Guilt Shall Not Escape Nor Innocence Suffer: A Theory of Optimal Prosecutor Behavior when Defendant Guilt is Uncertain

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GUILT SHALL NOT ESCAPE nor INNOCENCE SUFFER: A THEORY OF OPTIMAL PROSECUTOR BEHAVIOR WHEN DEFENDANT GUILT is UNCERTAIN*

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Abstract

This paper attempts to derive optimal prosecutor behavior with respect to plea bargaining (from a societal perspective) when defendant guilt is uncertain, and society cares both about incarcerating guilty individuals and not incarcerating innocent individuals. While defendant guilt is uncertain to everyone but the defendant, I assume that prior to trial, both prosecutors and defendants observe a signal of the defendant’s guilt, after which the prosecutor can make a take-it-or-leave-it plea bargain offer which the defendant can accept or reject. If rejected, the defendant goes to trial, where a jury observes a new signal of the defendant’s guilt. The key assumptions of the model are that the two guilt signals are correlated and are both informative (but noisy). Given this environment, I characterize optimal prosecutor behavior and show that it is always optimal to allow for sufficient prosecutorial discretion over the plea bargaining process so that plea offers can be tied to the initial guilt signal emitted by each defendant.

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1 Introduction

“The United States Attorney is the representative not of an ordinary party to a controversy but of a sovereignty whose obligation to govern impartially is as compelling as its obligation to govern at all; and whose interest, therefore, in a criminal prosecution is not that it shall win a case, but that justice shall be done. As such, he is in a peculiar and very definite sense the servant of the law, the twofold aim of which is that guilt shall not escape or innocence suffer.” *Supreme Court opinion in Berger v. United States* (as quoted by Reinganum (1988)).

A primary tension in any criminal justice system is between ensuring punishment for those defendants who are guilty of the crime for which they are accused, and ensuring that innocent individuals are not punished for crimes they did not commit.\(^1\) As long as it is possible that innocent individuals can be convicted for crimes they didn’t commit, society will always face this tension between wanting to increase sentences in order to punish the guilty, while simultaneously wanting to decrease sentences in order to impose minimal costs on the falsely convicted.

One possible way in which society might optimally manage this tension is via the plea bargaining process. For example, because accepting a plea bargain is generally a voluntary decision on the part of the defendant, and only the defendant generally knows for sure whether he is guilty or not, society may want use plea bargaining to more effectively sort the innocent from the guilty. Alternatively, plea bargaining may be used to mitigate the risk of imposing long sentences on those defendants who are more likely to be innocent. The question to be examined by this paper is, given juries and prosecutors have only imperfect information regarding defendant guilt, how should plea bargaining be applied? In particular, will society be better off by constraining prosecutors to treat

\(^1\)An area where this tension has become particularly acute is with respect to the death penalty. For example, in January of 2000, the then governor of Illinois placed a moratorium on all criminal executions due to concerns raised over the number of innocent individuals who may be on death row.
individuals arrested for the same crime equally (i.e. enable prosecutors to commit to a particular sentence length in plea negotiations), or by allowing prosecutors to have complete discretion over plea offers (i.e. allows prosecutors to adjust plea offers on an individual basis)?

In the environment I examine, I assume society becomes progressively worse off the longer the sentence imposed on an innocent individual, but becomes progressively better off the longer the sentence imposed on a guilty individual. Moreover, only defendants know for sure whether they are guilty or innocent, but after arrest and prior to a trial, the prosecutor observes and a noisy signal of the defendant’s guilt (i.e. the strength of the initial case against the defendant). I then attempt to incorporate the rules of discovery by assuming that this observed guilt signal must also be revealed to the defendant prior to trial. At this stage, the prosecutor can make a take-it-or-leave-it plea bargain sentence offer to the defendant, and the defendant can choose whether or not to accept this offer by pleading guilty and agreeing to serve the plea bargained sentence. If no plea bargain is struck, the defendant goes to trial, where a jury observes a new signal of guilt and decides whether or not to convict. If it chooses to convict, the defendant receives a sentence length that is dictated by an exogenous legislative body.

In addition to assuming both prosecutors and defendants observe some information regarding the strength of the case against the defendant prior to trial, the other key assumptions of the model are the following. First, the guilt signal initially observed by the prosecutor and defendant is assumed to be correlated with any subsequent guilt signal observed by the jury. Second, both the initial guilt signal and the subsequent guilt signal observed by the jury are assumed to be informative, meaning higher values of either of the signals are more likely to come from guilty defendants (i.e. stronger evidence is more likely to arise for guilty defendants than innocent defendants).

Given this environment, I show that it is always optimal (from a societal perspective) for prosecutors to have discretion over the plea bargain they offer, such that the offered plea bargain increases in the strength of the initial evidence against the defendant.
Furthermore, I show that for defendants who exhibit low initial guilt signals, it is optimal for prosecutors to offer relatively lenient plea bargain sentences such that both innocent and guilty defendants with these low initial guilt signals accept the plea. Alternatively, for defendants exhibiting high initial guilt signals, it is optimal for prosecutors to offer only exceedingly harsh plea bargain sentences such that all of the defendants with these high initial guilt signals choose to go to trial. Moreover, under some conditions, it is optimal for prosecutors to offer moderately harsh plea bargains to defendants who exhibit moderate initial guilt signals, such that only the guilty defendants with these moderate initial guilt signals choose to accept the plea offer.

An important implication coming from these results is that, when defendant guilt is inherently uncertain and both prosecutors and defendants observe some information regarding the strength of the pre-trial case, it is always optimal to allow for a broad range of plea bargain offers even for defendants arrested for the same crime, as well as for enough discretion on the part of prosecutors so that plea bargain offers can be tailored to the strength of the initial case against the defendant.

The remainder of the paper is organized as follows. Section 2 discusses how this paper relates to the previous literature on criminal justice and optimal prosecutor behavior. Section 3 presents the model and the primary conclusions regarding optimal prosecutor behavior. Section 4 discusses how this optimal behavior may change with different types of sentencing legislation. Finally, Section 4 summarizes and concludes.

