose shareholders generally want, and can also induce fraud in the low state, which shareholders may or may not want depending upon the relative levels of fine \( l \) and shareholder short term interest \( \pi \). Thus, the interesting problem arises when there is a tradeoff of sorts: what if the shareholders want effort exerted, but know that awarding the requisite compensation to induce effort may also result in more fraud than the shareholders prefer?

Before getting to that question, I first describe two important assumptions, which can together be summed up as (i) the gain to the firm from the manager’s effort is much larger than the cost of that effort, and (ii) the cost of effort is much greater than the manager’s personal cost of fraud. I give more detail on this
assumption in Appendix A, but describe their relevance briefly here.

First, in order to ensure that shareholders desire that managers exert effort, all else being equal, I assume that $c$ and $\bar{\alpha}$ are sufficiently small relative to $N \Delta \gamma H$, the overall gain from the manager’s exertion of effort. This seems to be true in large firms: if one examines executive compensation of the top corporate officers in large public firms such as GE and Apple, one finds that top executive compensation is well under a percent of overall costs. Despite the frenzied preoccupation with the levels of executive pay, it seems unlikely that the amount of such compensation currently affects shareholder welfare in a directly significant way for the vast majority of large firms. In order to further simplify the analysis, since $\bar{\alpha}$ and $c$ are already assumed to be small, I let $\bar{\alpha}/N \to 0$ and drop it from the shareholder’s payoff function altogether. This greatly reduces the arduousness of the algebra.

Second, in order to ensure that there is a meaningful tradeoff between effort and disclosure policy, I assume that $R/c \to 0$, which is sufficient to ensure that $\alpha_x \leq \bar{\alpha}_s$ and $\alpha_p \leq \bar{\alpha}_p$. In other words, the effect of this conditions is that managers are induced to lie (if the level of fine $l$ is low enough) before they are induced to exert effort. Even without this assumption, some tradeoff will typically exist: unless $R$ is high relative to $c$, there will be some level of $l$ small enough that $\alpha_x \leq \bar{\alpha}_s$. Note that if $R$ is indeed high, there is no problem presented by agency costs in the model: shareholders will always be able to induce either truthful or deceitful effort, depending upon their preference, although obtaining the optimal fraud level may not be possible. The game would then be equivalent to one in which the marginal benefit of effort $\Delta \gamma = 0$, which I discuss in Section [4.2].

With these assumptions in hand, I turn to the issue of how shareholders choose to award equity compensation. In doing so, shareholders will consider the joint effect of compensation upon effort and fraud. I therefore divide the analysis by which zone, M1 - M4, the manager is in. The results are summarized in the following proposition:

**Proposition 5**

- **In M1 and M2:** Equity compensation of $\alpha = \bar{\alpha}_p$ is an equilibrium if

\[
l \leq \frac{\Delta \gamma}{(1 - \gamma_1)(1 - \pi)} H + \frac{\pi}{1 - \pi} \gamma_0 H
\]

\[\text{6}I \text{ give the necessary condition in the appendix.}\]
Equity compensation of \( \alpha = 0 \) is an equilibrium if

\[
l \geq \frac{\Delta \gamma}{(1 - \gamma_1)(1 - \pi)} H + \frac{\pi}{1 - \pi} H
\]

If neither of these conditions are met, there exists a mixed strategy equilibrium where the shareholder mixes at rate \( s \), given as

\[
s = \frac{\gamma_0}{1 - \gamma_0} \cdot \frac{H[\Delta \gamma + (1 - \gamma_1) \pi] - l(1 - \gamma_1)(1 - \pi)}{l(1 - \gamma_1)(1 - \pi) - \Delta \gamma H}
\]

with a market price \( p_s \) of

\[
p_s = \frac{s(\gamma_1 - \gamma_0) + \gamma_0 H}{s(1 - \gamma_0) + \gamma_0 H}
\]

- **In M3:** The shareholder awards equity compensation of \( \alpha = \bar{\alpha}_p \) if

\[
k \geq \frac{\gamma_1(\pi_m - \pi) - \Delta \gamma (1 - \pi)(1 - \pi_m)}{\Delta \gamma \pi(1 - \pi_m) + \gamma_1(\pi_m - \pi)}
\]

where \( l = \frac{\pi_m}{1 - \pi_m} k H, k \in (\gamma_1, 1) \). The shareholder awards \( \alpha = 0 \) if

\[
l \geq \frac{\Delta \gamma}{(1 - \gamma_1)(1 - \pi)} H + \frac{\pi}{(1 - \pi)} H
\]

If neither condition is met, there exists a mixed strategy equilibrium where the shareholder mixes at rate \( s \) and price \( p_s \).

- **In M4:** Equity compensation of \( \alpha = \bar{\alpha} \) is always an equilibrium.

The proof follows.
3.4.1 Choice of compensation in zone M1

Assuming that $R/c \to 0$, when the fine $l \in [0, \frac{\pi p_x - \gamma H}{1 - \gamma_1}]$, the manager will report truthfully in the low state if $\alpha = 0$, will always lie in the low state given $\alpha > 0$, and will exert effort if $\alpha \geq \bar{\alpha}_p$. Since $\alpha_x < \alpha_p < \bar{\alpha}_p$, the shareholder has three choices: truthful reporting without effort ($\alpha \in [0, \alpha_x]$), mixed reporting without effort ($\alpha \in [\alpha_x, \alpha_p]$), false reporting without effort ($\alpha \in [\alpha_p, \bar{\alpha}_p]$), and false reporting with effort ($\alpha \geq \bar{\alpha}_p$). Hence, to minimize the payment to the manager in each case, the shareholder chooses among $\alpha \in \{0, \alpha_x, \alpha_p, \bar{\alpha}_p\}$.

Lemma 6 The non-separating, non-effort inducing levels of compensation, $\{\alpha_x, \alpha_p\}$, cannot be equilibria.

