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Plea Bargains Only for the Guilty

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# Plea Bargains Only for the Guilty

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## **Abstract**

A major concern with plea bargains is that innocent defendants will be induced to plead guilty. This paper argues that the law can address this concern by providing prosecutors with incentives to select cases in which the probability of guilt is high. By restricting the permissible sentence reduction in a plea bargain the law can preclude plea bargains in cases where the probability of conviction is low (L cases). The prosecutor will therefore be forced to – (1) select fewer L cases and proceed to trial with these cases; or (2) select more cases with a higher probability of conviction (H cases) that can be concluded via a less-costly plea bargain. As long as the probability of conviction is positively correlated with the probability of guilt, this selection-of-cases effect implies a reduced number of innocent defendants that accept plea bargains. We argue that the Federal Sentencing Guidelines in fact achieve, albeit inadvertently, this socially desirable selection effect. We further argue that more limited discretion in sentencing facilitates the selection-of-cases effect. In this respect, the Federal Guidelines are superior to some of the state-level guidelines that leave considerable room for discretion in sentencing.

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# PLEA BARGAINS ONLY FOR THE GUILTY

*Oren Bar-Gill\* and Oren Gazal\*\**

## Abstract

A major concern with plea bargains is that innocent defendants will be induced to plead guilty. This paper argues that the law can address this concern by providing prosecutors with incentives to select cases in which the probability of guilt is high. By restricting the permissible sentence reduction in a plea bargain the law can preclude plea bargains in cases where the probability of conviction is low (L cases). The prosecutor will therefore be forced to – (1) select fewer L cases and proceed to trial with these cases; or (2) select more cases with a higher probability of conviction (H cases) that can be concluded via a less-costly plea bargain. As long as the probability of conviction is positively correlated with the probability of guilt, this selection-of-cases effect implies a reduced number of innocent defendants that accept plea bargains. We argue that the Federal Sentencing Guidelines in fact achieve, albeit inadvertently, this socially desirable selection effect. We further argue that more limited discretion in sentencing facilitates the selection-of-cases effect. In this respect, the Federal Guidelines are superior to some of the state-level guidelines that leave considerable room for discretion in sentencing.

Keywords: Plea bargains, Sentencing guidelines.

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## 1. Introduction

About 95% of all convictions in the United States are secured with a guilty plea, most of them through plea bargaining.<sup>1</sup> Yet despite their prevalence, or perhaps due to it, plea bargains remain one of the most controversial practices in the criminal justice system.<sup>2</sup> The fear that innocent defendants would plead guilty animates the often heated debate over plea bargains.<sup>3</sup> And imposing sanctions on the innocent is not only morally wrong, it is also inefficient.<sup>4</sup>

Importantly, most of the responsibility for the wrongful convictions problem lies not on the plea bargain institution, but rather on the inherent inaccuracy of the adjudication process. In an ideal, error-free adjudication system no innocent defendant would ever plead guilty. In fact, given the imperfections of the system, it has been argued that plea bargains can only help the risk-averse defendant, guilty or innocent.<sup>5</sup>

Facing a credible threat by the prosecutor to proceed to trial, an innocent defendant may indeed benefit from a plea bargain. But the prosecutor cannot credibly

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<sup>1</sup> See U.S. Department of Justice, Bureau of Justice Statistics, *Compendium of Federal Justice Statistics, 2001*, 2 (2003) ("the proportion of convicted defendants who plead guilty increased from 87% during 1990 to 95% during 2001"); U.S. Department of Justice, Bureau of Justice Statistics, *State Court Sentencing of Convicted Felons, 2000*, 43 (2003) (95% of felony convictions in State courts were achieved through guilty plea).