2 Previous Literature

A variety of papers have theoretically examined the importance of accounting for imperfect information regarding guilt in the criminal justice system (Andreoni, 1991; Friedman and Wickelgren, 2003; Png, 1986; Rubinfield and Sappington, 1987; Gay et al., 1989). The papers most related to this one however, are Grossman and Katz (1983), Reinganum (1988), and Baker and Mezzetti (2001). Like this paper, these papers attempt to charac-
terize optimal prosecutor behavior in the face of uncertainty regarding defendant guilt. In each case, the authors assume that both innocent and guilty individuals are arrested, but the probability that a jury convicts an innocent individual is strictly less than the probability that a jury convicts a guilty individual (although innocent individuals are still assumed to be convicted by juries with a strictly positive probability). Moreover, these authors assume that society incurs utility from punishing guilty individuals, but incurs disutility from punishing innocent individuals, and prosecutors at least partially act as agents of society. Several very valuable insights concerning optimal prosecutor behavior arise from these models. Notably, under some conditions, it may be optimal to use plea bargaining as a sort of screening mechanism to separate (at least partially) the innocent from the guilty.

Reinganum (1988) and Baker and Mezzetti (2001) provide a further important innovation to Grossman and Katz (1983), in that they assume prosecutors observe some information correlated with the guilt of the defendant prior to going to trial (i.e. the strength of the case against the defendant). Reinganum’s proposed environment is arguably the more general however, in that the pre-trial information observed by prosecutors is continuous in nature rather than binary as assumed in Baker and Mezzetti. This proves to be important because by making the information binary, where one of the realizations excludes any possibility of guilt while the other is uninformative, means that all defendants who emit the innocent signal are released, and all the defendants who emit the uninformative signal appear identical to the prosecutor. Hence, other than in the most basic way, the Baker and Mezzetti model cannot be used to analyze the optimality of allowing prosecutors to have substantial discretion over plea offers, which is one of the primary interests of both Reinganum’s analysis as well as this paper.³ Fur-

²Grossman and Katz (1983) additionally discuss the optimality of plea bargaining as a system of insurance even when all defendants are guilty, as well as the implications of different levels of risk aversion between guilty and innocent defendants. However, these features are not examined in this paper.

³Specifically, in Baker and Mezzetti (2001), within a particular arrest charge, plea offers will neces-
thermore, Reinganum’s analysis is also closer in spirit to this one in that it attempts to derive optimal prosecutor behavior when prosecutors act as direct agents of society.\(^4\)

The primary dimension on which this paper extends the analysis of optimal prosecutor behavior is by allowing both prosecutors and defendants to observe some information regarding the strength of the case against the defendant prior to trial. In particular, unlike in Reinganum (1988), where both the defendant and the prosecutor had private information (the defendant about his guilt, the prosecutor about the strength of the case), in the model proposed here only the defendant has private information (about his guilt), but both have imperfect information about the likelihood of conviction at trial. In essence, this model tries to incorporate the rules of disclosure, where the evidence against a defendant must be revealed to the defense prior to trial, into the characterization of optimal prosecutor behavior when defendant guilt is uncertain.

3 Model

Assume the criminal justice system is imperfect, meaning both guilty and innocent individuals are arrested for each type of crime. Let \( t = G, I \) denote the two types of arrested defendants: those who are guilty and those who are innocent. While a defendant knows his type, the defendant’s type is not directly observable to others. Rather, only imperfect signals of guilt can be observed. Specifically, assume there are two random variables drawn for each defendant, \( s, \theta \in [s, \infty) \), such that both prosecutors and defendants observe \( s \) for each defendant and, if a defendant goes to trial, a jury observes \( \theta \) for each defendant. Assume both prosecutors and defendants observe \( s \) prior necessarily be identical across all defendants (except for those who emit the definitively innocent guilt signal, who are released).

\(^4\)By contrast, Baker and Mezzetti focus on optimal prosecutor behavior when prosecutors not only care about the interest of society, but also attempt to further individual goals. Namely, prosecutors in Baker and Mezzetti’s environment get direct disutility from losing a case at trial above and beyond any court costs, regardless of whether the defendant is guilty or innocent.
to any decisions, but neither learn \( \theta \) until a trial.\(^5\)

These signals can be thought of as the strength of the evidence against a defendant. There are many reasons why juries may observe different evidence than the evidence initially observed by the prosecutor. For example, the judge may fail to admit some evidence, witnesses may have different credibility to a jury than a prosecutor, the defense attorney may be a good or poor trial lawyer, or new evidence may arise during the course of trial.

In order to model these signals more precisely, I will make the following assumptions. First, I assume these signals are positively correlated, meaning a high initial guilt signal emitted prior to trial means that it is more likely that the jury will also observe a high guilt signal if the case goes to trial. More specifically, for any \( \theta' \in [s, \infty) \), let \( P(\theta \geq \theta'|t, s) \) denote the probability an individual of type \( t \), who emits a guilt signal of \( s \) prior to trial, will emit a guilt signal \( \theta \) greater than \( \theta' \) to the jury. Given this definition, the positive correlation assumption implies \( P(\theta \geq \theta'|t, s) \) is increasing in \( s \) for both types \( t \).

Second, I assume the two guilt signals are informative, meaning higher guilt signals are more likely to come from guilty individuals. More specifically, assume that \( P(\theta \geq \theta'|G, s) > P(\theta \geq \theta'|I, s) \) for any guilt initial signal \( s \), and that \( P(G|\theta) \) is strictly increasing in \( \theta \) (where \( P(G|\theta) \) is the probability that a defendant is guilty given a signal \( \theta \)) and \( P(G|s) \) is strictly increasing in \( s \) (where \( P(G|s) \) is the probability that a defendant is guilty given a signal \( s \)). Third, while correlated and informative, assume these signals generally fail to perfectly eradicate uncertainty. In particular, assume \( 0 < P(G|s) < 1 \) for all \( s > s \), \( P(\theta \geq \theta'|t, \infty) > 0 \), and \( \lim_{s \to \infty} P(\theta \geq \theta'|t, s) < 1 \). However, assume that at the extremes, the signals are perfectly informative, meaning \( P(G|s) = 0 \) and \( \lim_{s \to \infty} P(G|s) = 1 \).\(^6\)

\(^5\)While the model assumes that both prosecutors and defendants observe \( s \) simultaneously, with out loss of generality we can assume that prosecutors observe \( s \) first, but are required to truthfully reveal the \( s \) they observe to the defendant prior to end of plea bargaining negotiations.