Proof. In order for either $\alpha_x$ or $\alpha_p$ to be an equilibrium, the shareholder must not prefer to defect to $\bar{\alpha}_p$. We have as a necessary condition
\[
\frac{\Delta \gamma + (1 - \gamma_1) y_0 \pi}{(1 - \gamma_1)(1 - \pi)} H
\]

Lemma 7 The non-separating, effort-inducing level of compensation $\bar{\alpha}_p$ is an equilibrium if and only if
\[
l \leq \frac{\Delta \gamma + (1 - \gamma_1) y_0 \pi}{(1 - \gamma_1)(1 - \pi)} H
\]

Proof. In order for $\bar{\alpha}_p$ to be an equilibrium, the shareholder must prefer not to defect to $\alpha = 0, \alpha_x, \alpha_p$. It is apparent that defections to $\alpha_x$ and $\alpha_p$ are never feasible since it must always be that $\gamma_1 (\pi p_1 + (1 - \pi) H) + (1 - \gamma_1) (\pi p_1 - (1 - \pi) l) > \gamma_0 (\pi p_1 + (1 - \pi) H) + (1 - \gamma_0) y (\pi p_1 - (1 - \pi) l)$. In the pooling case where $y = 1$, the condition is always true since $\pi p_1 + (1 - \pi) H > \pi p_1 - (1 - \pi) l$ and $\gamma_1 > \gamma_0$. If $\pi p_1 - (1 - \pi) l > 0$, the RHS is maximized if $x = 1$, and hence a defection to mixing is dominated by a defection to pooling. If $\pi p_1 - (1 - \pi) l < 0$ (which must always be the case as $R/c \to 0$), then a defection to pooling is dominated by a defection to mixing ($\alpha_x$, in which $y = x \in (0, 1)$), which is in turn dominated by a defection to $\alpha = 0$ (where $y = 0$).

There is technically another option, which is to award $\alpha \in (\alpha_x, \alpha_p)$, which results in mixed disclosure by the manager and no effort. However, this is dominated by both $\alpha = 0$ when $R/c \to 0$ and by $\alpha = \bar{\alpha}_p$ even if $R/c \neq 0$.
As the only remaining necessary condition for $\bar{\alpha}_p$ to be an equilibrium, then, it must be that the shareholder would not defect to the truthtelling level of compensation ($\alpha = 0$).

$$\gamma_1(\pi p_1 + (1 - \pi) H) + (1 - \gamma_1)(\pi p_1 - (1 - \pi) l) \geq \gamma_0(\pi p_1 + (1 - \pi) H)$$

$$\bar{\alpha}_p \text{ in } M1 : l \leq \frac{\Delta \gamma}{(1 - \gamma_1)(1 - \pi)} H + \frac{\pi}{1 - \pi} \gamma_0 H$$

Lemma 8 The necessary condition for $\alpha = 0$ to be an equilibrium is $l \geq \frac{\Delta \gamma + (1 - \gamma_1) \pi}{(1 - \gamma_1)(1 - \pi)} H$.

Proof. In order for $\alpha = 0$ to be an equilibrium, it must be preferable to defections to $\alpha_x, a_p, \bar{\alpha}_p$. Defections to $\alpha_x, \alpha_p$ occur if the following does not hold: $\gamma_0 H \geq \gamma_0 H + (1 - \gamma_0) y (\pi H - (1 - \pi) l)$, where $y = 1$ if $\alpha = \alpha_p$ and $y = x$ if $\alpha = \alpha_x$. If $\pi H - (1 - \pi) l < 0$, then the inequality must be true. If $\pi H - (1 - \pi) l > 0$, then a defection to $\bar{\alpha}_p$ would be preferable. This leaves as the necessary condition that a shareholder does not prefer a defection to $\bar{\alpha}_p$:

$$\gamma_0 H \geq \gamma_1(\pi H + (1 - \pi) H) + (1 - \gamma_1)(\pi H - (1 - \pi) l)$$

$$\alpha = 0 \text{ in } M1 : l \geq \frac{\Delta \gamma}{(1 - \gamma_1)(1 - \pi)} H + \frac{\pi}{1 - \pi} H$$

Lemma 9 There exists a range of $l \in \left( \frac{\Delta \gamma + (1 - \gamma_1) \pi}{(1 - \gamma_1)(1 - \pi)} H, \frac{\Delta \gamma + (1 - \gamma_1) \pi}{(1 - \gamma_1)(1 - \pi)} H \right)$ in which there is no pure strategy equilibrium. In such a case, the shareholder mixes at rate $s = \frac{\gamma_0}{1 - \gamma_0} \cdot \frac{H[\Delta \gamma(1 - \pi) + (1 - \gamma_0) \pi - l(1 - \gamma_1)(1 - \pi)]}{(1 - \gamma_1)(1 - \pi) - \Delta \gamma H}$ and the market price is $p_s = \frac{1 - \pi}{\pi} \cdot \frac{(1 - \gamma_1) l - \Delta \gamma H}{1 - \gamma_0}$.

Proof. Conditions 6 and 7 are not exhaustive:

$$\frac{\Delta \gamma + (1 - \gamma_1) \gamma_0 \pi}{(1 - \gamma_1)(1 - \pi)} H < \frac{\Delta \gamma + (1 - \gamma_1) \pi}{(1 - \gamma_1)(1 - \pi)} H$$

If $l$ is in this range, the shareholder will choose to defect from $\bar{\alpha}$ to 0, and also from 0 to $\bar{\alpha}$. In order for the shareholder to not choose to defect either way, the shareholder mix between $\bar{\alpha}$ and 0 at a rate $a$ (where $a$
denotes the probability of awarding \( \tilde{a} \) such that the market price makes the shareholder indifferent between compensation strategies. This occurs when

\[
\gamma_1 (\pi s + (1 - \pi) H) + (1 - \gamma_1) (\pi s + (1 - \pi) l) = \gamma_0 (\pi s + (1 - \pi) H)
\]