<sup>2</sup> See, e.g., Tomas W. Church, *In Defense of "Bargain Justice"*, 13 L. & Soc'y Rev. 509 (1979); Conrad G. Brunk, *The Problem of Voluntariness Coercion in the Negotiated Plea*, 13 L. & Soc'y Rev. 527 (1979); Kenneth Kipnis, *Plea Bargaining: A Critic's Rejoinder*, 13 L. & Soc'y Rev. 555 (1979); Frank H. Easterbrook, *Criminal Procedure as a Market System*, 12 J. Legal Stud. 289 (1983); Stephen J. Schulhofer, *Criminal Justice Discretion as a Regulatory System*, 17 J. Legal Stud. 43 (1988); Robert E. Scott & William J. Stuntz, *Plea Bargaining as Contract*, 101 Yale L.J. 1909 (1992); Stephen J. Schulhofer, *Plea Bargaining as Disaster*, 101 Yale L.J. 1979 (1992); Ronald Wright & Marc Miller *The Screening/Bargaining Tradeoff*, 55 Stan. L. Rev. 29 (2002); Albert W. Alschuler, *Straining at Gnats and Swallowing Camels: The Selective Morality of Professor Bibas*, 88 Cornell L. Rev. 1412 (2003).

<sup>3</sup> See Albert W. Alschuler, *The Changing Plea Bargaining Debate*, 69 Cal. L. Rev. 652, 679-80 (1981); Michael O. Finkelstein, *A Statistical Analysis of Guilty Plea Practices in Federal Courts*, 89 Harv. L. Rev. 293, 309-10 (1975).

<sup>4</sup> See Louis Kaplow & Steven Shavell, *Accuracy in the Determination of Liability*, 37 J. L. & Econ. 1 (1994).

<sup>5</sup> See William M. Landes, *An Economic Analysis of the Courts*, 14 J. L. & Econ. 61 (1971); Scott and Stuntz, *supra* note 2, at 1938.

threaten to take every case to trial. Her budget constraint will generally allow for only a very small number of trials. The prosecutor's ex ante decision which cases to pursue is, therefore, of central importance. And since the prosecutor's goals will generally diverge from the social objective, there is a real danger that the prosecutor will choose the wrong cases.<sup>6</sup> Specifically, society's preference for wrongful acquittals over wrongful convictions<sup>7</sup> might not be reflected in the prosecutor's choice of cases.<sup>8</sup> And since plea bargains increase the number of cases the prosecutor can pursue within a given budget constraint, the plea bargain institution exacerbates the consequences of this divergence between social objectives and the prosecutor's private goals.

Can the law cure this divergence, or at least minimize it? This paper argues that it can, and in fact it already does, albeit inadvertently. The Federal Sentencing Guidelines allow a maximal sentence reduction of approximately 25% from the benchmark sentence for the offence (including relevant circumstances surrounding the offence) in return for a guilty plea. Under the Guidelines, a defendant can receive a two-level reduction in the offence level, translating into a sentence reduction of about 20-25%, if he "clearly demonstrate[s] acceptance of responsibility."<sup>9</sup> While "acceptance of responsibility" is not

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<sup>6</sup> Compare: Gene M. Grossman and Michael L. Katz, *Plea Bargaining and Social Welfare*, 73 *Amer. Econ. Rev.* 749 (1983) and Jennifer F. Reinganum, *Plea Bargaining and Prosecutorial Discretion*, 78 *Amer. Econ. Rev.* 713 (1988), who adopt the assumption that the prosecutor maximizes social welfare.

<sup>7</sup> This social preference is grounded both in moral theory – see Ronald M. Dworkin, *Principle, Policy, Procedure*, in Ronald M. Dworkin (ed.), *A Matter of Principle* 72 (1985), and in economic theory – see Oren Bar-Gill, *The Efficiency of Protecting the Innocent*, mimeo, Harvard Law School (2004).

<sup>8</sup> This will be true under many different assumptions regarding the structure of the prosecutor's objective function. In the formal model developed in Section 2, we assume, as is common in the law and economics literature, that the prosecutor maximizes the overall expected sanction across all chosen cases. Similar results would obtain if we assume that the prosecutor is driven mainly by a desire to win cases (i.e. to convict or to secure a guilty plea). See Albert W. Alschuler, *The Prosecutor's Role in Plea Bargaining*, 36 *U. Chi. L. Rev.* 50, 105-112 (1968) (describing prosecutors who care about maintaining high "batting averages"). See also *id.* at 58-64 (prosecutors do not generally decline to prosecute defendants whose guilt they doubt; rather they bring the greatest pressure to plead guilty to bear on such defendants).