\(^6\)For example, in some cases, a negative DNA test can reveal a defendant to be innocent, while a
Given the initially observed guilt signal $s$, the prosecutor decides the length of sentence he will offer each defendant as a plea bargain, denoted $z_p$. If the defendant accepts the plea bargain sentence, the defendant receives a sentence of $z_p$, and the judicial process ends for this defendant. If the defendant rejects this plea bargain, the defendant goes to trial, where a jury observes the subsequent guilt signal $\theta$ and decides whether or not to convict.\(^7\) If convicted by the jury, the defendant receives a sentence $\bar{z}$, where the prosecutor takes $\bar{z}$ as exogenously determined. If not convicted by the jury, the defendant receives a sentence of length zero.

Finally, assume individuals of type $t$ incur disutility of $-V_t(z)$ upon receiving a sentence length of $z$, where $V_t(0) = 0$, $V_t'(z) \geq 0$ and $V_t''(z) > 0$ for all $z > 0$ (i.e. individuals incur disutility from incarceration at an increasing rate). Moreover, assume $V_G'(0) \leq V_I'(0)$ and $V_G''(0) \geq V_I''(0)$. In words, assume innocent individuals get at least as much disutility from short sentences as guilty individuals.\(^8\) Furthermore, assume society’s disutility from incarcerating an innocent individual is proportional to that individual’s disutility. Specifically, if $-U_I(z)$ represents society’s disutility associated with incarcerating an innocent individual for $z$ months, assume $-U_I(z) = -\lambda V_I(z)$ for some parameter $\lambda > 0$. This means that the disutility society incurs from increasing the sentence imposed on an innocent individual is also increasing at an increasing rate.\(^9\) This component of society’s utility is depicted in figure 1. Alternatively, assume society incurs disutility of $-U_G(z)$ for imposing a sentence of only length $z$ on a guilty individual, where $-U_G(0) < 0$ and $U_G'(z) < 0$ and $U_G''(G) > 0$. In words, longer sentences imposed on guilty defendants increases society’s utility, but at a decreasing rate. This component of society’s preferences is depicted in figure 2. Finally, assume $-U_G'(0) > U_I'(0)$, or positive DNA test can show a defendant is indeed the culprit.

\(^7\)Assume the jury does not observe the initial guilt signal $s$.

\(^8\)Note however, that these assumptions allow guilty and innocent individuals to incur the same disutility with respect to sentence length.

\(^9\)Note that these societal preferences mean society does not get disutility from convicting an innocent individual if that individual is not punished.
that society’s marginal utility from imposing a marginally small sentence on a guilty individual exceeds society’s marginal disutility of imposing a marginally small sentence on an innocent individual.

The interest of this paper lies in characterizing optimal prosecutor behavior in this environment, given prosecutors are acting as direct agents of society. In other words, the primary question asked in this paper is what plea bargain sentence should be offered to each defendant, given each defendant’s initial guilt signal \( s \)? The remainder of this section derives optimal prosecutor behavior via backward induction, where jury behavior is characterized first, then defendant behavior is analyzed given this jury behavior, and finally optimal prosecutor behavior is derived given the derived defendant and jury behavior.

3.1 Jury Behavior

Let \( \pi(G|\theta) \) represent the jury’s beliefs concerning the probability that an individual who emits a guilt signal \( \theta \) at trial is guilty. Note that it will not necessarily be the case that \( \pi(G|\theta) = P(G|\theta) \). Indeed, if juries are perfectly rational, and there is considerable selection into which defendants go on to trial, then it will certainly not be the case that \( \pi(G|\theta) = P(G|\theta) \). However, it is not necessary for these probability of guilt functions used by the jury to actually correspond to the truth. All that is necessary for this model is that this perceived probability of guilt functions follow the same restrictions that were imposed on \( P(G|\theta) \) above.\(^{10}\)

Juries are assumed to be made up of members of society, meaning they will have the same preferences as society. Therefore, given a defendant deemed guilty by a jury will receive a sentence length of \( \tau \), the jury’s expected utility for convicting a defendant who emits a guilt signal of \( \theta \) is \( -\pi(G|\theta)U_G(\tau) - (1 - \pi(G|\theta))U_I(\tau) \). Alternatively, the

\(^{10}\)Note, these assumptions on \( \pi(G|\theta) \) certainly do not rule out the possibility that jury’s beliefs corresponds to the truth in equilibrium. I simply do not restrict jury’s beliefs to correspond to the truth, which greatly simplifies the analysis.
jury’s expected utility for not convicting a defendant who emits a guilt signal of $\theta$ is 
$-U_G(0)\pi(G|\theta)$. Therefore, a jury will convict an individual if he emits a guilt signal such 
that $-\pi(G|\theta)U_G(\pi) - [1 - \pi(I|\theta)]U_I(\pi) \geq -U_G(0)\pi(G|\theta)$. Re-arranging this expression 
we can see that a jury convicts a defendant if and only if 
\[
\frac{U_G(0) - U_G(\pi)}{U_I(\pi)} \geq 1 - \frac{\pi(G|\theta)}{\pi(G|\theta)}.
\] 
(1)

Since $\pi(G|\theta)$ is strictly increasing in $\theta$, $\pi(G|S) = 0$, and $0 < \pi(G|\theta) < 1$ for all $\theta > S$, 
equation (1) implies that there exists a $\theta^*$ such that the jury will convict a defendant if 
and only if the defendant emits an guilt signal at trial such that $\theta \geq \theta^*$. 

3.2 Defendant Behavior

Given a defendant believes a jury will convict anyone who emits a guilt signal at trial 
greater than $\theta^*$, a defendant of type $t$ will accept a plea bargain sentence length of $z_p$ if 
and only if $-V_t(z_p) \geq -P(\theta \geq \theta^*|t, s)V_t(\pi)$.\footnote{Once again, it is not necessary that the perceived probabilities of emitting a guilt signal greater than $\theta^*$ used by defendants correspond exactly to the truth. Rather, it is only necessary that the perceived probability probability is consistent with the assumptions made above, and prosecutors know these perceived probabilities used by defendants.} Re-arranging this expression we get 
\[
P(\theta \geq \theta^*|t, s) \geq \frac{V_t(z_p)}{V_t(\pi)}.
\] 
(2)

Since $V_t(z_p)$ is increasing in $z_p$, equation (2) implies that there exists a $z_t(s)$ such 
that a defendant of type $t$ who emits an initial guilt signal of $s$ should accept a plea 
bargain sentence $z_p$ as long as $z_p \leq z_t(s)$. Moreover, since $P(\theta \geq \theta^*|t, s)$ was assumed 
to be increasing in $s$, we know $z_t(s)$ will be increasing in $s$. In words, the higher the 
initial guilt signal, the higher the plea bargain sentence a defendant will be willing to accept. Similarly, since it was assumed that $P(\theta \geq \theta^*|G, s) > P(\theta \geq \theta^*|I, s)$ for all $s$, it
will be true that \( z_G(s) > z_I(s) \) for any given initial guilt signal \( s \).\(^{12}\) This means that for any initial guilt signal, guilty individuals will be willing to accept longer plea bargain sentences. Intuitively, for any given strength of pre-trial evidence, innocent individuals believe they will have a better chance that the actual evidence presented at trial will be insufficient for conviction.