\( \Leftrightarrow p_s = \frac{1 - \pi}{\pi} \cdot \frac{(1 - \gamma_1) l - \Delta \gamma H}{1 - \gamma_0} \)  

(8)

Because purchasers must always break even in expectation, the price paid will take into account the equilibrium rate at which the shareholder mixes. The price \( p_a \), may be written as a function of \( \alpha \), the rate at which the shareholder employs compensation of \( \tilde{a} \). The price is then the Bayesian updated of the probability of the firm’s being of high type given a high signal, times the payoff \( (H) \) when the firm is of high type:

\[
p_s = \frac{s (\gamma_1 - \gamma_0) + \gamma_0}{s (1 - \gamma_0) + \gamma_0} H
\]

(10)

Equations 8 and 10 allow one to solve for the rate of mixing \( \alpha \).

\[
s = \frac{\gamma_0}{1 - \gamma_0} \cdot \frac{H [\Delta \gamma + (1 - \gamma_1) \pi] - l (1 - \gamma_1) (1 - \pi)}{l (1 - \gamma_1) (1 - \pi) - \Delta \gamma H}
\]

(11)

With some algebra one can show that the the only way \( s > 0 \) is for the numerator and denominator to both be positive, which implies as a condition that \( l \in \left( \frac{\Delta \gamma}{(1 - \gamma_1) (1 - \pi)}, \frac{\Delta \gamma + (1 - \gamma_1) \pi}{(1 - \gamma_1) (1 - \pi)} H \right) \). In addition, one can verify that in order for \( s < 1 \), it must also be that \( l > \frac{\Delta \gamma}{(1 - \pi)(1 - \gamma_1)} H + \frac{\pi}{1 - \pi} \gamma_0 H \). Hence the shareholder mixes when \( l \in \left( \frac{\Delta \gamma}{(1 - \gamma_1) (1 - \pi)} H + \frac{\pi}{1 - \pi} \gamma_0 H, \frac{\Delta \gamma + (1 - \gamma_1) \pi}{(1 - \gamma_1) (1 - \pi)} H + \frac{\pi}{1 - \pi} H \right) \), the boundaries of which correspond to the upper bound for awarding \( \tilde{a}_p \) and the lower bound for \( \alpha = 0 \) as pure strategies, respectively.

Remark 10 Nowhere in zone M1 does the level of the manager’s short term interest, \( \pi_m \), have any effect on either the likelihood of fraud or the likelihood that effort will be exerted.

3.4.2 Choice of compensation in zone M2

In zone M2, the manager will tell the truth if \( \alpha = 0 \), will mix disclosure at rate \( x \) if \( \alpha = \alpha_x \), and will always lie in the low state if \( \alpha = \alpha_p \). The same logic as in zone M1 holds here to show that \( \alpha_x \) cannot be an
equilibrium. The shareholder is left to choose between $\alpha = 0$ and $\alpha = \bar{\alpha}_p$, with conditions identical to Eqs. 6 - 11 in the zone M1 analysis.

### 3.4.3 Choice of compensation in zone M3

In zone M3, managers will play a mixing strategy: given that the firm is of low type, a manager receiving compensation of $\bar{\alpha}$ will falsely disclose high value with probability $x$, which results in price $p_x$. If compensation is $\alpha = 0$, the manager always tells the truth, resulting in a price given a high signal of $H$ and zero otherwise. If the level of fine $l$ is high enough, either $\alpha = \bar{\alpha}$ or $\alpha = 0$ may be an equilibrium.

In order for $\bar{\alpha}$ to be an equilibrium in M3, it must be that the shareholder prefers $\bar{\alpha}$ to a defection to any of $\alpha \in \{0, \alpha_x, \alpha_p\}$. That is,

$$\gamma_1 (\pi p_x + (1 - \pi) H) + (1 - \gamma_1) x (\pi p_x - (1 - \pi) l) \geq \gamma_0 (\pi p_x + (1 - \pi) H) + (1 - \gamma_0) y (\pi p_x - (1 - \pi) l)$$

where $y = \begin{cases} 0 & \text{if defection to } \alpha = 0 \\ x & \text{if defection to } \alpha = \alpha_x \\ 1 & \text{if defection to } \alpha = \alpha_p \end{cases}$

One can immediately dismiss $\alpha_x$, since the RHS of the inequality must be maximized at either $y = 0$ or $y = 1$, depending on whether $\pi p_x - (1 - \pi) l \geq 0$. Under the assumption $R/c \to 0$, it must be that $\pi p_x - (1 - \pi) l < 0$, and consequently it must also be that $\alpha_p$ is not a viable defection, either. It is only left to consider when a shareholder might consider a defection to $\alpha = 0$.

To make this less complicated, I take the limit of the expression as $R/c \to 0$ and plug in terms for $p_x$ and $x$. It is somewhat easier algebraically to proceed by letting $l = \pi_m kH$, where $k \in (\gamma_1, 0)$. In terms of $k$, we get the following condition:
This means that the fine must be sufficiently large in order to maintain $\alpha_p$ as an equilibrium. This is because net shareholder payoffs, as given in Eq.12, are actually increasing in $l$ because, while an increase in $l$ subjects the shareholder to greater liability in the event of false disclosure, an increase in $l$ has the salutary effect of decreasing the rate $x$ at which the manager mixes and increasing the price $p_x$ that the shareholder receives for selling his share. Note that this the condition is only meaningful agency costs are sufficiently large. Where $\pi_m = \pi$, for instance, the RHS of Eq.13 is negative, and any level of fine $l$ is sufficient to maintain $\alpha_p$ as an equilibrium. An intuitive interpretation of this condition is that if the fine is large relative to the degree of agency cost, then shareholder and manager interests are well enough aligned that the manager’s mixing strategy $x$ is acceptably palatable to the shareholder. In zone M3, then, it is only when agency costs are relatively high that $\alpha_p$ may fail to be an equilibrium. When they are low, shareholders will always award $\alpha_p$.