<sup>9</sup> See *U.S.S.G.* § 3E1.1. In some cases the defendant can receive an additional one level reduction.

equivalent to “pleading guilty,”<sup>10</sup> in practice only defendants that plead guilty are considered eligible for these sentence reductions. By restricting the prosecutor’s ability to offer a significantly reduced sentence as part of a plea bargain, the Guidelines induce the selection of stronger cases in which the defendant is more likely to be guilty.

To see how the Sentencing Guidelines reduce the number of innocent defendants accepting a plea bargain, divide the universe of cases into two sub groups: cases with a high ( $\geq \frac{3}{4}$ ) probability of conviction (H cases), and cases with a low ( $< \frac{3}{4}$ ) probability of conviction (L cases). The plea bargain sanction would have to be lower in the L cases, often lower than three-quarters of the sentence that the defendant would have received at trial, if convicted. Under the Guidelines, however, such a plea bargain would be unenforceable. Accordingly, the prosecutor would have to choose between trying L cases and substituting L cases with H cases.<sup>11</sup> As demonstrated below, the rule adopted by the Sentencing Guidelines leads to the selection of fewer L cases, and to a smaller overall number of cases. As long as the probability of conviction is positively correlated with the probability of guilt, this implies a reduced number of innocent defendants that accept plea bargains.<sup>12</sup>

We show that the selection-of-cases effect that reduces the number of innocent defendants entering into plea bargains is most powerful when the benchmark sentence is well defined. Our analysis thus supports the limited sentencing discretion permitted under

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<sup>10</sup> See *United States v. Bennett*, 161 F.3d 171 (3rd Cir.1998).

<sup>11</sup> Bibas recently observed that fixed discounts may “deter prosecutions of the possible innocent.” Stephanos Bibas, *Plea Bargaining Outside the Shadow of Trial*, 117 Harv. L. Rev. 2464, 2536 (2004).

<sup>12</sup> This result must be qualified if we believe that innocent individuals are systematically more risk averse than guilty individuals (see, e.g., Robert E. Scott & William J. Stuntz, *A Reply: Imperfect Bargains, Imperfect Trials, and Innocent Defendants*, 101 Yale L.J. 2011, 2012 (1992)). Such heterogeneity in the degree of risk aversion implies that innocent individuals would be willing to accept plea bargains with higher, not lower, sentences. This qualification notwithstanding it seems that the direct effect of innocence on the expected sanction will generally outweigh the effect of heterogeneous risk preferences.

the Federal Guidelines. The broader discretion permitted under state-level sentencing guidelines while not eliminating the selection-of-cases effect does dilute the force of the selection effect. Discretion in sentencing clearly serves an important social purpose. The cost of greater discretion, however, cannot be ignored.

The remainder of the paper is organized as follows. Section 2 formally derives the selection-of-cases result. Section 3 offers concluding remarks, focusing on possible implementation problems and on the deterrence implications of the selection effect.

## 2. Model

Let  $p$  denote the probability of conviction, and  $s$  denote the expected sentence following a conviction—the benchmark sentence. Let  $\Omega$  denote the universe of cases, where each case is characterized by its  $(p, s)$  pair. The universe of cases,  $\Omega$ , can be divided into two, mutually exclusive sub-groups: cases with a high probability of conviction, i.e. with  $p \geq a \in (0,1)$  (H cases):  $\Omega_H \equiv \{(p, s) | (p, s) \in \Omega, p \geq a\}$ , and cases with a low probability of conviction, i.e. with  $p < a$  (L cases):  $\Omega_L \equiv \{(p, s) | (p, s) \in \Omega, p < a\}$ . Let  $q = ps$  denote the expected sanction, and let  $f_H(q)$  and  $f_L(q)$  represent the distribution of cases, according to expected sanction, in  $\Omega_H$  and  $\Omega_L$ , respectively.