Finally, note that since \( P(\theta > \theta^* | I, \bar{s}) > 0 \), it will be true that \( z_I(\bar{s}) > 0 \). Similarly, since \( \lim_{s \to \infty} P(\theta > \theta^* | I, \bar{s}) < 1 \), it will be true that \( \lim_{s \to \infty} z_I(s) < \bar{z} \).

### 3.3 Prosecutor Behavior

The defendant behavior derived above suggests three relevant strategies for prosecutors with respect to the plea bargain sentence to offer a defendant with an initial guilt signal \( s \). If the prosecutor offers a plea bargain sentence length \( z_p \leq z_I(s) \), the defendant will accept the plea bargain whether he is guilty or innocent. If the prosecutor offers a plea bargain sentence length \( z_I(s) < z_p \leq z_G(s) \), the defendant will accept the plea bargain if he is guilty and reject the plea bargain if he is innocent. Finally, if the prosecutor offers \( z_G(s) < z_p \) the defendant will choose to go to trial whether he is guilty or innocent.

To derive optimal prosecutor behavior with respect to a defendant who emits an initial guilt signal \( s \), I will first derive the prosecutor’s optimal behavior within each of the three strategies discussed above. Then, given the optimal behavior for each strategy, I derive which of the three strategies is optimal overall for each initial guilt signal \( s \).

**(i) Offer a Plea Bargain that Both Innocent and Guilty Defendants Accept**

(i.e. \( z_p \leq z_I(s) < z_G(s) \))

As discussed above, one strategy for the prosecutor is to offer a plea bargain sentence \( z_p \) such that both innocent and guilty defendants accept it. The prosecutor’s utility using this strategy for a defendant who emits an initial guilt signal \( s \) will equal \( E[U(z_p) | z_p \leq z_I(s) < z_G(s)] \).\(^{12}\)

\(^{12}\)Note that this result also relies on the assumption that \( z_G'(0) \leq z_I'(0) \) and \( z_G''(0) \geq z_I''(0) \), as this means \( \frac{V_G(z_p)}{V_G'(z_p)} \leq \frac{V_I(z_p)}{V_I'(z_p)} \) for any \( z_p \).
\begin{align*}
    z_I(s) &= -P(G|s)U_G(z_p) - [1 - P(G|s)]U_I(z_p) \quad \text{(noting that } P(I|s) = 1 - P(G|s)).
\end{align*}

Given 

\([-U_G(0) > U_I(0)],\) it will be true that \(E[U(z_p|z_p \leq z_I(s))]\) is an inverse u-shaped function of \(z_p,\) meaning for any \(s > \underline{s},\) it will have a unique maximum \(z^*(s)\) where

\[\frac{-U_G'(z^*(s))}{U_I'(z^*(s))} = \frac{1 - P(G|s)}{P(G|s)}.
\]

Note that the right-hand side of the above expression is decreasing in \(s,\) and \(\frac{-U_G'(z^*(s))}{U_I'(z^*(s))}\) is decreasing in \(z^*(s),\) meaning \(z^*(s)\) must be increasing in \(s\) in order to maintain equality.

Moreover, since \(P(G|s) = 0,\) we know \(z^*(\underline{s}) = 0,\) and since \(\lim_{s \to \infty} P(G|s) = 1,\) we know \(\lim_{s \to \infty} z^*(s)\) will eventually be bounded by \(\overline{z}.\)

The above argument implies that if the prosecutor chooses to offer a plea bargain such that every defendant who initially emits a guilt signal of \(s\) accepts, it is optimal to offer a plea bargain of length \(z^*(s)\) as defined above. However, this optimal offer is only relevant if both guilty and innocent defendants accept it, meaning \(z^*(s)\) is only relevant for defendants with guilt signals \(s\) such that \(z^*(s) < z_I(s).\) The question then becomes, for which \(s\) will this expression will be true?

Recalling that \(z_I(\underline{s}) > 0,\) we know that at least for \(\underline{s},\) it will be true that \(z^*(\underline{s}) < z_I(\underline{s}).\) However, recalling that \(\lim_{s \to \infty} z_I(s) < \overline{z},\) we also know that \(\lim_{s \to \infty} z^*(s) > \lim_{s \to \infty} z_I(s).\) This means there exists a threshold \(s^*,\) such that \(z^*(s) < z_I(s)\) if \(s < s^*\) and \(z^*(s) \geq z_I(s)\) if \(s \geq s^*\).

Therefore, if the prosecutor chooses to offer a plea bargain such that every defendant will accept it, he should offer a sentence length of \(z_p = z^*(s)\) if the defendant emits a guilt signal of \(s < s^*,\) or \(z_p = z_I(s)\) if the defendant emits a guilt signal of \(s \geq s^*.\)

This means the prosecutor’s utility from using this strategy and behaving optimally will

\[\text{http://law.bepress.com/alea/15th/art34}\]
equal

\[ E[U(z)z_p = z^*(s), s] = -P(G|s)U_G(z^*(s)) - [1 - P(G|s)]U_I(z^*(s)), \]  

(4)

if \( s < s^* \), and

\[ E[U(z)z_p = z_I(s), s] = -P(G|s)U_G(z_I(s)) - [1 - P(G|s)]U_I(z_I(s)), \]  

(5)

if \( s \geq s^* \).

(ii) Offer a Plea Bargain that Only Guilty Defendants will Accept (i.e. \( z_I(s) < z_p \leq z_G(s) \))

As discussed previously, when \( z_I(s) < z_p \leq z_G(s) \), only a guilty defendant will accept the plea bargain offer. Therefore, the prosecutor’s utility when offering a plea bargain in this range will equal

\[ E[U(z)z_p = z_I(s), s] = -P(G|s)U_G(z_I(s)) - [1 - P(G|s)]U_I(z_I(s)), \]  

(6)

if \( s \geq s^* \).