A high enough fine $l$ makes $\alpha = 0$ an equilibrium. The condition for this to obtain is

$$\gamma_0 H \geq \gamma_1 H + (1 - \gamma_1)(\pi H - (1 - \pi)l) : \text{ (versus } \alpha = \bar{\alpha}_p)$$

and

$$\gamma_0 H \geq \gamma_0 H + (1 - \gamma_0) y (\pi H - (1 - \pi)l) : \text{ (versus } \alpha = \alpha_x, \alpha_p)$$

One can show that $\alpha_x, \alpha_p$ need not be considered, since if $\pi H - (1 - \pi)l < 0$, the shareholder would not defect, while if $\pi H - (1 - \pi)l > 0$, the shareholder which is identical to the condition (Eq. 7) for awarding zero equity in the pure pooling cases of zone M1 and M2. This gives the equilibrium condition

$$\alpha = 0 \text{ in M3 : } l \geq \frac{\Delta \gamma}{(1 - \gamma)(1 - \pi)} H + \frac{\pi}{(1 - \pi)} H$$

(14)

The bigger the gain from an exertion of managerial effort ($\Delta \gamma$), the greater must $l$ be in order to sustain the non-effort equilibrium.

What happens in zone M3 when the fine is such that neither $\alpha = 0$ nor $\alpha = \bar{\alpha}$ is an equilibrium? In such a case, there exists no pure strategy equilibrium: shareholders would defect from $\bar{\alpha}$ to 0 and back again. There is, however, at least one mixed strategy equilibrium: the shareholder can mix at rate $a$ between $\alpha = 0$ and $\alpha = \bar{\alpha}$ such that the equilibrium price, $p_a$, makes the shareholder indifferent between the two
compensation strategies. The conditions for shareholder mixing are in fact identical to those in zones M1 and M2 (Eqs. 8 - 11); this is so because the shareholder’s mixing has the effect of raising the price above the manager’s mixing price, \( p_a > p_x \), and for any price greater than \( p_x \), the manager strictly prefers to lie in the low state.

**Lemma 11** When the shareholder mixes between 0 and \( \bar{\alpha}_p \), the manager will always choose to pool when equity compensation is \( \bar{\alpha}_p \) and will report truthfully when it is 0.

**Proof.** Given that equity compensation is \( \bar{\alpha} \), the manager chooses to report falsely in the low state if the payoff from lying exceeds the payoff from telling the truth (which equals zero). From this, one finds that the manager reports falsely in the low state (and will not mix) if, and only if, the price is above \( p_x \):

\[
p > \frac{1 - \pi_m}{\pi_m} I = p_x
\]  

(15)

The shareholder mixes only when he is indifferent between awarding \( \bar{\alpha} \) and awarding 0, or, formally, when Eq. 8 holds. One can show that 8 implies 15. First, if the shareholder prefers to mix instead of playing a pure strategy (such as \( \alpha = \bar{\alpha} \)), it must be true that the shareholder’s payoff from mixing exceeds the expected payoff from playing pure \( \bar{\alpha} \):

\[
\bar{\gamma}(\pi p_a + (1 - \pi) H) + (1 - \bar{\gamma})(\pi p_a - (1 - \pi) l) = \gamma(\pi p_a + (1 - \pi) H) \\
> \bar{\gamma}(\pi p_x + (1 - \pi) H) + (1 - \bar{\gamma})x(\pi p_x - (1 - \pi) l)
\]

Since this is true everywhere, it must also be true when \( x = 1 \):

\[
\bar{\gamma}(\pi p_a + (1 - \pi) H) + (1 - \bar{\gamma})(\pi p_a - (1 - \pi) l) > \bar{\gamma}(\pi p_x + (1 - \pi) H) + (1 - \bar{\gamma})(\pi p_x - (1 - \pi) l)
\]

Rearranging and canceling terms,

\[
p_a > p_x
\]

That is, the equilibrium price \( p_a \) when the shareholder mixes is greater than the equilibrium price when the
manager mixes, \( p_x \). Because \( p_a > p_x \), it cannot be that the manager will be indifferent between lying and telling the truth when the firm is of low type. In particular, she will strictly prefer lying, since the payoff from lying \( (\pi_m p_a - (1 - \pi_m) l) \) is strictly greater than the payoff from telling the truth \( (0) \).

### 3.4.4 Zone M4

If the fine is high enough such that \( l \geq \frac{\pi_m}{1 - \pi_m} H \), the manager always prefers to tell the truth when the the firm is of low type. Since \( \pi_m \geq \pi \), this means that the shareholder must also prefer to tell the truth since \( l \geq \frac{\pi_m}{1 - \pi_m} H \geq \frac{\pi}{1 - \pi} H \). Hence, the shareholder will always award equity compensation of \( \alpha = \bar{\alpha} \), and the manager will always exert effort and report truthfully.

### 4 Analysis: the relationship between fraud, effort, and agency costs

What does this model tell us about the effect of agency costs on fraud, compensation, and effort? While the parameters of the model allow for quite a lot of variation in incentives and behavior, one can still draw several general insights. As I will discuss below in more detail:

1. Fraud and effort are positively correlated, and fraud is more likely to occur as the gains from effort increase.

2. The incidence of fraud-inducing compensation \( \bar{\alpha} \) is increasing as agency costs are reduced.

3. A reduction in agency costs leads to an increase in the incidence of fraud when agency costs are high, and leads to either a decrease or no change in the incidence of fraud when agency costs are low (and any decrease is attributable to a change in managerial preferences, not resolution of the agency conflict).

4. The two separate types of sanctions – corporate fine \( l \) and personal managerial penalty \( R \) – have distinct effects on the choice of effort, fraud, and compensation.
4.1 The relation between fraud and effort

Fraud and effort are positively correlated. Shareholders never award $\alpha_x, \alpha_p$, and as a consequence the only time that fraud never occurs is when shareholders award $\alpha = 0$ and $e = 0$, or when the manager happens to be in zone M4.