As is conventional in the law and economics literature on plea bargaining, we assume that the prosecutor's private goal is to maximize the sum of expected sanctions

subject to a budget constraint,  $B$ .<sup>13</sup> A plea bargain costs the prosecutor  $c$ , while a trial costs  $c + x$ .<sup>14</sup>

For simplicity, assume that if a plea bargain is reached, the agreed sentence equals  $ps$ .<sup>15</sup> Assume that due to risk-aversion and/or litigation costs, without the restriction imposed by the Sentencing Guidelines, all cases end in a plea bargain. Finally, to make things interesting, assume that without the Guidelines' restrictions, some of the selected cases are in  $\Omega_H$  and some are in  $\Omega_L$ .<sup>16</sup>

The following lemma summarizes the effect of the Sentencing Guidelines on the prosecutor's selection of cases.

*Lemma:* A legal rule that renders a plea bargain unenforceable unless it specifies a sentence exceeding  $a$  times the benchmark sentence for the offence, i.e., a rule restricting the permissible sentence reduction to  $(1-a) \cdot s$ , will lead the prosecutor to select fewer L cases, a higher, lower, or unchanged number of H cases, and a lower overall number of cases.

*Remark:* The intuition for this result, which is proved in the Appendix, is as follows. Since pursuing L cases under the Sentencing Guidelines requires a costly trial, the prosecutor will select fewer L cases. The effect on the number of H cases is ambiguous.

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<sup>13</sup> See, e.g., Landes, *supra* note 5, at 63.

<sup>14</sup> In our formal model we assume that all trials are equally costly and all plea bargains are equally costly. This clearly unrealistic assumption is made for expositional purposes only. Our main results continue to hold under more realistic differential costs assumptions.

<sup>15</sup> The analysis remains qualitatively unchanged if we assume only that the plea-bargain sentence is positively correlated with  $ps$ .

<sup>16</sup> If without judicial scrutiny, all the cases selected by the prosecutor are in  $\Omega_H$ , judicial scrutiny will have no effect on the selection of cases.

There are two possible scenarios. Under the first scenario, the Guidelines lead to the selection of more H cases ( $q_H^S < q^{NS}$ ). Since the prosecutor can only pursue an L case through a costly trial, she may well prefer more H plea bargains with a lower per-case sanction over fewer L trials with a higher per-case expected sanction. Under the second scenario, the Guidelines lead not only to the selection of fewer L cases, but also to the selection of fewer H cases ( $q_H^S \leq q^{NS}$ ). If the prosecutor still chooses a significant number of L cases, the added trial costs might force her to take-on fewer H cases. The overall number of cases clearly declines under the second scenario. It also declines under the first scenario, since the prosecutor at most substitutes one L case for one H case.

Based on the preceding lemma, the following proposition establishes the desirability of the restrictions imposed by the Sentencing Guidelines.

**Proposition:** As long as the probability of conviction is positively correlated with the probability of guilt, the Sentencing Guidelines, by rendering unenforceable plea bargains with sentence reductions exceeding  $1 - \mathbf{a}$  of the benchmark sentence for the offense, will reduce the number of innocent defendants that accept plea bargains.

**Remarks:** The intuition for this result, whose formal proof is omitted, is as follows. The Sentencing Guidelines affect the prosecutor's selection of cases in two ways. First, they induce substitution from L cases to H cases. Second, they reduce the overall number of cases that the prosecutor can pursue. As long as the probability of conviction is positively correlated with the probability of guilt, the first effect results in a reduced number of

innocent defendants accepting a plea bargain. The second effect reduces the number of innocent defendants that are pursued by the prosecutor, and therefore also reduces the number of innocent defendants accepting a plea bargain.<sup>17</sup>

### 3. Concluding Remarks

We conclude by discussing several issues pertaining to the implementability and the deterrence consequences of the sentencing principles analyzed in this paper.

(1) *Uncertainty with respect to the Benchmark Sentence*: A key feature of the Federal Guidelines is the narrow range within which a defendant's sentence must be set (based on the offense level, the defendant's criminal history and a few additional factors). This strictness of the Federal Guidelines allows for an accurate determination of the benchmark sentence for the offence ( $s$  in our model). A common estimate of the benchmark sentence, shared by the prosecutor, the defendant and the court, is critical for a regime that restricts plea bargain sentences relative to the benchmark sentence for the offence. Therefore, the limited discretion allowed under the Federal Guidelines facilitates the socially desirable impact of the guidelines on the prosecutor's selection of cases.