(iii) Offer a Plea Bargain No Defendants Accept (i.e. \( z_I(s) < z_G(s) < z_p \))

When \( z_p > z_G(s) \), no defendants accept the plea bargain. Therefore, the prosecutor is indifferent to what he offers (since nobody will accept it anyway). For concreteness,
say that the prosecutor offers \( z_p = \overline{z} \), which can be interpreted as not offering a plea bargain at all. However, none of the analysis depends on what plea bargain sentence the prosecutor offers when it exceeds \( z_G(s) \).

Since no defendants accept a plea bargain in this range, the prosecutor’s utility from offering a sentence in this range is

\[
E[U(z)|z_p > z_G(s), s] = -P(G|s)P(\theta \geq \theta^*|G, s)U_G(\overline{z}) - [1 - P(G|s)]P(\theta \geq \theta^*|I, s)U_I(\overline{z}).
\] (7)

Now, given the prosecutor’s utility from acting optimally in each of the three relevant ranges, we can now determine optimal prosecutor behavior by simply by comparing his optimized utility across the different plea bargain sentence ranges. This leads to Proposition 1.

**Proposition 1:** There exists two initial guilt signal levels \( s_1 \) and \( s_2 \), such that if \( s_1 < s_2 \), optimal prosecutor behavior with respect plea bargaining will be the following:

(i) For defendants with \( s \leq s_1 \), offer \( z_p = z^*(s) \);

(ii) For defendants with \( s_1 < s \leq s_2 \), offer \( z_p = z_G(s) \);

(iii) For defendants with \( s > s_2 \), offer \( z_p = \overline{z} \);

**Proof:** In Appendix.

In words, Proposition 1 says that when society cares both about imprisoning guilty individuals and not imprisoning innocent individuals, then, under certain restrictions (i.e. \( s_1 < s_2 \)), prosecutors should offer relatively lenient plea bargains to defendants who initially appear to have relatively weak evidence against them so that all defendants in this range accept the plea; offer relatively harsh plea bargains to defendants who appear to have a moderate amount of evidence against them so that only the innocent defendants in this range still choose to pursue a trial; and finally, offer extremely harsh
(if any) plea bargains to defendants with very strong evidence against them so that all of the defendants in this range will go to trial.

Clearly, the optimal prosecutor behavior as described by Proposition 1 is subject to the caveat that it is optimal only if \( s_1 < s_2 \). As can be confirmed in the proof of Proposition 1, it will always be true that \( s_1 < s^* \) (but \( s_1 \) will generally rise as \( s^* \) rises), but it will not necessarily be true that \( s_1 \leq s_2 \), where \( s_2 \) solves

\[
\frac{U_G(0) - U_G(z_G(s_2))}{U_G(0) - U_G(\overline{z})} = P(\theta > \theta^*|G, s_2). \tag{8}
\]

This leads to Corollary 1.

**Corollary 1:** When \( s_1 > s_2 \), there exists an initial guilt signal level \( s_3 \), such that optimal prosecutor behavior with respect plea bargaining will be:

(i) For defendants with \( s \leq s_3 \), offer \( z_p = z^*(s) \);

(ii) For defendants with \( s \geq s_3 \), offer \( z_p = \overline{z} \).

**Proof:** In Appendix.

In words, Corollary 1 says that when \( s_1 > s_2 \), it is optimal to offer those defendants with relatively low initial guilt signals a plea sentence that increases in the initial guilt signal, but is lenient enough that both guilty and innocent defendants will accept it. Alternatively, for defendants who emit relatively high initial guilt signals, offer a prohibitively harsh plea bargain that no defendants will accept.

It is informative to compare the optimal prosecutor behavior as derived in Proposition 1 and Corollary 1 to the optimal prosecutor behavior as derived in Reinganum (1988), in which only the prosecutor observes information about the strength of the case prior to trial. In that model, it is always optimal for the prosecutors to offer sentence lengths of zero to defendants who had very weak initial evidence against them. Somewhat similarly, Proposition 1 and Corollary 1 reveal that, in this environment, it
is optimal for prosecutors to offer defendants with very weak initial evidence against them a plea bargain sentence length of $z^*(s)$, which will be of length zero or very short for defendants who emit a very low $s$ (i.e. short enough that all defendants will accept it).

Furthermore, in Reinganum’s (1988) environment, optimal prosecutor behavior with respect to defendants who are not offered a sentence length of zero depended on the efficiency of the arrest process. If the arrest process was relatively inefficient (i.e. does not do a good job screening out the innocent), then it is optimal for prosecutors to offer plea bargain sentences that are increasing in the initial guilt signal of the defendant, subject to the plea bargain sentence being low enough such that all defendants accept the offer (including the innocent ones). Alternatively, if the arrest process was relatively efficient (i.e. does a good job screening out the innocent), then it is optimal to limit prosecutorial discretion. Namely, it is optimal to allow prosecutors to only be able to offer a fixed plea bargain sentence to all defendants whose cases are not dropped, where this offer is sufficiently severe such that only guilty individuals accept the plea bargain.

In other words, depending on the parameters, when the prosecutor does not reveal the strength of the initial evidence against the defendant to the defendant, Reinganum (1988) shows that it is either optimal to allow the prosecutor to have the discretion to lower the plea bargain sentence offered to defendants with weaker initial evidence against them, or restrict prosecutorial discretion such that prosecutors can only offer a single plea bargain, where this offer effectively separates the innocent from the guilty.

\[15\]
Recall that it may be that $z^*(s)$ equals zero even for some $s$ close to by strictly greater than $z$.

\[16\]
Only defendants with extremely weak initial evidence against them will have their case dropped.

\[17\]
As mentioned above, this “separating” plea offer is a feature that also comes out of both Grossman and Katz’s (1983) analysis, where prosecutors do not observe any information relevant to a defendant’s guilt prior to trial. Similarly, the equilibrium derived in Baker and Mezzetti’s (2001) analysis, where initial guilt signals were either perfectly informative about innocence or perfectly uninformative, exhibits partial separation over some range of crime severity.
By contrast, in the model developed here, where defendants also observe information regarding the relative strength of the initial evidence against them prior to trial, allowing prosecutors to decrease the plea bargains offered to defendants with weaker initial evidence against them, and using plea bargaining to separate the guilty from the innocent, are no longer mutually exclusive. In fact, it will always be optimal for the prosecutor to offer shorter plea bargain sentences to those defendants with weaker initial evidence against them, even when it is also optimal for the prosecutor to use plea bargaining to separate the innocent from the guilty.