Further, fraud is more likely to occur when the gains from effort are higher. From an inspection of the manager’s disclosure conditions, one can see that a greater fine $l$ is required to maintain separation as $\gamma_1$ increases ($l \geq \frac{\pi_m}{1-\pi_m} \gamma_1 H$). The rate at which the manager mixes, $x = \frac{\gamma_1}{1-\gamma_1} \frac{k}{l}$, is also a positive function of effort, since in equilibrium the only time the manager mixes is when effort is exerted. Consider also the conditions for awarding the fraud-and-effort-inducing compensation package $\tilde{\alpha}$ in M1/M2 and M3, as given by Eqs.6 and 13. Letting $\gamma_1 = \gamma_0 + \omega$, these may be rewritten in terms of the gain to effort:

$$6' : \quad l \leq \frac{\omega}{(1-\gamma_0 + \omega) (1-\pi)} H + \frac{\pi}{1-\pi} \gamma_0 H$$
$$13' : \quad k \geq \frac{(\omega + \gamma_0) (\pi_m - \pi) - \omega (1-\pi) (1-\pi_m)}{(\omega + \gamma_0) (\pi_m - \pi) + \omega \pi (1-\pi_m)}$$

Taking the derivatives with respect to $\omega$, one can verify that the RHS of 6' is increasing in $\omega$, while the RHS of 13' is decreasing, meaning that each condition is getting more slack as the gains to effort increase. (One can also verify that the conditions to maintain the truthtelling equilibria, $\alpha = 0$, of Eqs. 7 and 14 grow more restrictive as $\omega$ increases). Hence, holding everything else constant, increases in the marginal product of managerial effort should tend to induce a greater incidence of lying from managers given that firm type is low, and should also tend to increase the propensity of shareholders to award compensation that induces the manager to commit fraud.

This relationship has important regulatory implications. Consider the perspective of an outsider who sees that fraud is committed and the compensation paid to managers, but cannot observe effort. The outsider in this model would see that all firms that commit fraud have fraud-inducing compensation packaged ($\alpha = \tilde{\alpha}$), and would see that all firms that commit fraud have ex post valuations that are low ($\eta = L$). Because poor performance is correlated both with fraud and high compensation, the outsider may make the additional leap to conclude that high compensation causes fraud (which is true) and that high compensation and fraud cause
the low ex post performance (which is false). The outsider may believe that one could increase overall welfare by enacting policies that reduce or eliminate the incidence of fraud; for instance, by prohibiting compensation packages of $\tilde{\alpha}$. However, that has the unintended consequence of eliminating effort and reducing overall firm value by a factor of $\Delta \gamma$. While there is likely some offsetting benefit from the elimination of fraud (such as viability of follow-on projects and improved liquidity), such a policy is likely to be an overall detriment.

This same point is true for an outsider who wishes to raise corporate fines $l$. Suppose that substantial agency costs exist ($\pi_m >> \pi$), and that the outsider can choose one of two fine levels $l_1 < l_2$. Suppose further that $l_1, l_2 \leq \frac{\pi_m}{1-\pi_m} \gamma_0 H$, meaning that the manager will want to commit fraud no matter what the fine level so long as $\alpha > 0$. Finally, suppose that $l_1 < \frac{\Delta \gamma}{(1-\gamma_\phi)(1-\pi)} H + \frac{\pi}{1-\pi} \gamma_0 H < \frac{\Delta \gamma}{(1-\gamma_\phi)(1-\pi)} H + \frac{\pi}{1-\pi} H < l_2$.

This means that the shareholder will award $\alpha_p$ with $l_1$ but will award $\alpha = 0$ with $l_2$. That is, while a move from $l_1$ to $l_2$ will eliminate fraud, it will also eliminate effort and destroy $\Delta \gamma H$ of value. If agency costs are high, increases in fines will be relatively ineffective in deterring manager’s fraud decision given $\alpha > 0$, but will deter shareholders from awarding the efficient compensation package $\tilde{\alpha}$.

### 4.2 Reducing agency costs increases fraud-inducing compensation

In this model, it is true that large performance-based compensation packages (i.e., $\tilde{\alpha}$) lead to fraud. One interesting thing that this model generates, though, is that the incidence of fraud-inducing compensation is actually increasing as agency costs decline. Consider what happens as $\pi_m$ declines, taking as our starting point when $\pi_m$ and $l$ are such that the manager is in zone M1/M2. Assume that $\tilde{\alpha}_p$ is an equilibrium (Eq.6 is satisfied). The fact that $\tilde{\alpha}_p$ is an equilibrium in M1/M2 implies that it must also be an equilibrium in M3. In fact, one can show formally that, in general, the awardance of $\tilde{\alpha}$ is non-declining as $\pi_m$ declines toward $\pi$ (i.e., as agency costs decrease), as I do in the following three propositions:

**Proposition 12** As $\pi_m$ declines to $\pi$, if the shareholder awards $\tilde{\alpha}$ in zone M1/M2, the shareholder also awards $\tilde{\alpha}$ in zone M3 and M4.

**Proof.** Eq.6 gives as the condition for $\tilde{\alpha}_p$ in M1/M2 that $\Delta \gamma (\pi p_1 + (1-\pi) H) + (1-\gamma_\phi)(\pi p_1 - (1-\pi) l) \geq 0$, while Eq.12 gives as the condition for $\tilde{\alpha}_p$ in M3 that $\Delta \gamma (\pi p_x + (1-\pi) H) + (1-\gamma_\phi) x (\pi p_x - (1-\pi) l) \geq 0$. Since $p_x > p_1$ and $\pi p_1 - (1-\pi) l < 0$ (which must be true since by assumption since the manager being in
zone M3 means that \( l > \frac{\pi}{1-\pi} \gamma_1 H > \frac{\pi}{1-\pi} \gamma_1 H \), it follows that Eq.6 implies Eq.12. Finally, as \( \pi_m \) decreases to the point that the manager is in zone M4, \( \bar{\alpha}_s \) is always an equilibrium. ■

**Proposition 13** As \( \pi_m \) declines to \( \pi \), if the shareholder played mixed strategy \( s \) in M1/M2, the shareholder will either mix at rate \( s \) or play a pure strategy of \( \bar{\alpha} \) in M3.