But even under less strict guidelines, such as the state-level sentencing guidelines, a selection-of-cases effect, albeit a weaker one, exists. If the relevant sentencing guidelines leave considerable discretion, such that the benchmark sentence may fall

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<sup>17</sup> While we focus on the reduction in the number of innocent defendants who plead guilty, the restrictions imposed by the Sentencing Guidelines also reduce the average sentence imposed on the innocent defendants who plead guilty despite these restrictions. To the extent that the Guidelines induce substitution from L cases to H cases, they result in plea bargains characterized by both a lower probability of innocence and a lower sanction. We acknowledge, however, that to the extent that the Guidelines induce prosecutors to try (rather than plea) L cases, they will hurt the innocent defendants who are deprived of the plea bargain option. The tradeoff is between fewer innocent people facing the risk of prosecution and a greater risk—the trial risk—for those few who are forced to stand trial. *Cf.* Scott & Stuntz, *supra* note 12, at 2013.

anywhere within a  $[\underline{s}, \bar{s}]$  range, still plea bargain sentences below  $a \cdot \underline{s}$  would be avoided. Accordingly, sufficiently weak L cases where the likelihood of guilt is especially low would not be selected. Still, the analysis in this paper identifies an advantage of strict sentencing guidelines.

(2) *Charge Bargaining and Fact Bargaining*: Even absent uncertainty regarding the benchmark sentence, the selection-of-cases effect would disappear if charge bargaining or fact bargaining were permitted. While such circumvention should not be underestimated, the problem is not unique to the selection-of-cases result. Rather, the prevention of charge and fact bargaining is crucial to the efficacy of any sentencing guidelines.<sup>18</sup> And, accordingly, the Federal Rules of Criminal Procedure and the Federal Guidelines themselves explicitly restrict the enforceability of such agreements.<sup>19</sup>

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<sup>18</sup> See Stephen J. Schulhofer & Ilene H. Nagel, *Plea Negotiations Under the Federal Sentencing Guidelines: Guideline Circumvention and Its Dynamics in the Post- Mistretta Period*, 91 Nw. U. L. Rev. 1284 (1997). See also William J. Stuntz, *Plea Bargaining and Criminal Law's Disappearing Shadow*, 117 Harv. L. Rev. 2548 (2004). A more fundamental objection to our selection-of-cases argument questions the link between the expected trial outcome and the plea bargain sentence, even absent charge and fact bargaining. See Bibas, *supra* note 11.

<sup>19</sup> See, Fed.R.Crim.P. 11(c)(5) (allowing the court to review charge bargains and fact bargains and accept or reject the agreement); U.S.S.G. § 6B1.2(a) (Standards for Acceptance of Plea Agreements - Policy Statement: "[T]he court may accept the agreement if the court determines, for reasons stated on the record, that the remaining charges adequately reflect the seriousness of the actual offense behavior and that accepting the agreement will not undermine the statutory purposes of sentencing or the sentencing guidelines. However, a plea agreement that includes the dismissal of a charge or a plea agreement not to pursue a potential charge shall not preclude the conduct underlying such charge from being considered under the provisions of § 1B1.3 (Relevant Conduct) in connection with the count(s) of which the defendant is convicted"). See also the recent policy guidelines issued by the Attorney General John Ashcroft, *Justice Department Policy Concerning Charging Criminal Offenses, Disposition of Charges and Sentencing*, memo issued (U.S. Dep't of Justice Sept. 29, 2003) (instructing prosecutor to pursue the most serious readily provable offences, and prohibiting fact bargaining or any other "plea agreement that result in the sentencing court having less than a full understanding of all readily provable facts relevant to sentencing"). Another effective way to prevent fact bargaining and to restrict charge bargaining is to prohibit any pre-charge negotiations. A comparative perspective is informative. While in the U.S. it is very common for the prosecutor to be involved in the criminal investigation, thus opening the door to pre-charge negotiations, this practice is uncommon in other common law countries, like England and Israel. In these legal systems the prosecutor is rarely involved in the police investigation. The indictment is often based only on the information provided in the dossier, and charges are brought before the prosecutor even meets the defendant (or his attorney). Consequently, plea bargaining takes place only after the indictment is filed. See JOHN SPRACK, *EMMINS ON CRIMINAL PROCEDURE* 251 (9<sup>th</sup> Edition, 2002). For other ways of limiting prosecutors' power to charge bargain - see William J. Stuntz, *The Pathological Politics of Criminal Law*,