However, more similarly to the results from Reinganum’s (1988) environment, the optimality of using the separating strategy may depend on the efficiency of the arrest process. To see why, note that a more efficient arrest process will mean relatively fewer arrested individuals are innocent, generally causing $P(G|s)$ to increase for all $s > s_*$. As can be seen from equation 3, this will mean $z^*(s)$ will increase for all $s > s_*$. 

By itself, this would mean that $s^*$ decreases as the arrest process becomes more efficient, meaning $s_1$ generally decreases as the arrest process becomes more efficient, suggesting that using the separating strategy tends to become optimal as the arrest process becomes more efficient. However, in this environment, there also may be a countervailing effect associated with a more efficient arrest process. Namely, if juries recognize that the arrest process is more efficient, then $\pi(G|\theta)$ will also be greater for all $\theta > s_*$, implying $\theta^*$ will get smaller as the arrest process becomes more efficient. As can be confirmed in equation (2), this in turn will mean $z_I(s)$ will be higher for all $s > s_*$. Therefore, as the arrest process becomes more efficient, $z^*(s)$ rises, but so will $z_I(s)$ if juries beliefs are adjusted to account for this greater efficiency. Hence, it is unclear whether $s^*$ (i.e. the threshold marking where $z^*(s)$ exceeds $z_I(s)$) increases or decreases as the arrest process becomes more efficient. Since we know that $s_1$ will generally track $s^*$, it will not necessarily be the case that an increasingly efficient arrest process will eventually cause the separating strategy to become optimal. This indeterminacy arises here because jury behavior is allowed to adjust to the efficiency of the arrest process,
something not possible in Reinganum’s (1988) environment.

Finally, in Reinganum’s (1988) environment, when it is optimal for prosecutors to offer plea bargains that separate the guilty from the innocent, it will be the case that only the innocent defendants choose to go to trial. This is problematic because a necessary condition for this equilibrium to hold is that juries become increasingly likely to convict a defendant in the guilt signal he emits a trial. However, if only innocent individuals go to trial, such jury behavior necessarily requires irrational beliefs on the part of juries.\textsuperscript{18} This issue is not necessarily a problem in the environment used here however, as all defendants who emit very high initial guilt signals will go to trial when prosecutors act optimally. Therefore, even if plea bargaining perfectly separates the guilty from the innocent among those who emit moderate initial guilt signals, as long as there is a relatively high fraction of guilty defendants who emit very high initial guilt signals, there will always be a substantial mix of guilty and innocent individuals going to trial. Hence, it will be reasonable for juries to believe a defendant is more likely to be guilty as the guilt signal at trial increases.

\section{4 How Does Optimal Prosecutor Behavior Change with Sentence Length Legislation?}

We can also use this model to consider how optimal prosecutor behavior will change given different sorts of changes in sentencing legislation. For example, legislators could increase the maximum sentence imposed on criminals who are convicted of a particular crime (i.e. legislators could increase \( \tau \)). Alternatively, legislators could impose a mandatory minimum sentence, such that anyone who is convicted or pleads guilty to a particular crime must receive a sentence greater than or equal to some lower bound \( \delta \). This section examines how optimal prosecutor behavior will respond to such changes in sentencing legislation.

\textsuperscript{18}This issue also arises in Grossman and Katz (1983). Alternatively, a key feature of Baker and Mezetti’s (2001) environment is that this issue does not arise in the equilibria they derive.
4.1 Increasing the Maximum Sentence

One clear effect of increasing $\bar{\pi}$ will be that it will increase $\theta^*$ (if jury beliefs are held constant), as can be seen in equation (1). Looking at the left-hand side of equation (2), such an increase in $\theta^*$ will lower $z_t(s)$ for any $s$ and any type $t$, all else equal. However, as can be seen in the right-hand side equation (2), an increase in $\bar{\pi}$ will increase $z_t(s)$ for any $s$ and any type $t$. Therefore, it is unclear whether $z_t(s)$ for any $s$ and any type $t$ will increase or decrease following an increase in $\bar{\pi}$. However, given the offsetting effects, one could argue that an increase in $\bar{\pi}$ is likely to lead to only small changes in $z_t(s)$ for either type $t$. Given $z_t(s)$ stays relatively unaffected by changes in $\bar{\pi}$, and since an increase in $\bar{\pi}$ would also not affect $\theta^*$, $s_1$ would likely stay largely unaffected by increases in $\bar{\pi}$.

By itself, the above result would suggest that increasing the maximum sentence for a crime would not make it any more or less likely for it to be optimal for prosecutors to separate the guilty from the innocent via plea bargaining. However, recall that $s_2$ is defined as the threshold such that for any $s \geq s_2$, it will be true that

$$\frac{V_G(z_G(s))}{V_G(\bar{\pi})} \geq \frac{U_G(0) - U_G(z_G(s))}{U_G(0) - U_G(\bar{\pi})}.$$ 

Given our assumptions about $V_G(z)$ and $U_G(z)$, both sides of the above expression decrease as $\bar{\pi}$ increases. Hence, it is unclear whether $s_2$ increases or decreases as $\bar{\pi}$ increases. However, since $V_G(z)$ is increasing and convex, while $U_G(z)$ is decreasing and convex, it will generally be true that increasing $\bar{\pi}$ will increase $V_G(\bar{\pi})$ more than $U_G(0) - U_G(\bar{\pi})$. This in turn means that $s_2$ may very plausibly increase as $\bar{\pi}$ increases. Hence, while not necessarily the case, increasing the maximum sentence may make it more likely for it to be optimal to use plea bargaining to not only shorten the sentences.

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\textsuperscript{19}It is worth noting that in this section these changes in sentencing legislation are assumed to be exogenous and not necessarily optimal. It is a project for further research to simultaneously analyze optimal prosecutor behavior and optimal sentence length.
of those who are more likely to be innocent, but also as a way of partially sorting the guilty from the innocent as described in Proposition 1.

4.2 Imposing a Mandatory Minimum Sentence

Another possible type of sentencing legislation is the introduction of a mandatory minimum sentence targeting a particular crime. In the context of this model, such legislation can be modelled as a sentence $z$, such that anyone convicted for or pleading to a particular crime is required to serve a sentence of $z$ or greater. Clearly, this law will not affect prosecutor behavior with respect to defendants whose initial guilt signals are such that their plea sentence offer as prescribed by the optimal policy exceeds $z$. However, for those defendants whose initial guilt signals are low enough such that their optimal plea offers are less than $z$, it may be optimal for prosecutors to drop the charges (or effectively imposing a sentence length of zero). However, another strategy may be to prosecute and/or allow the defendant to plead to a different crime unaffected by the mandatory minimum sentence law. In other words, for those defendants exhibiting very low initial guilt signals, it may be optimal for the prosecutor to circumvent the law by altering the charges for which the defendant is prosecuted.