**Proof.** From Proposition [5], the shareholder mixes in M1/M2 if and only if \( l \in \left( \frac{\Delta \gamma}{(1-\gamma_1)(1-\pi)} H + \frac{\pi}{1-\pi} \gamma_0 H, \frac{\Delta \gamma}{(1-\gamma_1)(1-\pi)} H + \frac{\pi}{1-\pi} H \right) \). Also from Proposition [ ], the shareholder plays a pure strategy of \( \alpha = 0 \) in M3 only if \( l \geq \frac{\Delta \gamma}{(1-\gamma_1)(1-\pi)} H + \frac{\pi}{(1-\pi)} H \). From inspection, then, it must be that if the shareholder mixes in M1/M2, the shareholder must either mix or play pure \( \bar{\alpha} \) in M3. ■

**Proposition 14** As \( \pi_m \) declines to \( \pi \), there is a level of \( \pi_m = \pi^*_m > \frac{l}{H+l} \) (i.e., in zone M3) where the shareholder awards \( \bar{\alpha} \) for any \( \pi_m \leq \pi^*_m \) so long as \( \Delta \gamma > 0 \). That is, the shareholder always awards \( \bar{\alpha} \) at some point in zone M3, and will award \( \bar{\alpha} \) for all higher levels of \( \pi_m \) as well.

**Proof.** Suppose that \( \pi_m = \frac{l}{H+l} \). At this point, the manager is on the border between zones M3 and M4 and hence will commit fraud at the rate \( x = 0 \). The shareholder always chooses to award \( \bar{\alpha} \) at this point since it induces the manager to exert effort but does not change disclosure strategy. Now suppose \( \pi_m \) increases by the very small increment \( \varepsilon \). The shareholder’s marginal loss from an increase in \( \pi_m \) is

\[
\frac{\partial EU}{\partial \pi_m} = \frac{\gamma_1}{1-\pi_m} H,
\]

while the loss from switching to \( \alpha = 0 \) is \( \Delta \gamma H \). The shareholder will continue to award \( \bar{\alpha} \) where \( \varepsilon \frac{\gamma_1}{1-\pi_m} H < \Delta \gamma H \). It must be the case for \( \varepsilon \) sufficiently small and \( \Delta \gamma > 0 \) that \( \varepsilon \frac{\gamma_1}{1-\pi_m} H < \Delta \gamma H \), since by assumption \( \pi_m = \frac{l}{H+l} < 1 \) and hence \( \frac{\gamma_1}{1-\pi_m} H < \infty \). ■

What this means is that the incidence of fraud-inducing compensation weakly increases as agency costs decrease (in the sense of \( \pi_m \to \pi \)). Even if \( \bar{\alpha} \) is not an equilibrium in M1/M2, it may become an equilibrium in M3 (for agency costs sufficiently low in M3, \( \bar{\alpha}_p \) is always an equilibrium), and \( \bar{\alpha}_p \) is always becomes one in M4.

Note that the results of Propositions [12] and [13] above do not rely on the gain from efficiency being greater than zero. In general, the shareholders will play \( \bar{\alpha} \) at some point in M3 so long as shareholders prefer some level of fraud, i.e., \( l < \frac{\pi}{1-\pi} H \). Suppose \( \gamma_1 = \gamma_0 = \gamma \) and \( \pi_m, \pi \), and \( l \) are such that \( \frac{\pi}{1-\pi} \gamma H < \frac{\pi}{1-\pi} \gamma_1 H < \frac{\pi}{1-\pi} \gamma H \).
$l < \frac{s - H\gamma}{1 - \pi}H$ and $l < \frac{s - H}{1 - \pi}H$. This means that the shareholder would prefer some fraud but, on the whole, the manager will, if granted equity compensation, commit a super-optimal amount of fraud, such that the shareholder will not choose to award the equity compensation all the time; instead, the shareholder mixes at rate $s = (\pi H - (1 - \pi)l) / (1 - \pi)l$. Consider then what happens as $\pi_m \to \pi$: by continuity of $x$, there will be a point where the manager mixes disclosure at exactly the same rate that the shareholder would want, and hence the shareholder plays a pure strategy of $\alpha = \bar{\alpha}_p$. If $l \geq \frac{s - H}{1 - \pi}H$, then the shareholder will award $\alpha$ only when the manager moves into $M4$ as $\pi_m$ declines toward $\pi$. As in the case where $\Delta \gamma > 0$, the incidence of fraud-inducing compensation $\bar{\alpha}$ is weakly increasing as $\pi_m \to \pi$.

We can sum up Propositions [12]-[14] in the following Corollary:

**Corollary 15** There are three possibilities for changes in shareholder behavior as $\pi_m$ declines toward $\pi$ and where $\Delta \gamma > 0$:

1. The shareholder awards $\bar{\alpha}$ in $M1/M2$ and continues to play $\bar{\alpha}$ thereafter,

2. The shareholder plays mixed strategy $s$ in $M1/M2$ and switches to $\bar{\alpha}$ at some point in $M3$ and plays $\bar{\alpha}$ thereafter,

3. The shareholder awards $\alpha = 0$ in $M1/M2$, switches to $\bar{\alpha}$ in $M3$ and plays $\bar{\alpha}$ thereafter.$^8$

### 4.3 Reducing agency costs leads to more fraud when agency costs are high

With Corollary [15]’s summary of shareholder behavior as agency costs decrease, one can then map the overall incidence of fraud using the manager’s disclosure and effort behavior as given in Propositions [2] and [4].