(3) *Deterrence*: The overall effect of the sentencing restrictions imposed by the Sentencing Guidelines on deterrence is indeterminate. If the probability of guilt and the probability of conviction are strongly correlated, these sentencing restrictions can lead prosecutors to pursue fewer innocent defendants and more guilty defendants, thus enhancing deterrence.<sup>20</sup> However, judicial scrutiny can also force the prosecutor to pursue fewer guilty defendants, thus compromising deterrence.<sup>21</sup>

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100 MICH. L. REV. 505, 594-95 (2001) (proposing constitutional limits on the sentencing implications of the charges listed in the indictment as well as restrictions on mandatory minimum sentences as a way to limit the benefits to prosecutors from overcharging and from charge bargaining).

<sup>20</sup> To take an extreme case: if a guilty defendant will always face  $p > a$ , and only an innocent defendant can face  $p < a$ , then setting free defendants with  $p < a$  enhances deterrence.

<sup>21</sup> If the probability of guilt and the probability of conviction are only weakly correlated, the overall deterrence effect will be smaller, but still ambiguous. Such weak correlation, however, implies that the criminal justice system suffers from problems that are more fundamental than those caused by the plea bargain institution.

## Appendix

The appendix contains the proof of the lemma.

*Proof:* We prove that the prosecutor will select fewer L cases. The reasoning supporting the remaining parts of the lemma is provided in the text.

Let  $q_H$  and  $q_L$  represent the threshold values of the expected sanction, such that the prosecutor selects cases with  $q \geq q_H$  in  $\Omega_H$  and cases with  $q \geq q_L$  in  $\Omega_L$ . Without the restrictions imposed by the Sentencing Guidelines, the prosecutor solves:

$$(1) \max_{q_H, q_L} \left\langle \int_{q_H}^{\bar{q}} qf_H(q) dq + \int_{q_L}^{\bar{q}} qf_L(q) dq \right\rangle \text{ s.t. } \int_{q_H}^{\bar{q}} cf_H(q) dq + \int_{q_L}^{\bar{q}} cf_L(q) dq \leq B.$$

With the restrictions imposed by the Guidelines, the prosecutor solves:

$$(2) \max_{q_H, q_L} \left\langle \int_{q_H}^{\bar{q}} qf_H(q) dq + \int_{q_L}^{\bar{q}} qf_L(q) dq \right\rangle \text{ s.t. } \int_{q_H}^{\bar{q}} cf_H(q) dq + \int_{q_L}^{\bar{q}} (c+x)f_L(q) dq \leq B.$$

The Lagrangian is:

$$L = \int_{q_H}^{\bar{q}} qf_H(q) dq + \int_{q_L}^{\bar{q}} qf_L(q) dq - I \left[ \int_{q_H}^{\bar{q}} cf_H(q) dq + \int_{q_L}^{\bar{q}} (c+x)f_L(q) dq - B \right],$$

and the FOCs are:  $q_H = Ic$  and  $q_L = I(c+x)$ , implying  $I > 0$ .<sup>22</sup>

Since (1) is identical to (2) with  $x = 0$ , we have  $q_H^{NS} = q_L^{NS} = q^{NS}$  (and  $I^{NS} = q^{NS}/c$ ). When  $x > 0$ , we have  $q_H^S < q_L^S$  (and  $I^S = q_H^S/c = q_L^S/(c+x)$ ). We can now prove that  $q_L^S > q^{NS}$  by contradiction. If  $q_L^S \leq q^{NS}$ , then  $q_H^S < q_L^S \leq q^{NS}$ , which violates the budget constraint. QED

<sup>22</sup> The FOCs are derived from:

$$\frac{\partial L}{\partial q_H} = -q_H f(q_H) + I c f(q_H) = f(q_H) [-q_H + I c] = 0$$

$$\frac{\partial L}{\partial q_L} = -q_L f(q_L) + I(c+x) f(q_L) = f(q_L) [-q_L + I(c+x)] = 0$$