Interestingly, such behavior appears to be occurring with respect to several three-strikes type mandatory minimum sentencing laws passed in several states throughout the 1990s. In particular, prosecutors appear to become significantly more likely to lessen an initial felony charge to a misdemeanor, when conviction for the initial felony charge would lead to sentencing under a three-strikes type law (Bjerk, 2004).

5 Summary and Conclusion

Given the nature of crime, it is generally the case that the actual guilt or innocence of any given defendant cannot be known with certainty by anybody but the defendant. Rather, only signals correlated with guilt, namely the evidence against a defendant, are apparent
to outside observers such as police, attorneys, prosecutors, judges, and juries. Moreover, the disclosure rules of our judicial system require prosecutors to reveal the evidence they have against a defendant to the defendant prior to trial. Given this evidence, prosecutors then generally have the ability to strike plea bargains with defendants. The goal of this paper was to characterize optimal prosecutor behavior regarding plea bargain offers in this environment, when the prosecutor is an agent of society and society cares not only about incarcerating the guilty, but also not punishing the innocent.

The results of this analysis show that, in the environment proposed here, it is optimal for prosecutors to make their plea bargain offers increasingly severe the stronger the initial evidence appears to be against the defendant. Moreover, for defendants who appear to have relatively weak evidence against them, prosecutors should offer relatively lenient enough plea bargains such that all defendants will accept the offer, while for defendants who appear to have very strong evidence against them, prosecutors should offer only excessively harsh plea bargains such that all of these defendants choose to go to trial. However, it may be optimal for prosecutors to offer relatively harsh (but not exceedingly so) plea offers to those defendants who appear to have moderate evidence against them, such that only the guilty defendants in this category accept the offer.

These results provide some useful guideposts for thinking about issues and concerns related to plea bargaining. Namely, having a large fraction of criminal cases resolved through plea bargains, and/or having wide disparity in plea bargain sentences for the same initial charge, are not necessarily reasons for policy interventions or even concern. In fact, this paper shows that these outcomes are likely to arise when prosecutors are acting in society’s best interest and society cares both about punishing the guilty and not punishing the innocent. Furthermore, giving prosecutors the discretion to tailor their plea offers to the characteristics of the defendant (namely the initial evidence against him or her) is necessarily a feature of any optimal prosecution policy.

In this way, this paper is meant to be more prescriptive than descriptive. This is not to say that this model does not correctly model true prosecutor behavior in the
criminal justice system, indeed the hope is that it does. However, determining whether prosecutors generally act in accordance with the optimal behavior as derived by this model is an empirical question and provides an important avenue for further research.
Appendix

Proof of Proposition 1:

The sequence of the proof will be as follows. First, I will prove that there exists an $s_1$ such that if $s < s_1$, the prosecutor is made better off by offering $z^*(s)$ rather than either $z_G(s)$ or $z_I(s)$, while if $s \geq s_1$, the prosecutor is made better off by offering $z_G(s)$ rather than either $z^*(s)$ or $z_I(s)$. Then, I show that there exists an $s_2$ such that if $s > s_2$, then the prosecutor is made better off by offering $\bar{z}$ rather than $z_G(s)$. Given this, it will be true that as long as $s_1 < s_2$, it will be optimal for the prosecutor to offer $z_p = z^*(s)$ if $s < s_1$, $z_p = z_G(s)$ if $s_1 \leq s \leq s_2$, and $z_p = \bar{z}$ if $s > s_2$, which will confirm Proposition 1.

Say a defendant emits an initial guilt signal $s \geq s^*$. We know from equation (5) that if the prosecutor chooses to offer this defendant a plea that the defendant will accept whether he is guilty or innocent, we know the prosecutor’s optimized utility will be $E[U(z)|z_p = z_I(s), s \geq s^*] = -P(G|s)U_G(z_I(s)) - [1 - P(G|s)]U_I(z_I(s))$. Alternatively, from equation (6), we know that if the prosecutor offers this defendant a plea such that the defendant will only accept it if he is guilty, the prosecutor’s optimized utility will be $E[U(z)|z_p = z_G(s), s \geq s^*] = -P(G|s)U_G(z_G(s)) - [1 - P(G|s)]P(\theta \geq \theta^s|I, s)U_I(\bar{z})$.

Recalling the definition of $z_I(s)$ as given by equation (2), we know $V_I(z_I(s)) = P(\theta \geq \theta^s|I, s)V_I(\bar{z})$. Moreover, since $U_I(z) = \lambda V_I(z)$, we know $U_I(z_I(s)) = P(\theta \geq \theta^s|I, s)U_I(\bar{z})$. Substituting in this equality into the expression for $E[U(z)|z_p = z_G(s), s \geq s^*]$ as given above, we find that

$$E[U(z)|z_p = z_I(s), s \geq s^*] - E[U(z)|z_p = z_G(s), s \geq s^*] = P(G|s)[U_G(z_G(s)) - U_G(z_I(s))].$$

Since $z_G(s) > z_I(s)$, and since $U_G(z)$ is decreasing in $z$, the above expression implies $E[U(z)|z_p = z_I(s), s \geq s^*] - E[U(z)|z_p = z_G(s), s \geq s^*] < 0$. Therefore, it will always be the case that it is optimal to offer a plea bargain that only the guilty accept (i.e. $z_G(s)$) rather than a plea bargain no defendants accept, for defendants with $s \geq s^*$.
Now say a defendant emits an initial guilt signal \( s < s^* \). We know from equation (4) that if the prosecutor chooses to offer this defendant a plea that the defendant will accept whether he is guilty or innocent, we know the prosecutor’s optimized utility will be \( E[U(z)|z_p = z^*(s), s < s^*] = -P(G|s)U_G(z^*(s)) - [1 - P(G|s)]U_I(z^*(s)) \). As before, if the prosecutor offers this defendant a plea such that the defendant will only accept it if he is guilty, the prosecutor’s optimized utility will be \( E[U(z)|z_p = z_G(s), s < s^*] = -P(G|s)U_G(z_G(s)) - [1 - P(G|s)]P(\theta \geq \theta^*|I, s)U_I(\bar{z}) \). Therefore, the resulting utility difference between these two strategies will equal

\[
[ -P(G|s)U_G(z^*(s)) - [1 - P(G|s)]U_I(z^*(s))] - \\
[ -P(G|s)U_G(z_G(s)) - [1 - P(G|s)]P(\theta \geq \theta^*|I, s)U_I(\bar{z})].
\]