In general, there are two effects, depending on how great the degree of agency conflict is:

---

$^8$It is never the case that shareholders would switch from $\alpha = 0$ to a mixed strategy of $s$ in $M3$. This is because mixed strategy $s$ achieves the same result in $M1/M2$ as it does in $M3$: the manager’s disclosure practice is exactly the same. This is also true of pure strategy $\alpha = 0$, and hence changes in $\pi_m$ cannot switch the shareholder’s preference between 0 and $s$ strategies.
**Proposition 16** When agency costs are "large" in the sense that shareholders choose not to award $\bar{\alpha}$ as a pure strategy, incremental decreases in agency costs can only have the effect of increasing the incidence of fraud.

**Proof.** From Corollary [15], agency costs are "large" in the cases where the shareholder plays either $s$ or 0 in M1/M2. In these "large" cases, the shareholder will at some point switch to $\bar{\alpha}$ in M3 as $\pi_m \rightarrow \pi$. From Propositions [2] and [4], compensation of $\alpha = 0$ induces no fraud and no effort, while $\bar{\alpha}$ induces both fraud at rate $x$ and effort in M3. Therefore, the only possible effect of a small decrease in $\pi_m$ when agency costs are large is that the incidence of fraud increases from either 0 or $s(1 - \gamma_1)$ to $x(1 - \gamma_1)$. This result also holds as $\pi \rightarrow \pi_m$. ■

**Proposition 17** When agency costs are "small" in the sense that shareholders play $\bar{\alpha}$ as a pure strategy, incremental decreases in the manager's short term interest have the effect of decreasing incrementally the incidence of fraud, while incremental increases in shareholders' short term interests have no effect on fraud.

**Proof.** Once agency costs are "small" such that shareholders have chosen to award the equity inducing compensation $\bar{\alpha}$, decreases in $\pi_m$ cannot lead to more fraud. Rather, if those decreases occur while the manager is in zone M1/2 or M4, it causes no change in fraud levels. If $\pi_m$ decreases in zone M3, the manager chooses to commit incrementally less fraud as $x$, which is a positive function of $\pi_m$ from Eq.5, declines. ■

Figure 2 below presents these Propositions graphically. The top line represents the case (1) where shareholders award $\bar{\alpha}$ in M1/2 and thereafter, (2) the middle line represents a mixing strategy of $s$ in M1/M2 which switches to $\bar{\alpha}$ in M3 and thereafter, and (3) the bottom line represents a strategy of $\alpha = 0$ in M1/M2 which switches to $\bar{\alpha}$ in M3 and thereafter.

One thing to note is that the declines in the incidence of fraud when agency costs are small are not really due to a decrease in agency costs per se. Rather, these declines are due to the change in the manager's preferences, not to any resolution of the divergence of interest between the manager and shareholders. If agency costs were to decrease because the shareholders' short term interest approaches that of the manager ($\pi \rightarrow \pi_m$), there would be no corresponding decrease in the incidence of fraud.

This result runs counter to the commonly stated assertion that corporate fraud, in the sense of overstating the firm's value by managers, is the product of agency costs as between managers and shareholders, and that
reducing agency costs will reduce the incidence of fraud. In the context of the model, this is not generally true. Instead, where agency costs are very high, reducing them can only increase the level of fraud, while the effect of reducing agency costs when they are already small is ambiguous.

4.4 Corporate fines ($l$) vs. manager penalties ($R$)

Even if $R$ is large, a large personal fine on the manager succeeds in limiting fraud only to the extent that it makes effort without fraud achievable ($\bar{\alpha} < \alpha_x, \alpha_p$), and then only to the extent that shareholders do not desire fraud. So long as $N\Delta \gamma H$ is large compared to $R$, shareholders will simply increase $\alpha$ to achieve the desired effect. Effort will always be awarded (which is a good thing), but to the extent that shareholders desire to incentivize fraud, they can simply increase the level of equity compensation until $\alpha_x, \alpha_p$ is reached. Put another way, so long as $R$ is small relative to the size of the firm, changes in $R$ may affect shareholder incentives to commit fraud by changing the bundling of fraud and effort compensation policies, but changes in $R$ do not affect shareholders’ incentives to commit or ability to effectuate fraud via compensation $\alpha_x, \alpha_p$.

In contrast, changes in the fine $l$ have an effect upon the ability of the shareholders to incentivize fraud by making the manager more fraud averse. For a sufficiently high $l$ relative to $\pi_m$, shareholders are unable to induce the manager to commit fraud via equity compensation. Additionally, increases in $l$ affect shareholder
incentives directly, which increases in $R$ do not.

5 Conclusion

This model describes the tradeoff inherent when shareholders choose the level of equity compensation to pay to the manager. Performance based compensation induces effort, but also induces fraud. Hence, fraud and effort may of necessity go together.

While shareholders themselves may desire some degree of fraud in light of their own short term interests, the case where managers’ interests are more short term than those of the shareholders presents a divergence of interest where managers will tend to commit more fraud than shareholders would want.

As agency costs decrease and the interests of shareholders and managers become more aligned in terms of short term interest, shareholders will tend to award a higher level of performance based compensation. In fact, as the interests of managers approach those of shareholders, shareholders will unambiguously award more performance based compensation. Reductions in agency costs actually increase the incidence of fraud when agency costs are high, but may reduce the incidence of fraud (or have no effect) when agency costs are already low.

Finally, this model has implications for the choice of vicarious liability as opposed to managerial fines. Managerial fines are unable to deter fraud that arises from the incentives of shareholders, while vicarious liability can be an effective deterrent.