Rearranging the above expression and recalling that \( U_I(z_I(s)) = P(\theta \geq \theta^*|I, s)U_I(\bar{z}) \) we get

\[
P(G|s)[U_G(z_G(s)) - U_G(z^*(s))] + [1 - P(G|s)][U_I(z_I(s)) - U_I(z^*(s))].
\]

Since \( U_G(z) \) is decreasing in \( z \), and we know that \( z^*(s) \leq z_I(s) < z_G(s) \) for \( s \leq s^* \), meaning the first term in the above expression is always negative for \( s \leq s^* \). Alternatively, since \( U_I(z) \) is increasing in \( z \), and we know that \( z^*(s) \) gets increasingly smaller than \( z_I(s) \) as \( s \) gets smaller, meaning the second term gets increasingly positive as \( s \) decreases.

Hence, for equation (9) to be positive, \( s \) will have to fall below some threshold \( s_1 \), where \( s_1 \) is strictly less than \( s^* \). This confirms that if \( s < s_1 \), the prosecutor is made better of by offering \( z^*(s) \) rather than \( z_G(s) \) or \( z_I(s) \), while if \( s \geq s_1 \), the prosecutor is made better of by offering \( z_G(s) \) rather than \( z^*(s) \) or \( z_I(s) \). Moreover, it is also worth noting that a higher \( s^* \) generally implies that either \( z^*(s) \) is lower for each \( s \) and/or \( z_I(s) \) is higher for each \( s \), implying the right side of equation (9) will be greater for any any \( s < s^* \). Therefore, a higher \( s^* \) will generally mean a higher \( s_1 \).
Alternatively, from equations (6) and (7), we know that the prosecutor will be better off offering $\pi$ rather than $z_G(s)$ as long as

$$-[P(G|s)P(\theta \geq \theta^*|G,s)U_G(\pi) - [1 - P(G|s)]P(\theta \geq \theta^*|I,s)U_I(\pi)] - [P(G|s)U_G(z_G(s)) - [1 - P(G|s)]P(\theta \geq \theta^*|I,s)U_I(z_G(s))] > 0.$$ 

Rearranging the above expression and cancelling terms, we get

$$P(\theta > \theta^*|G,s) > \frac{U_G(0) - U_G(z_G(s))}{U_G(0) - U_G(\pi)}.$$ 

Now, recall from equation (2) that $P(\theta > \theta^*|G,s) = \frac{V_G(z_G(s))}{V_G(\pi)}$, meaning equation (10) can be re-written as

$$\frac{V_G(z_G(s))}{V_G(\pi)} > \frac{U_G(0) - U_G(z_G(s))}{U_G(0) - U_G(\pi)}.$$ 

Since $U_G(z)$ is decreasing and convex in $z$, we know the right-hand side of equation (11) will be increasing, but concave in $z_G(s)$. Alternatively, since $V_G(z)$ is increasing and convex in $z$, we know the left-hand side of equation (11) is increasing but convex in $z_G(s)$. Therefore, since $z_G(s)$ is increasing in $s$, there exists some $s_2$ such that equation (11) holds if and only if $s > s_2$. This, in turn, implies that for $s > s_2$ the prosecutor is made better off by offering $\pi$ rather than $z_G(s)$, but if $s \leq s_2$ the prosecutor is made better off by offering $z_G(s)$ rather than $\pi$.

Therefore, we know that as long as $s_1 < s_2$, it will be optimal for the prosecutor to offer $\pi(s)$ if $s < s_1$, offer $z_G(s)$ if $s_1 \leq s \leq s_2$, and offer $\pi$ if $s > s_2$, which confirms Proposition 1.

**Proof of Corollary 1:**

From the proof of Proposition 1 above, we know that it will be optimal for the prosecutor to offer $z_G$ only if $s_1 \leq s < s_2$. Therefore, if $s_1 > s_2$, it will never be optimal to offer $z_G(s)$. 

25
Given it will never be optimal for the prosecutor to offer $z_G(s)$ if $s_1 > s_2$, we can determine optimality by simply comparing equations (4) (since it will never be optimal to offer $z_I(s)$ as shown above) and (7). Specifically, if $s_1 > s_2$, it will only be optimal to offer $\exists$ if the following equation holds

$$\left[-P(G|s)P(\theta \geq \theta^*|G,s)U_G(\exists) - [1 - P(G|s)]P(\theta \geq \theta^*|I,s)U_I(\exists)\right] -$$

$$\left[-P(G|s)U_G(z^*(s)) - [1 - P(G|s)]U_I(z^*(s))\right] > 0.$$

Rearranging the above expression we get

$$P(G|s)[U_G(z^*(s)) - P(\theta \geq \theta^*|G,s)U_G(\exists)] +$$

$$[1 - P(G|s)][U_I(z^*(s)) - P(\theta \geq \theta^*|I,s)U_I(\exists)] > 0. \quad (12)$$

Since we know that at $s$, it will be the case that $P(\theta \geq \theta^*|G,s) > 0$, $P(\theta \geq \theta^*|I,s) > 0$, $P(G|s) = 0$, $z^*(s) = 0$, and since $U_I(z)$ is increasing in $z$, we know that equation (12) will not hold at $s$.

On the other hand, as $s$ approaches infinity, $P(G|s)$ goes to one, $z^*(s)$ goes to (or is capped by) $\exists$, while $P(\theta \geq \theta^*|G,s)$ and $P(\theta \geq \theta^*|I,s)$ are both bounded strictly below one. This implies that there exists some threshold level $s_3$ such that equation (12) holds if and only if $s > s_3$. Hence, if $s_1 > s_2$, it is optimal for the prosecutor to offer $z^*(s)$ if $s \leq s_3$, and to offer $\exists$ if $s > s_3$, confirming Corollary 1.
References


Figure 1 - Underlying shape for $U_I(z)$ function and depiction of $-U_I(z)$ (i.e. how societal utility decreases as the sentence length $z$ imposed on an *innocent* defendant increases).

Figure 2 – Underlying shape for $U_G(z)$ function and depiction of $-U_G(z)$ (i.e. how societal utility increases as the sentence length $z$ imposed on a *guilty* defendant increases).