6 APPENDIX A: Assumptions on $R$ and $c$

I describe in greater detail the assumption regarding the relative sizes of $R$ and $c(e)$. The assumption, briefly stated, is that equity compensation sufficient to induce effort of $e = 1$ will result in:

- pooling in zone M1 and M2 (that is, where $l < \frac{\pi_m - \gamma_1 H}{1 - \pi_m}$), and

- mixing in zone M3 \( (l \in [\frac{\pi_m - \gamma_1 H}{1 - \pi_m}, \frac{\pi_m - \gamma_1 H}{1 - \pi_m}]) \)

The first assumption requires that $\alpha_x > \alpha_e$ and $\tilde{\alpha}_x > \alpha_e$, which means that the equity compensation necessary to induce effort in the separating equilibrium ($\tilde{\alpha}_x$) is greater than the equity compensation that
induces mixing behavior \((\alpha_x)\), and that the equity compensation necessary to induce effort in the mixing equilibrium \((\tilde{\alpha}_x)\) is greater than the level of equity compensation that induces pooling \((a_p)\). Consider the following example: \(l < \frac{\pi_m - \gamma_1 H}{1 - \pi_m} \). Starting with an equity compensation of \(\alpha = 0\), the manager will separate \((\alpha < \alpha_x)\), but will not exert any effort \((\alpha < \tilde{\alpha}_s)\). Since shareholders desire effort, they could incrementally try raising the manager’s equity compensation. However, since by assumption \(\tilde{\alpha}_s > \alpha_x\), before the manager is induced to exert effort, the manager will switch from truthful disclosure to a mixed strategy. Suppose that the shareholders continue to raise the manager’s compensation. Once again, before effort is induced, the manager will switch from mixing to purely false disclosure (pooling), since \(\tilde{\alpha}_x > a_p\). If shareholders continue to raise the manager’s equity compensation, they will reach the point, \(\alpha_p\), where the manager is induced to undertake effort.

The second assumption requires only that \(\tilde{\alpha}_s > \alpha_x\).

Formally, the first assumption \((\tilde{\alpha}_s > \alpha_x \text{ and } \tilde{\alpha}_x > a_p)\) requires both the following two conditions to be met:

1. \[\tilde{\alpha}_s = \frac{c}{\Delta \gamma H} > \alpha_x = \frac{R}{\pi_m H - (1 - \pi_m) l} \]
   \[l < \frac{\pi_m H - \Delta \gamma H R}{\Delta \gamma H R (1 - \pi_m)} \]

2. \[\tilde{\alpha}_x = \frac{c - \Delta \gamma R}{\Delta \gamma (1 - \pi_m) (H + l)} > \frac{R}{\pi_m \gamma H - (1 - \pi_m) l} = a_p \]
   \[l < \frac{\pi_m H - \Delta \gamma (1 - \pi_m (1 - \gamma_1))}{\Delta \gamma (1 - \pi_m (1 - \gamma_1))} \cdot \frac{R}{1 - \pi_m} \]

Suppose that \(R/c \neq 0\). Does that mean that there is no tradeoff between effort and fraud? Rearranging terms in (2), there exists an \(l > 0\) such that satisfies condition (2) so long as \(\pi_m c \geq \Delta \gamma R (1 - \pi_m (1 - \gamma_1)) \gamma_1^{-1}\).

From rearranging (1), there exists an \(l > 0\) satifying (1) so long as \(\pi_m c \geq \Delta \gamma R\), which is a less strict condition than (2). Hence, the necessary condition for there to be some tradeoff is \(\pi_m c \geq \Delta \gamma R\), and for it to be a greater tradeoff in that the manager fully pools in zone M1/M2 before exerting effort, \(\pi_m c \geq \Delta \gamma R (1 - \pi_m (1 - \gamma_1)) \gamma_1^{-1}\), which at its maximum as \(\gamma_1 \to 1\) is \(\pi_m c \geq (1 - \gamma_0) R\). Since man-
agers of large firms tend to be paid a lot of money (which should be proportional to $c$), but suffer personal penalties quite rarely, this condition does not seem to be particularly unreasonable.

7 Appendix B: Type 1 error

One can show that type 1 error can be readily incorporated into the model (indeed, it does not affect any of the results). The main effect is to reduce the effective magnitude of the penalty. Suppose, for instance, that $\Pr(R|\eta = \eta') = \theta$. Then $E[R|\eta' = L = \eta] = \theta R$. This models a failure of the punisher to correctly observe that the manager disclosed low when assessing whether to impose a personal sanction on the manager.\(^9\)

This is reasonable in light of the complexity of disclosure and the difficulty courts have in evaluating whether firms disclosed accurately given the ultimate outcome. We can write the separating and pooling conditions as follows:

\[
\text{separation : } -\theta R \geq \alpha [\pi_m H + (1 - \pi)(L - l)] - R \\
\text{pooling : } \alpha [\pi_m \gamma e H - (1 - \pi_m)l] - R > -\theta R
\]

One can then rewrite the conditions utilizing a $(1 - \theta) R$, that is, the effect of the reputational sanction is diminished by the likelihood of type 1 error.

For the effort conditions of the manager, the conditions for separation and mixing effort changes (the possibility of getting wrongly punished creates an additional incentive to exert effort), though the compensation required in the pooling case for effort ($\bar{\alpha}_p$) is unchanged.

\[
\text{effort if separating : } \bar{\alpha}_s \geq \frac{c - \Delta \gamma \theta R}{\Delta \gamma H} \\
\text{effort if pooling : } \bar{\alpha}_p \geq \frac{c - \Delta \gamma R}{\Delta \gamma (1 - \pi_m) (H + l)} \\
\text{effort if mixing : } \bar{\alpha}_x \geq \frac{c - (1 + \theta) \Delta \gamma R}{\Delta \gamma (1 - \pi_m) (H + l)}
\]

\(^9\)An alternative form of type 1 error would be a failure by the regulator to correctly perceive the firm’s type, which could result in overdeterrence in that managers of high type firms may choose to disclose their type as low.
Overall, the effect of this form of type 1 error is that the incentives to disclose truthfully are reduced, but the incentives to exert effort are increased, since the manager is effectively being punished for low cashflows. None of the shareholder’s choice of compensation decisions will be affected.